Multiscale aspects of parameter extraction in left-handed materials

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OUTLINE

- Introduction : context – quantum devices / photonic nanostructures
- Metamaterials : negative refraction
- Metallic double negative media : from ab-initio approach to homogeneization techniques
  - Left-handed structure : from negative permittivity and negative permeability to negative refraction
  - 1-d examples : a backward wave based finline device, a CL terahertz transmission line
  - 2-d approach : a NIR SRR array
  - Toward homogeneization : band structure engineering
- Dielectric artificial structures
  - Photonic crystals
  - Toward a perfect lens in optics…
- Conclusions and prospects

Introduction (1)

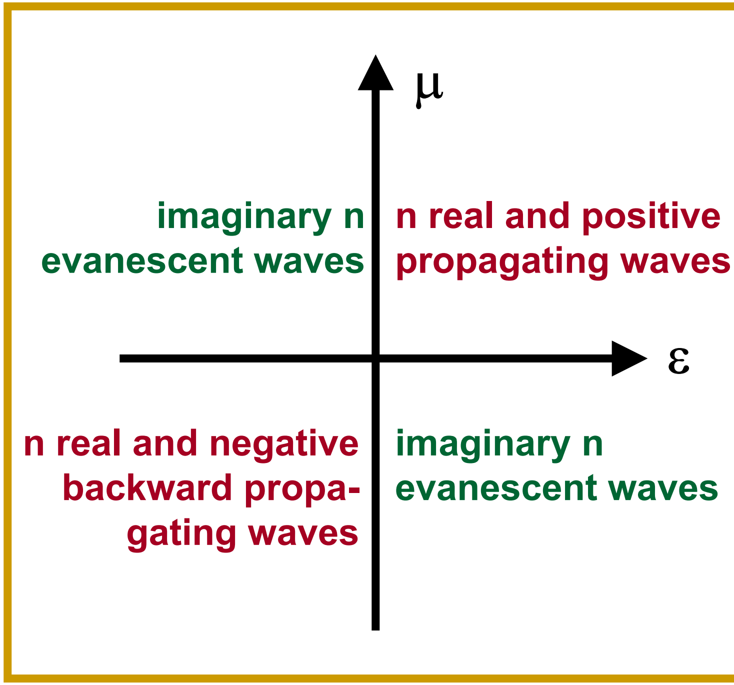
- Wave propagation in artificial media…
  - in electronics :
    - from bulk semiconductor materials to heterostructures, quantum wires and boxes → band structure engineering
    - the crystal gives the electronic properties of constitutive materials
  - for electromagnetism :
    - The idea is to create **« artificially »** crystals to control electromagnetic waves propagation → band structure engineering
    - of interest : the structuration level fixes the spectral range of use of such a material (conversely : the wavelength of an electron in a semiconductor is not really scalable)
    - also we have the possibility to explore 1D,2D and 3D artificial crystals…

Metamaterial (1)

- What is a metamaterial ?
  - Artificial metallic, dielectric or metallo-dielectric structure
  - Patterning on a subwavelength scale
    - Smallest intrinsic dimensions : mm for microwaves, µm for Terahertz frequency and nm for optics
  - Ab-normal electromagnetic properties not encountered in nature
- Double-negative media :
  - A class of metamaterial with simultaneously a negative permittivity and a negative permeability
  - Such material supports backward waves and shows negative refraction when interfaced with a classical material

Metamaterial (2)

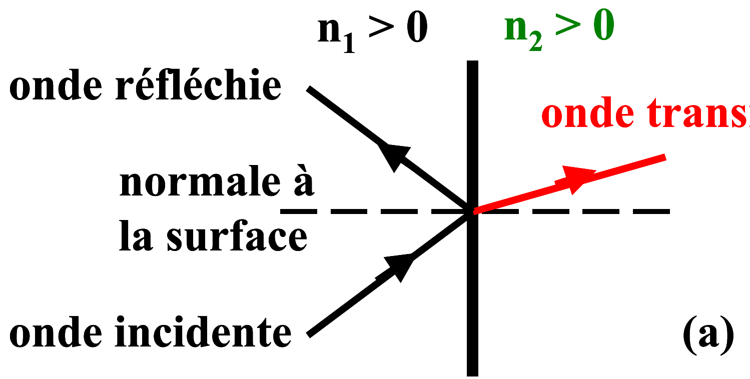
- Historical background
  - In 1969 : theoretical approach
    - V. Veselago claims that a material which exhibits $\varepsilon<0$ and $\mu<0$ supports propagating waves and can be characterized if isotropic by a constant refractive index n which is negative.
  - In 1999 : magnetic activity without magnetic medium
    - J. Pendry proposes a metallic structure which shows around a resonance negative value of permeability
  - In 2000 : double-negative medium
    - D. Smith et al presents experimental evidence of transmission recovering in a composite metallic structure designed to show negative refraction in microwaves.



Metamaterial (3)

- Consequence of a negative refractive index : generalization of Snell-Descartes law.

