

Toeplitz Order in the Area of Uncertainty

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Abstract

The area of Uncertainty Principle in Harmonic Analysis (UP) was founded in 1920s by Norbert Wiener. It stems from a simple rule stating that 'A function (measure, distribution) and its Fourier transform cannot be small simultaneously'. For various mathematical meanings of smallness this statement leads to deep and important problems of analysis. At present the area of UP spans across many fields including questions on completeness, sampling and uniqueness in function spaces, multiple classical and abstract versions of the moment problem, estimates of singular integrals and spectral problems for differential equations.

The idea to apply Toeplitz operators to study Riesz bases in model spaces of analytic functions first appeared in the seminal work by Khrushev, Nikolski and Pavlov in 1980s. In our papers with N. Makarov we tried to extend the Toeplitz approach to other problems of UP. In the last 10 years such methods led to solutions of some of the classical problems and revealed hidden connections between distant areas of analysis and mathematical physics.

In my course I will try to survey the basics of the Toeplitz approach and discuss recent developments. The new part of the course is the point of view based on partial order of the set of inner functions induced by Toeplitz operators. The study of the Toeplitz order includes several well-known open problems of analysis and points to new possibilities for further research.