Polynomial approximation for the Heston model.

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Abstract

Stochastic volatility models like the Heston model are frequently used in the pricing of derivatives. The solution of the corresponding partial differential equations is often given as a semi-explicit integral formula. We consider a different approach that takes into consideration that the underlying domain is not bounded and the initial data is not square-integrable. We study the problem in a weighted Sobolev space for finite-dimensional subspaces spanned by Hermite and Laguerre polynomials. The approximated smooth solutions give us new insight into the behavior of the solution especially for small volatility close to zero.

p-Laplacian - maximum and comparison principles, uniqueness versus nonuniqueness.

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Abstract

We will present some recent results on maximum principles for the Cauchy problem for the parabolic p-Laplacian. In the first part of the talk we discuss the weak comparison principle for a parabolic p-Laplacian problem. While the weak comparison principle holds provided the reaction function $h(x, t, u)$ is nondecreasing and Lipschitz continuous with respect to u, we show that the weak comparison principle does not hold when the reaction function is not Lipschitz continuous even for x from an arbitrarily small part of the spatial domain. Namely, using the method of sub- and supersolutions we construct a nontrivial nonnegative weak solution to an initial-value problem which also possesses the trivial solution. This is a joint work with V. Bobkov, P. Girg, L. Kotrla and P. Takac. In the second part we consider continuous nonnegative solutions to a doubly nonlinear parabolic problem with the p-Laplacian with zero Dirichlet boundary conditions. For simplicity we assume that both the initial data and the reaction function are continuous and nonnegative and the reaction function does not depend on u. We show that for $1 < p < 2$ the speed of propagation is infinite in the sense that for any fixed time the solution is either everywhere positive or identically zero. In particular, if the initial data are nonzero at at least one point, then for small positive time the solution is positive in the whole domain, i.e., the strong maximum principle holds. This is a joint work with P. Girg, L. Kotrla and P. Takac.

On the Fredholm-type theorems and sign properties of solutions for the (p, q)-Laplacian.

Vladimir Bobkov

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Abstract

We will discuss some recent results on the Dirichlet problem for the nonhomogeneous equation $-\Delta_p u - \Delta_q u = \alpha|u|^{p-2}u + \beta|u|^{q-2}u + f(x)$ in a bounded domain, where α and β are parameters. Namely, we will present assumptions on α and β which guarantee the resolvability of the considered problem. Moreover, we will describe several curves on the (α, β)-plane allocating sets of parameters for which the problem has or does not have positive or sign-changing solutions, provided f is of a constant sign. The talk is based on joint work with M.Tanaka (Tokyo University of Science).

Creativity in non linear Computation according to Dickson and Watkins.

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Abstract

About a century ago, L. Dickson (University of Chicago) devised a sequence of nested algebras A_k, $k \geq 0$, of vectors in R^{2^k}, based on recursively defined $+$, $\times$ and conjugation from $A_0 = \mathbb{R}$ so that $\mathbb{R} \subset A_1 = \mathbb{C} \subset A_2 = H \subset \cdots$. Some 50 years later, and quite independently, W. Watkins (Ca. State University at Northridge) computed the list of numbers whose base 2 representation is given by the 2^k first rows of the arithmetic triangle mod 2 introduced by Sierpiński. He observed that the list displays, for $k \geq 1$ and by increasing magnitude, 1 and all square-free products of specific numbers which can be 1 or any of the k first Fermat numbers from 3 up to $2^{2^{k1}} + 1$. This unconspicuous remark points to a possibly fertile connection between the 2^k basis vectors of A^k and the 2^k rows of the triangle. The talk presents first a review of some aspects of the inherent creativity of the dick- sonian definition. Then it shows how the connection with arithmetic mod 2 via Sierpiński does expand the creative potential.

Positive solutions for a class of superlinear semipositone problems.

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Abstract

We will summarize the results concerning the solvability of a class of superlinear semipositone problems for positive solutions. Both semilinear and quasilinear problems will be discussed.

Existence of positive and nodal solutions for quasilinear elliptic equations with Steklov nonlinear boundary conditions of critical growth. Joint work with Leadi Liamidi.

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Abstract

In this work we study the existence positive and nodal solutions of the following p-laplacian concave-convex problem with Steklov boundary conditions on a bounded regular domain $\Omega \subset \mathbb{R}^N$, $N > 1$,

$$\begin{cases} -\Delta_p u + V(x)|u|^{p-2}u = 0 & \text{in } \Omega; \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} = \lambda a(x)|u|^{p-2}u + b(x)|u|^{p_*-2}u & \text{on } \partial\Omega \end{cases}$$

with given functions a, V possibly indefinite, $b \geq 0$, $1 < p < N$ and $p_* := \frac{p(N-1)}{N-p}$ is the critical exponent for the trace map $W^{1,p}(\Omega) \to L^q(\partial\Omega)$. $\nu = \nu(x)$ denotes here the outward normal derivative at the point $x \in \partial\Omega$.

We will prove the existence of up 2 positive solutions and 2 nodal solutions if a chanhes sign.