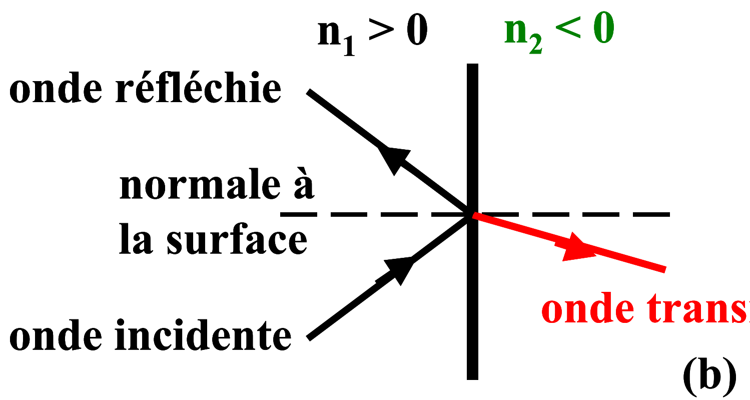
  - (a) positive refraction



with θ_1 : incidence angle
and θ_2 : angle between the transmitted wave and the normal direction

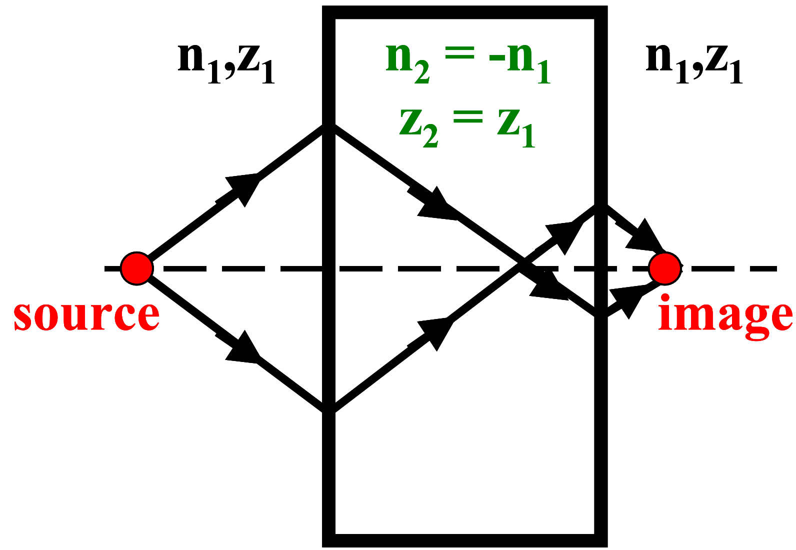
$$n_1.\sin\theta_1 = n_2.\sin\theta_2$$

  - (b) negative refraction



Metamaterial (4)

- Leading application : **the flat « superlens »**



- Effects :
  - near field focusing
  - evanescent wave amplification

  beyond the diffraction limit
  - translation invariance

en général : $n^2 = \varepsilon.\mu$; $z^2 = \varepsilon/\mu$

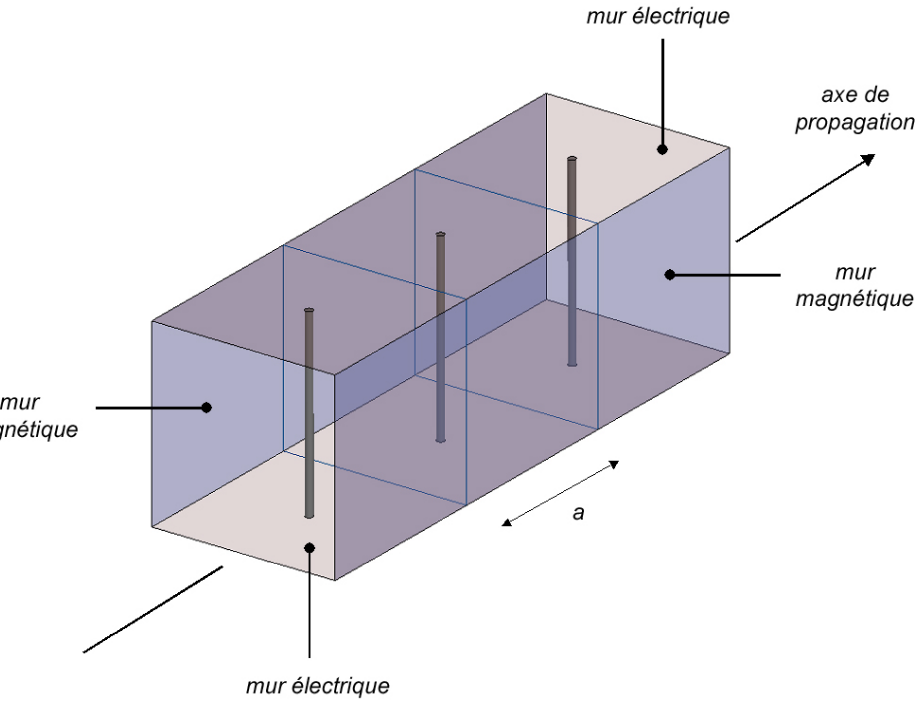
si $\varepsilon>0$ et $\mu>0$ alors $n>0$ et $z>0$

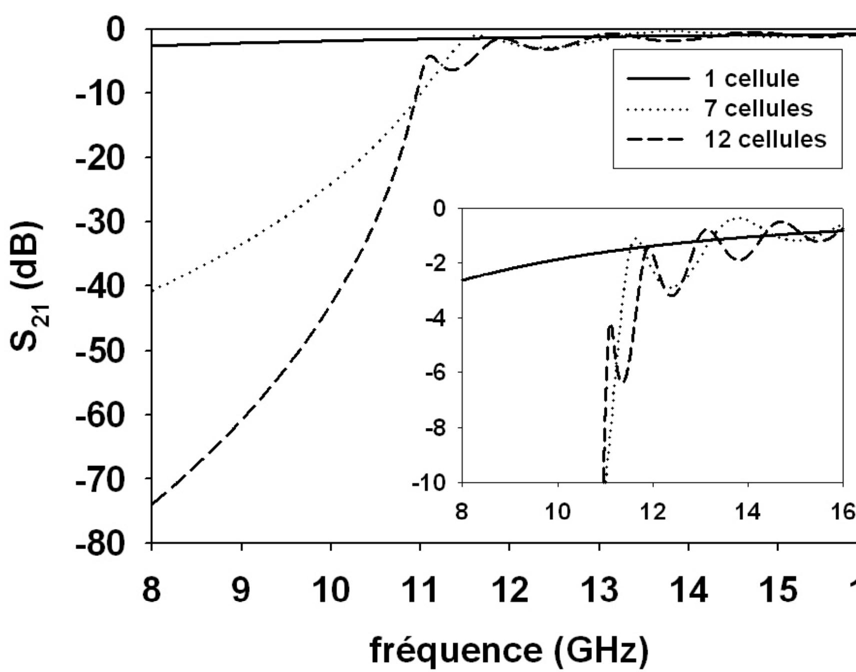
si $\varepsilon<0$ et $\mu<0$ alors $n<0$ et $z>0$

