Dimension reduction of cyclostationary random functions

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Abstract

Cylclostationary random functions, also refered as periodically correlated random processes, have been explored since the 1960's, are widely explored, on one hand for theoretical developments of their properties and treatment, and on another hand for modelizations of concrete applications. Indeed, cyclostationarity occurs in processes where some statistics present periodicity. These periodic phenomena are encountered in various situations, due for example to modulation in signal theory, rotation in mechanics, revolution of planets, sanguine pulse in medicine, seasonality in economics, or pulsation of stars in astronomy.

In this talk, we consider a random function $(X_t)_{t\in\mathbb{R}}$. We say that it is cyclostationary when $\mathrm{cov}(X_t, X_{t'}) = \mathrm{cov}(X_{t+\Delta}, X_{t'+\Delta})$, for a given Δ of $\mathbb{R}$, and for any (t, t') of $\mathbb{R}\times\mathbb{R}$. We transform it into a stationary series, and then we proceed to the dimension reduction of this stationary series, with the Principal Components Analysis in the frequency domain (cf. Brillinger [1] and Boudou and Dauxois [2]). Then we show how to reconstruct a summary of the intitial cyclostationary function.

We give an example on simulated data. The set of indices can be extended to $\mathbb{R}^k$ (cf. Boudou and Viguier-Pla [3]).

References

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