Beyond the unique continuation: flat solutions for reactive slow diffusions, the infinite and Hardy potential for Schrödinger equation and $2-d$ electron beams.

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Abstract

Solutions with compact support for some nonlinear elliptic and parabolic equations, and many other free boundary problems, are formulations for which the unique continuation property fails. In such problems, which have attracted the attention of many specialists, and among them this lecturer, the solution u and its normal derivative vanish on a region of the boundary (which leads to the definition of a flat solution of the corresponding equation). In this talk I will present, in a very sketched way, some recent results in this direction, trying to show how many open problems of this nature still remain as a source of current research. More specically, I will report on some results concerning the following problems:

I) Stable flat solutions of $u_t - \Delta u^m + u^a = \lambda u^b$ for $0 < a < b < m$ under the stability condition $2(m+a)(m+b) - N(m-a)(m-b) < 0$ (joint work with J. Hernández and Y. Sh. Ilyasov),

II) Flat solutions to $i\frac{\partial \psi}{\partial t} = -\Delta\psi + V(x)$ in $\mathbb{R}^N$; for $V(x) \geq Cd(x;\partial\Omega)^{-2}$ for some bounded domain (my research continued in collaboration with J. M. Rakotoson, D. Gmez-Castro, R. Temam and J. L. Vzquez),

III) Partially at solutions $(u(x;0) = \frac{\partial u}{\partial y}(x;0) = 0$ for $x \in (-a;0))$ to $\frac{partial^2 u}{\partial y^2} = \frac{j(x)}{\sqrt{u(x;y)}}$, $x \in (-a;a)$; $y \in (0;1)$; for suitable $j(x)$, independent of y (formulation raised to me by H. Brezis and J. Lebowitz).

p-Laplacian - history, mathematical models from hydrology and natural gas extraction, experiments.

Petr Girg

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Abstract

In the first part of this talk we will discuss evolution of mathematical models of turbulent flow in porous media. The approach will be purely phenomenological and not from the Navier-Stokes equations. The governing equation will follow from the continuity equation and an empirical constitutive law (nonlinear generalization of the Darcy law). As the nonlinear generalization of Darcy law, we use the power law discovered by Smreker and verified by laboratory experiments by Kroeber, Forchheimer, Izbash and Missbach in 1880's - 1930's. We will focus on pioneering work on the development of these mathematical models due to Dupuit, Forchheimer, Zhukovskii, Christianovitch, Leibenson and Barenblatt in 1880's - 1950's. This historical review was a joint work with J. Benedikt, L. Kotrla and P. Takac. In the second part of this talk, we will discuss some very recent experiments done by D. Vesely and Z. Vesely on real porous media used in civil engineering.

Elliptic systems with a gradient term having natural growth.

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Abstract

We consider the problem $-\Delta u = g(u)|\nabla u|^2 + f(x,u)$ in Ω, $u > 0$ in Ω, $u = 0$ on $\partial\Omega$, where Ω is a smooth bounded domain in $\mathbb{R}^N$. Using a Kazdan-Kramer change of variable, this problem can be reduced to a semilinear problem without gradient term, which can then be approachable by variational methods. However various new difficulties arise, in particular with respect to the Ambrosetti-Rabinowitz condition. We investigate some of these difficulties.

In the second part of the talk, we consider the situation of a system. Using again a Kazdan-Kramer change of variables, this system can be transformed into a system without gradient term. In some cases, this latter system can be handled by an upper-lower solution approach, by variational methods, or by blow-up. The blow-up procedure leads to the study of new Liouville type theorems.

Joint work with D. de Figueiredo (Campinas, Brasil), H. Ramos Quoirin (Cordoba, Argentina) and P. Ubilla (Santiago, Chile).

p-Laplacian - applications of the theory to mathematical models.

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Abstract

We will apply maximum and comparison principles to problems from turbulent filtration of natural gas in porous rock and groundwater filtration in gravel. In particular, we will focus on a model of turbulent filtration of natural gas in a porous rock due to Leibenson. We will discuss connections between our recent results and the classical results obtained by Barenblatt in 1950' for this equation. In the case of mathematical models of turbulent filtration of water in gravel, we will mainly focus on an example of filtration flow between two drainage channels. Our maximum principles have also some implication on the choice of suitable numerical methods for these problems.

On very weak solutions of semilinear elliptic PDEs with singular integral Neumann boundary data

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Abstract

In this talk, I would like to show how to use mixed fractional Sobolev spaces [4] to prove existence of very weak solutions $u \in L^p(\Omega)$, $1 < p < \infty$, to singular semilinear elliptic PDEs

$$-\Delta u + c(\cdot, u) = 0 \quad \text{in } \Omega \quad \text{with } c(x,u) := -\int_{\partial\Omega} b(x,y,u)\, dS(y)$$

on bounded C^2-domains $\Omega \subset \mathbb{R}^N$ subject to singular integral Neumann boundary conditions

$$\frac{\partial u}{\partial \vec{n}} + Iu = 0 \quad \text{on } \partial\Omega \quad \text{with } (Iu)(y) := \int_{\Omega} b(x,y,u(x))\, dx$$

for a Carathéodory function $b : \Omega \times \partial\Omega \times \mathbb{R} \to \mathbb{R}$ as kernel, which is assumed to be sublinear w.r.t. u with a subcritical singularity as $\Omega \ni x \to y \in \partial\Omega$. This work follows up a collaboration with Jean-Michel Rakotoson [1] initiated by Peter Takáč, and in fact, Peter Takáč was the first who introduced fractional Sobolev spaces to me.