Metallic double negative media (1)

Left-handed structure : from negative permittivity and negative permeability to negative refraction

- Negative permittivity : diluted metallic wire network





Metallic double negative media (2)

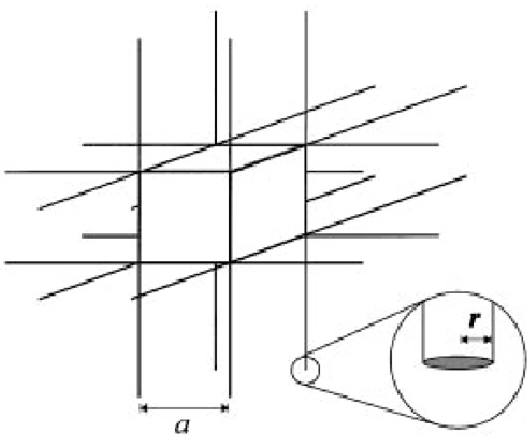
Left-handed structure : from negative permittivity and negative permeability to negative refraction

- Analysis :
  - Drude model for metals : $$\varepsilon(f) = 1 - \frac{f_p^2}{f(f + i\gamma)}$$
    - f_p : plasma frequency of the metal
    - γ : losses

$$\varepsilon < 0 \text{ for } f < f_p$$

  - For a diluted metal (wire array)
  - f_p is geometry dependant – it can be adjusted from microwaves to optics

Example :

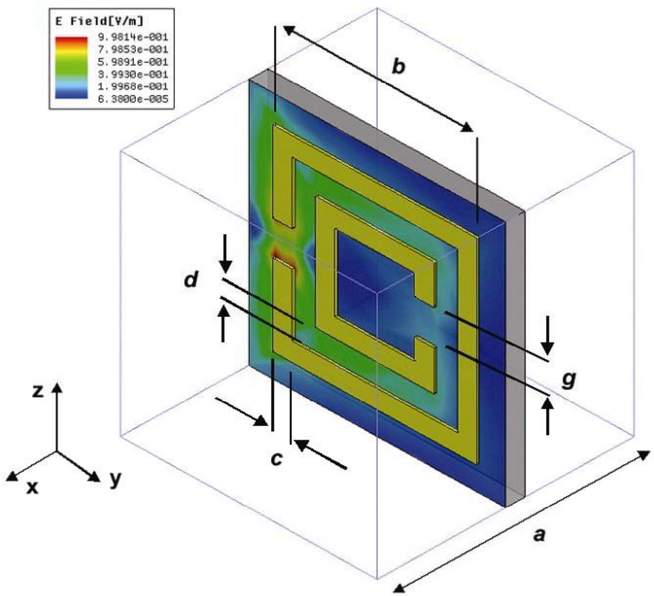


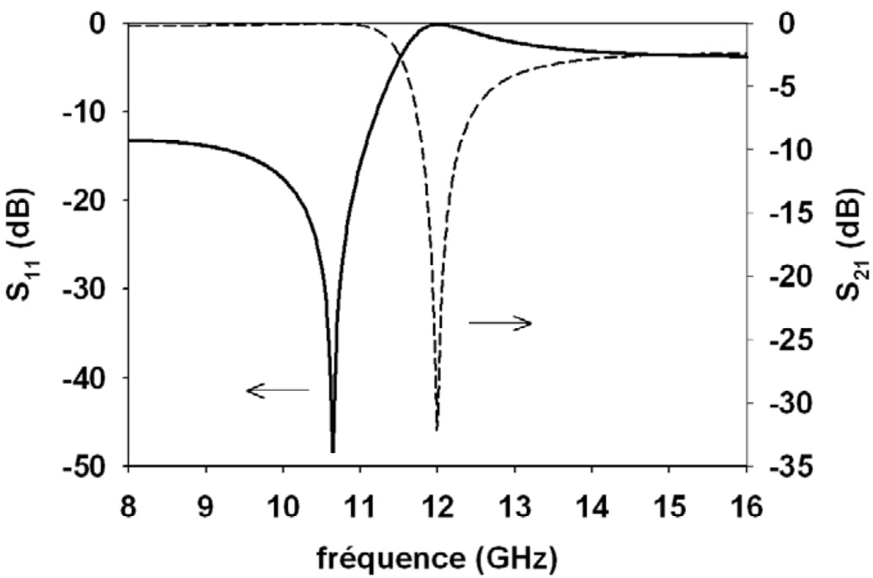
$$f_p^2 = \frac{c_0^2}{2\pi a^2 \ln(a / r)}$$

Metallic double negative media (3)

Left-handed structure : from negative permittivity and negative permeability to negative refraction

- Negative permeability : resonator array (based on split rings – SRR)





