Introduction to micromagnetics

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The aim of this course is to present the modern mathematical tools used in the study of micromagnetics. This topic lies at the interface between Partial Differential Equations (PDE), Calculus of Variations and Mathematical Physics. More precisely, micromagnetics is the continuum theory of magnetic moments underlying the description of magnetic structure and it is based on a variational principle that is nonconvex, nonlocal and multiscale. The plan is to analyse the pattern formation in ferromagnetic materials, in particular, domain walls, vortices, skyrmions etc. This study raises fundamental mathematical questions (regularity, uniqueness, symmetry, stability, asymptotic analysis etc.) for which various techniques will be presented coming from elliptic PDEs, theory of harmonic maps, scalar conservation laws, Γ-convergence, theory of Ginzburg-Landau etc.

The plan of the course is the following:

0. Introduction to the micromagnetic model:
Some physics of the model, Pattern formation, Functional analysis framework, Variational formulation.

1. Study of minimizers / critical points:
Existence, Euler-Lagrange equations, Regularity, Theory of $\mathbb{S}^2$-valued harmonic maps.

2. Asymptotic analysis:
Γ-convergence, Applications to micromagnetics.

3. Small magnetic monodomains:
Theory of Stoner-Wohlfarth, Stability, Hysteresis curve.

4. Large magnetic body limit:
Relaxation principle, Description of the limit model.

5. Thin film ferromagnetic regime:
Separation of energy scales, Asymptotics of the magnetostatic energy, Formation of vortices and domain walls.

6. Analysis of magnetic vortices:
Theory of Ginzburg-Landau, Vortices, Symmetry, Jacobian, Renormalized energy, Skyrmions.

7. Analysis of domain walls:
Néel walls, Symmetry, Stability, Renormalized energy.

8. Bloch walls:
Γ-convergence, Symmetry, Calibration method.

The lectures are supplemented with several lists of exercices so that the students practice the methods introduced in the course. The course is primarily addressed to PhD students. However, it may also be useful for Master students and young researchers interested on the mathematical aspects of Micromagnetics. A good level in Analysis is required though (it would be recommended that the students have already followed the functional analysis and elliptic PDE course).

Information about the course will be given as early as possible to all relevant PhD programs in the Rome area (PhD in Mathematics in Sapienza, Rome Tor Vergata and Rome 3, and PhD in Mathematical Models for Engineering in Sapienza) and also, to the Italian Calculus of Variations community (through the newsletter run by SNS Pisa). The course will also be advertised in due time on the weekly Math newsletter of the Rome area.

References:

1. Bethuel, Brezis and Hélein, *Ginzburg-Landau vortices*, Birkhäuser Boston, 1994.
2. DeSimone, Kohn, Müller and Otto, *Recent analytical developments in micromagnetics*, Science of Hysteresis, Elsevier, G. Bertotti and I Magyergyoz, Eds., 2005.
3. Hang and Lin, *Static Theory for Planar Ferromagnets and Antiferromagnets*, Acta Mathematica Sinica **17** (2001), 541-580.
4. Hubert and Schäfer, *Magnetic domains*, Springer, 1998.
5. Ignat and Monteil, *A DeGiorgi type conjecture for minimal solutions to a nonlinear Stokes equation*, Comm. Pure Appl. Math. **73** (2020), 771-854.
6. Ignat and Moser, *Separation of domain walls with nonlocal interaction and their renormalised energy by* Γ*-convergence in thin ferromagnetic films*, J. Differential Equations **339** (2022), 395-475.
7. Melcher, *Chiral skyrmions in the plane*, Proc. R. Soc. Lond. Ser. A **470** (2014), 17 pp.