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Parabolic systems associated with a traffic flow on two–lane. An uniqueness result.

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Abstract

The main aims of this talk are modeling of traffic flow on two–lane highway, form the of point of view of macroscopic scale, describing by a non linear parabolic system and to prove a comparison result among weak solutions for this class of nonlinear parabolic systems with zero–order coupling which. We develop a method based in doubling of the time variable. This technique is inspired in a method introduced by S.N. Kruzhkov [6] in order to prove a L^1–contraction property for entropy solutions of hyperbolic problems.

Macroscopic models where traffic is viewed as a compressible *fluid* formed by vehicles. The traffic flow can be characterized by macroscopic parameters like the mean velocity or the mean flow and without inflow from and outflow of cars. That is, on a stretch of highway with *no entries or exits*, the number of cars is conserved, so that we have (see, e. g., [5]) for traffic flow on only one line that

$$\rho_t + (\rho w)_x = 0. \tag{1}$$

where $\rho(x,t)$ be the *density of cars* at the location x and time t on a highway, i.e., the number of cars per unit length of the road, and let $w(x,t)$ denote the *flow speed*, that is, the speed of the car which reaches position x at time t. The two variables, traffic density ρ and car velocity w, are related by only one equation (1).

We extend these ideas to the case of two–lane highway going in one direction. Our model allows cars to pass of lane and that the drivers continuously adjust their speed towards what they consider to be the ideal value under the local traffic conditions. Let ρ_1 be the density in the slower lane, ρ_2 be the density in the faster lane, and we assume $w_1 = V_1(\rho_1)$ and $w_2 = V_2(\rho_2)$ (constitutive law) be the corresponding flow speeds. A good driver checks the traffic conditions on the road ahead (and behind), compensating for possible changes in the flow. It looks then natural to take $w_i = V_i(\rho_i) - \kappa_i \frac{\rho_{ix}}{\rho_i}$, where κ_i is a positive quantity which tells how much the change in the number of cars is considered by the driver. Form these assumption, we get the following doubly nonlinear parabolic system

$$\left\{ \begin{array}{lcl} \frac{\partial}{\partial t}\rho_1 - \frac{\partial}{\partial x}\left(\kappa_1 \frac{\partial}{\partial x}\rho_1 - V_1(\rho_1)\right) & = & F(t,x,\rho_1,\rho_2), \\ \frac{\partial}{\partial t}\rho_2 - \frac{\partial}{\partial x}\left(\kappa_2 \frac{\partial}{\partial x}\rho_2 - V_2(\rho_2)\right) & = & -F(t,x,\rho_1,\rho_2), \end{array} \right. \tag{2}$$

The net effect of time changes in density and space changes in flow at a point x and a time t are equal to the exchange between the two lanes and it is described by function F (see, e. g., [4], [5]). Several types of functions F can be chose: a model in which lane changes were proportional to the square of the density in the lane that was the source of departing vehicles and the first power of the emptiness (unused space) of the lane–admitting vehicles, can be expressed by $F(\rho_1,\rho_2) := -\alpha_1\rho_1^2[\gamma_2 - \rho_2] + \alpha_2\rho_2^2[\gamma_1 - \rho_1]$ where γ_i are the maximum allowable densities in the respective lanes [4].

Finally, we prove a comparison result for a this type of nonlinear parabolic systems.

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Potential-Capacity , properties and applications to some Schrödinger type problems.

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Abstract

In this talk, we shall consider a notion of capacity related to a potential function V, denoted C_{apV} . The motivation of such consideration is intimately linked to the mathematical model by P´osch-Teller introduced in the mathematical literature by Idelfonso Díaz. So, I will take as a potential model

$$V_q(x) = \frac{1}{|sin|x||^q} \text{ for } q > 0.$$

Such capacity has common properties as for classical Sobolev capacity, the main difference resides on the interaction of V and the set on which we consider its capacity. Namely, we will give sufficient conditions to ensure that a compact K will have a zero or a positive capacity . We shall describe various applications of such notion as the removable singularities, or finnding necessary and sufficient conditions of the existence of a solution of an equation of the form :

$$-\Delta\omega + U\nabla\omega + V\omega = f$$

under a generalized Dirichlet condition.

On Lewy-Stampacchias inequalities.

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Abstract

Our aim is to propose some recent results on Lewy-Stampacchias inequalities for elliptic, parabolic and stochastic obstacle problems.