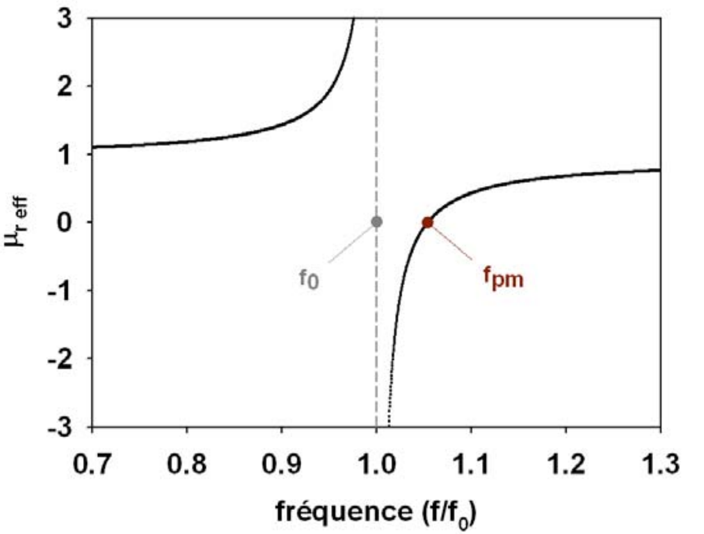
Metallic double negative media (4)

Left-handed structure : from negative permittivity and negative permeability to negative refraction

- Analysis
  - Resonant model for the permeability $$\mu(f) = 1 - \frac{F}{1 + i\frac{\gamma}{f} - \left(\frac{f_0}{f}\right)^2}$$
    - F : form factor
    - γ : damping term
    - f_0 : resonance frequency
  - A magnetic plasma frequency

can be defined as : $\mathbf{f_{pm}^2 = f_0^2 / (1\text{-}F)}$

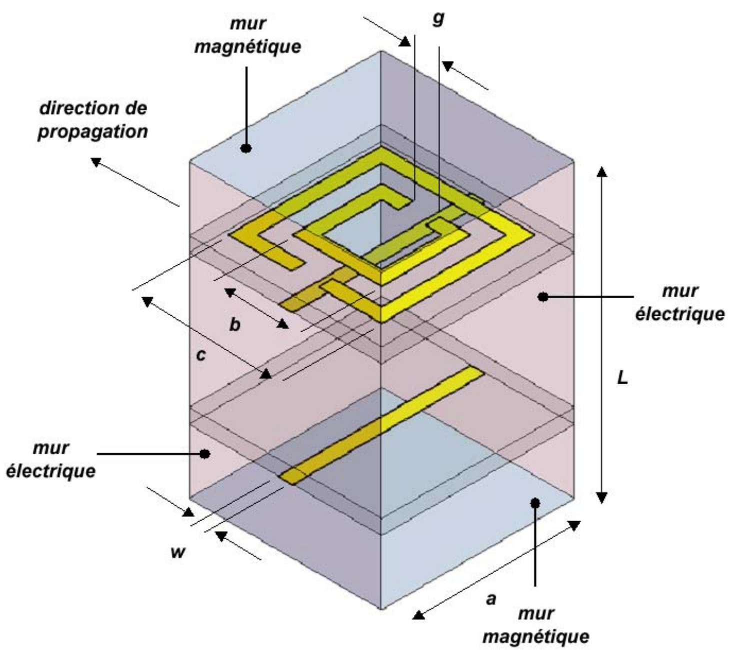
$$\mu < 0 \; for \; f_0 < f < f_{pm}$$

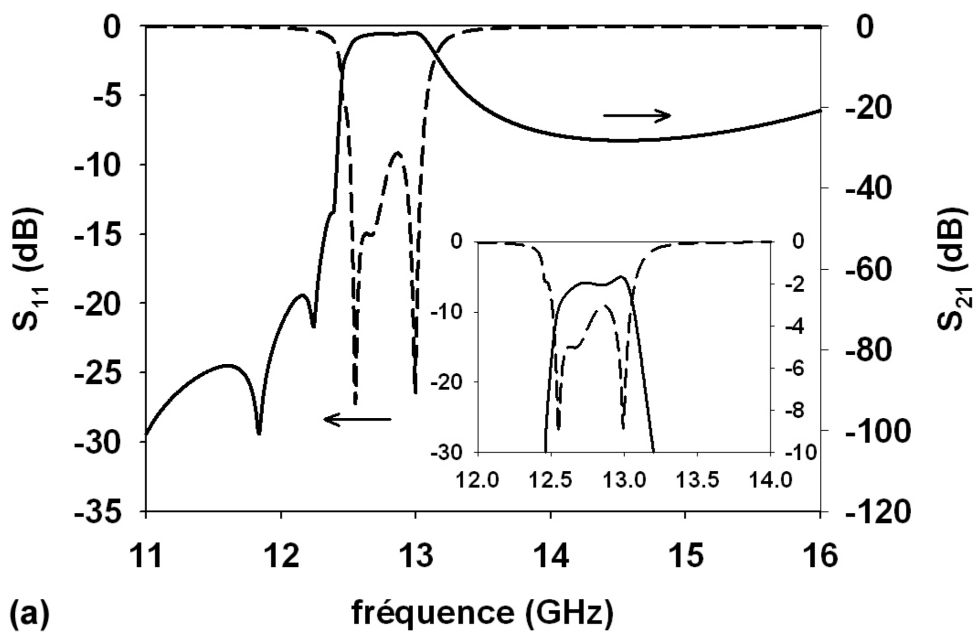


Metallic double negative media (5)

Left-handed structure : from negative permittivity and negative permeability to negative refraction

- Left-handed material : combination between a wire network and a resonator array



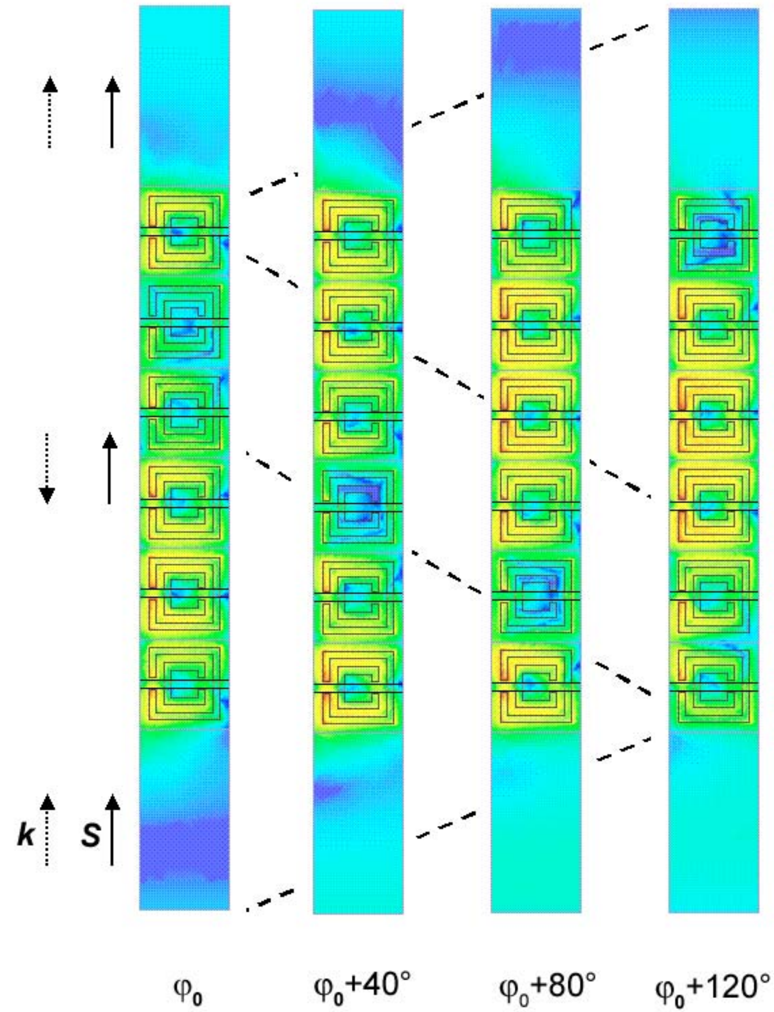


(a)

Metallic double negative media (6)

Left-handed structure : from negative permittivity and negative permeability to negative refraction

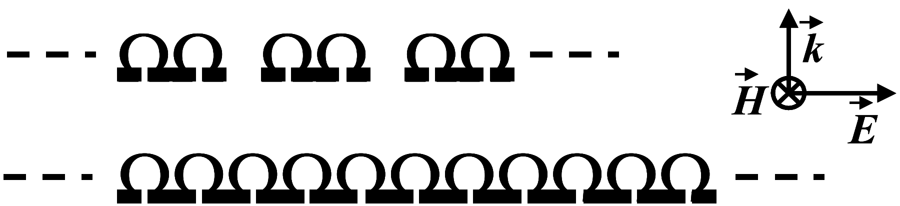
- Phase velocity and group velocity of opposite sign – Reversal of wave vector propagation direction with respect to Pointyng vector

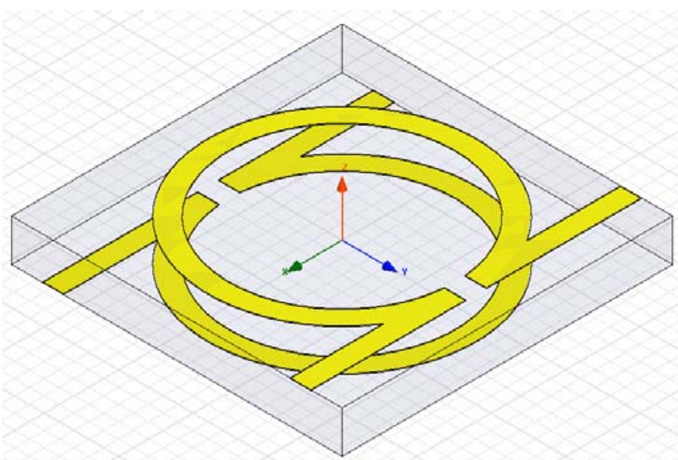


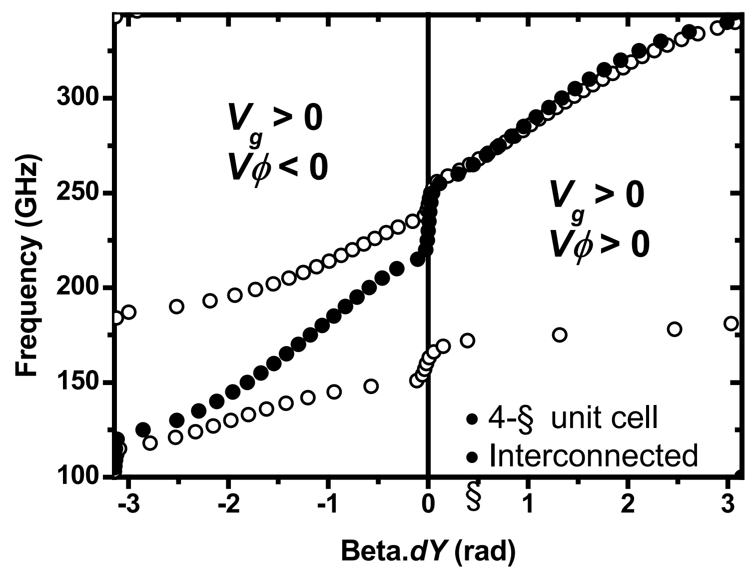
Metallic double negative media (7)

Left-handed structure : from negative permittivity and negative permeability to negative refraction

- New combined approach : the Ω letter
  - Idea : both ε < 0and μ <0 are given with a single pattern



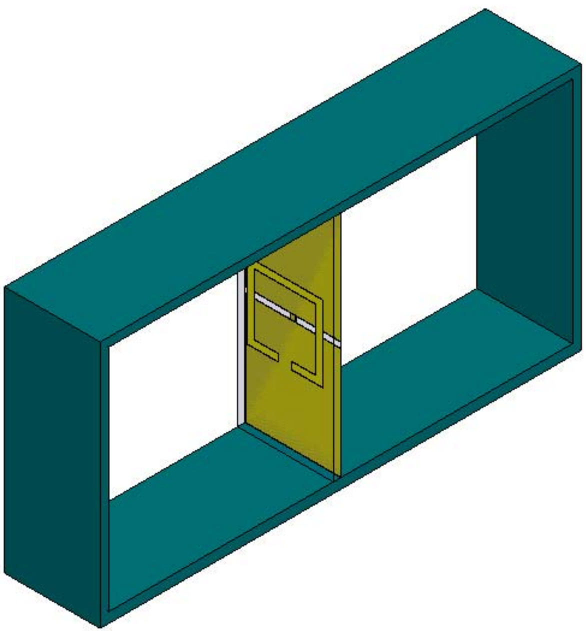


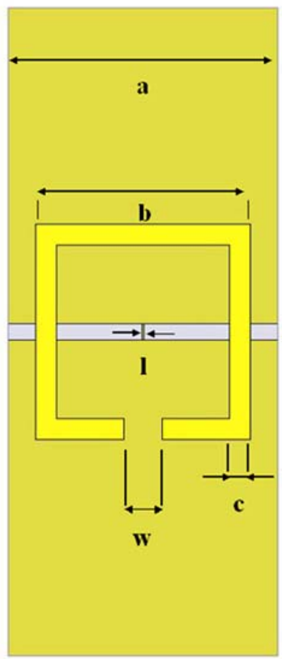


Metallic double negative media (8)

1-d example : a backward wave based finline device

- Another way to interpret LHM : BACKWARD WAVES
  - prototype suitable for characterization in microwaves



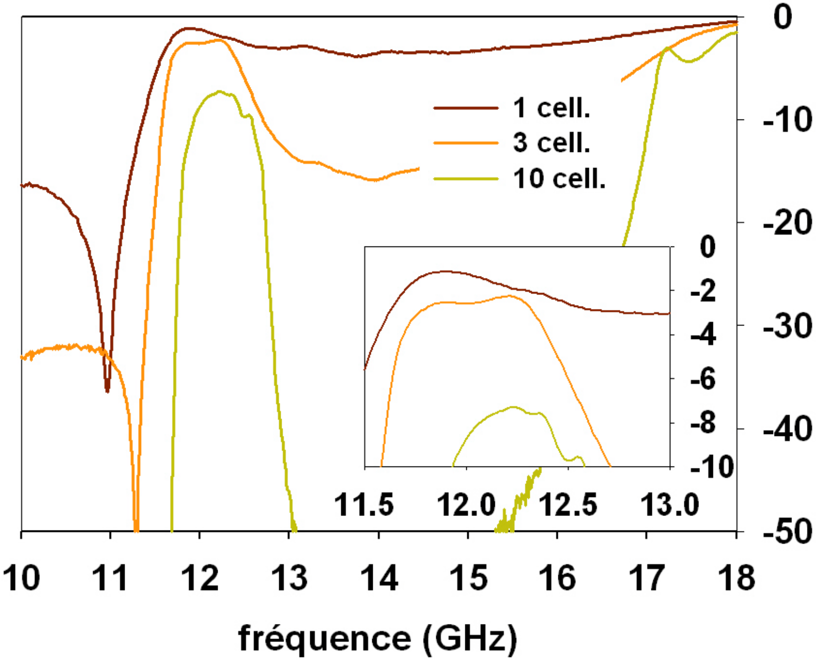


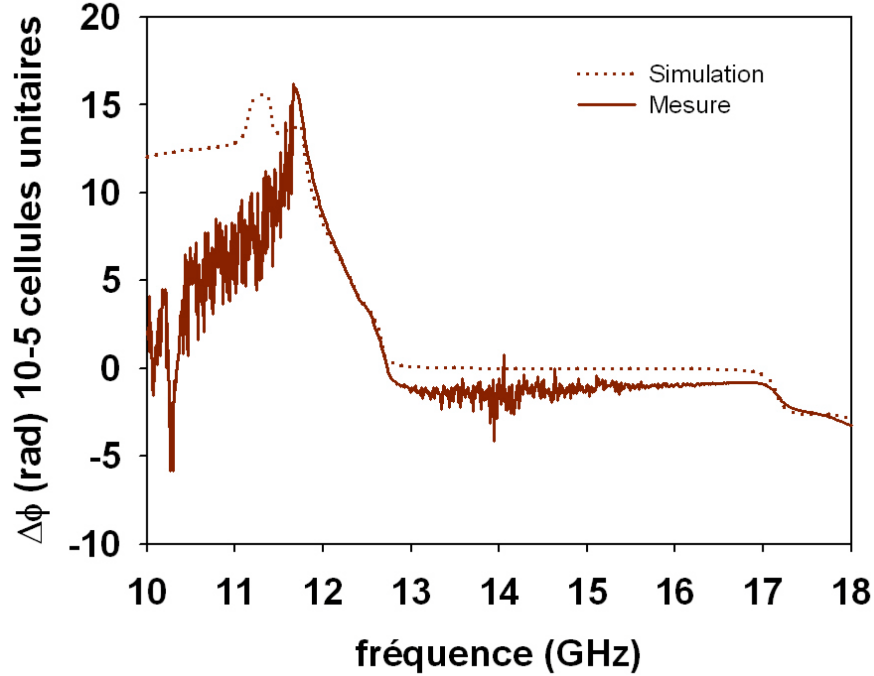
finline prototype from Barcelona University – Omicron society

Metallic double negative media (9)

A 1-d example : a backward wave based finline device

- Characterization

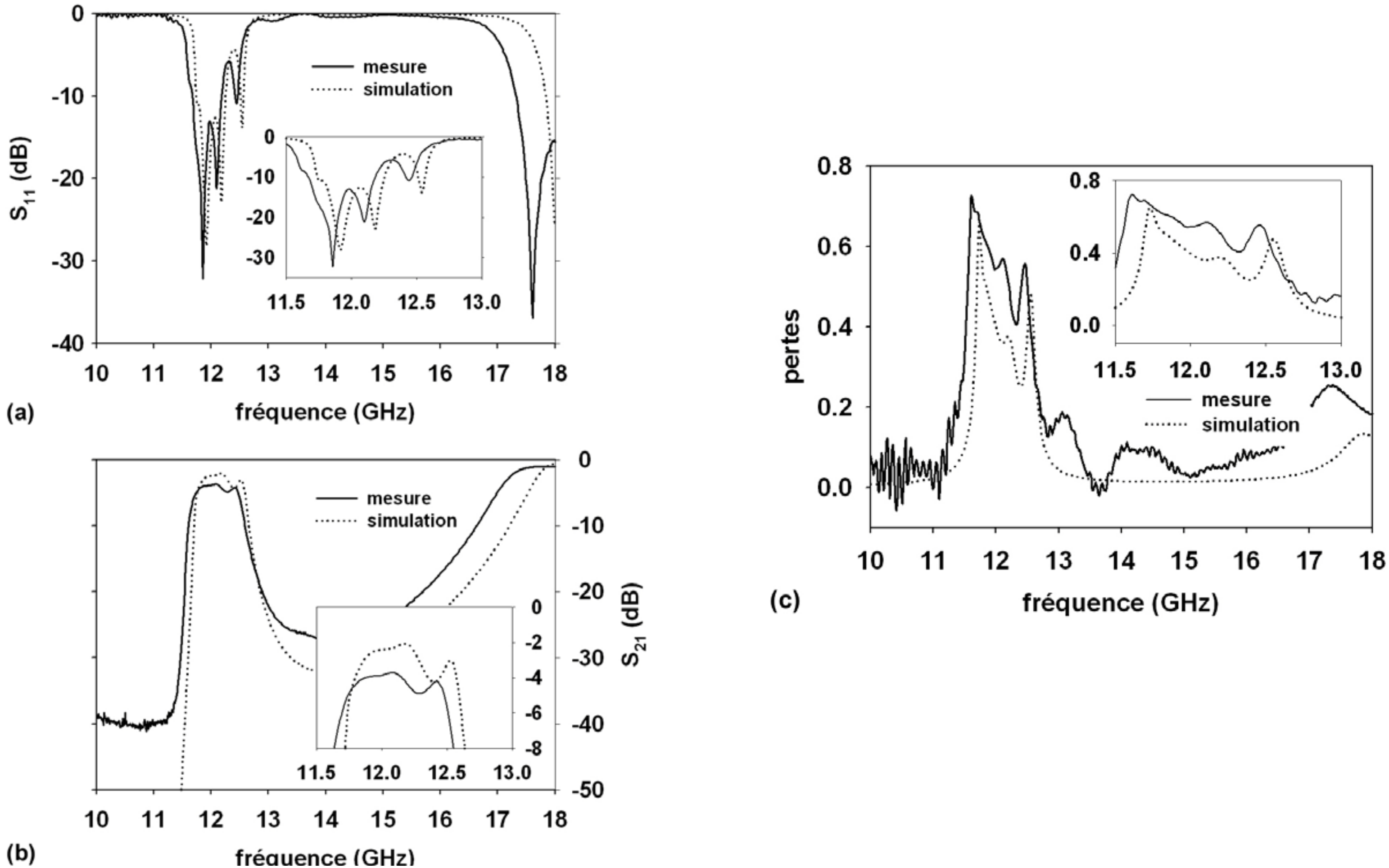




Metallic double negative media (10)

1-d examples : a backward wave based finline device

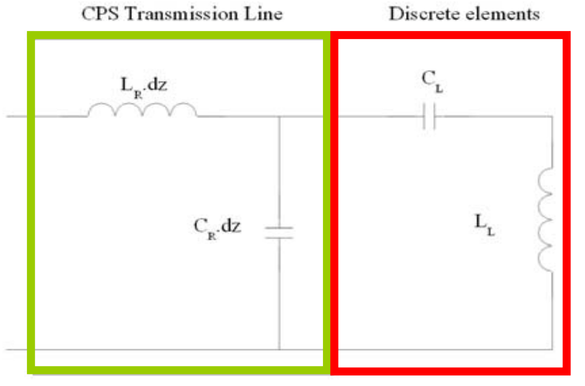
- Comparison with theory

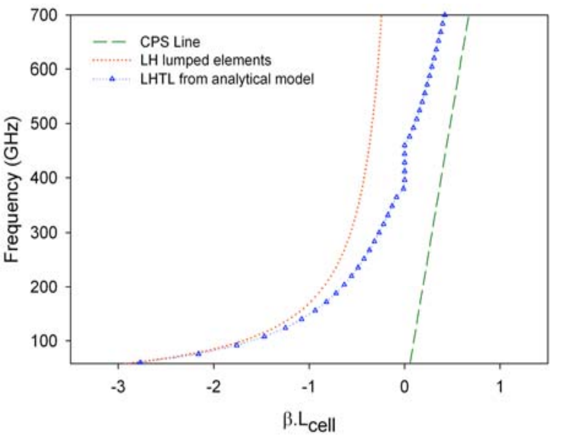


Metallic double negative media (11)

1-d examples : CL terahertz transmission line

- Distributed C-L approach





« RH »

$$\omega = \frac{\beta}{\sqrt{LC}}$$

$$v_\varphi = \frac{1}{\sqrt{LC}}$$

« LH »

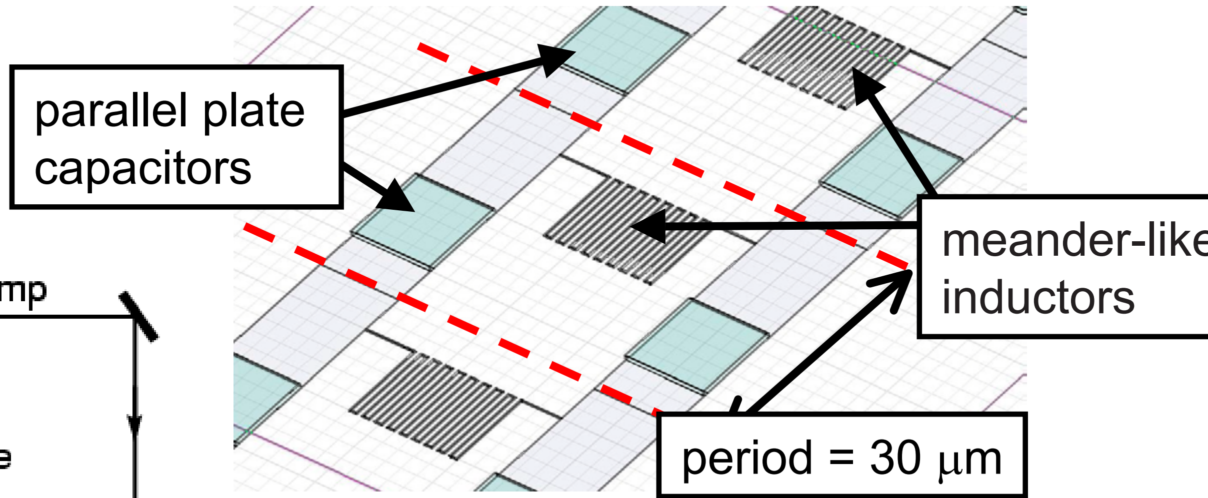
$$\omega = -\frac{1}{\beta\sqrt{L'C'}}$$

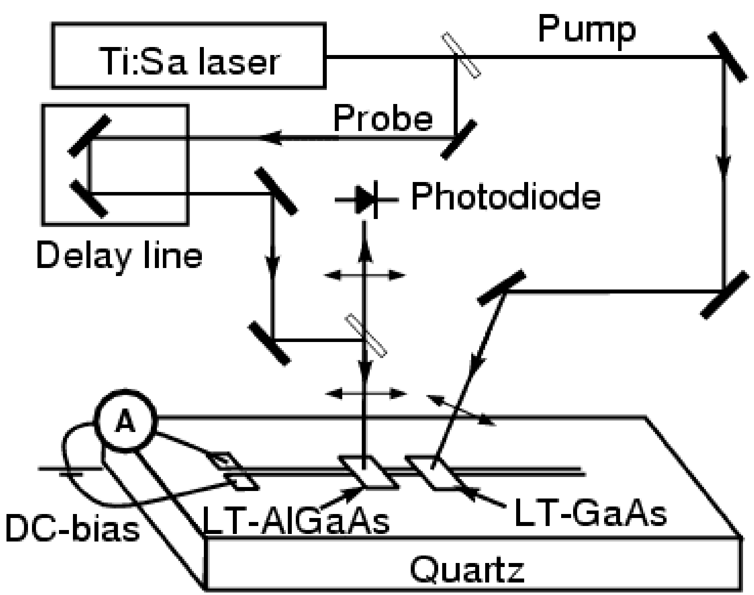
$$v_\varphi = -\frac{1}{\beta^2\sqrt{L'C'}}$$

Metallic double negative media (12)

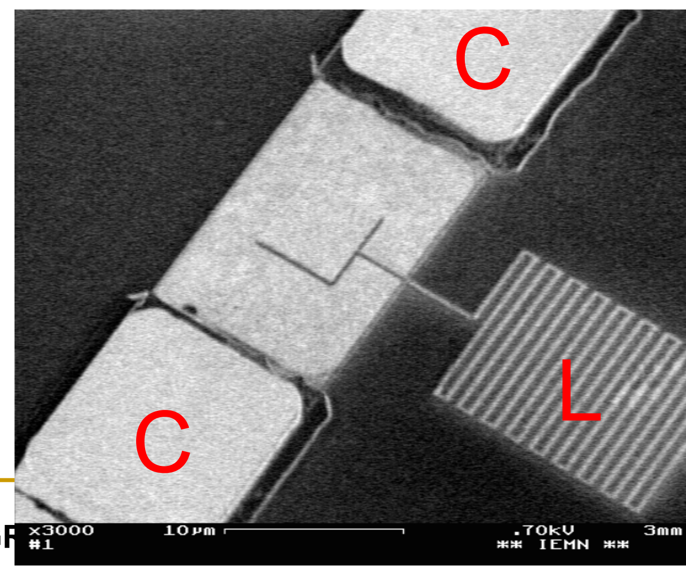
1-d examples : CL terahertz transmission line

- CL stripline at terahertz frequencies



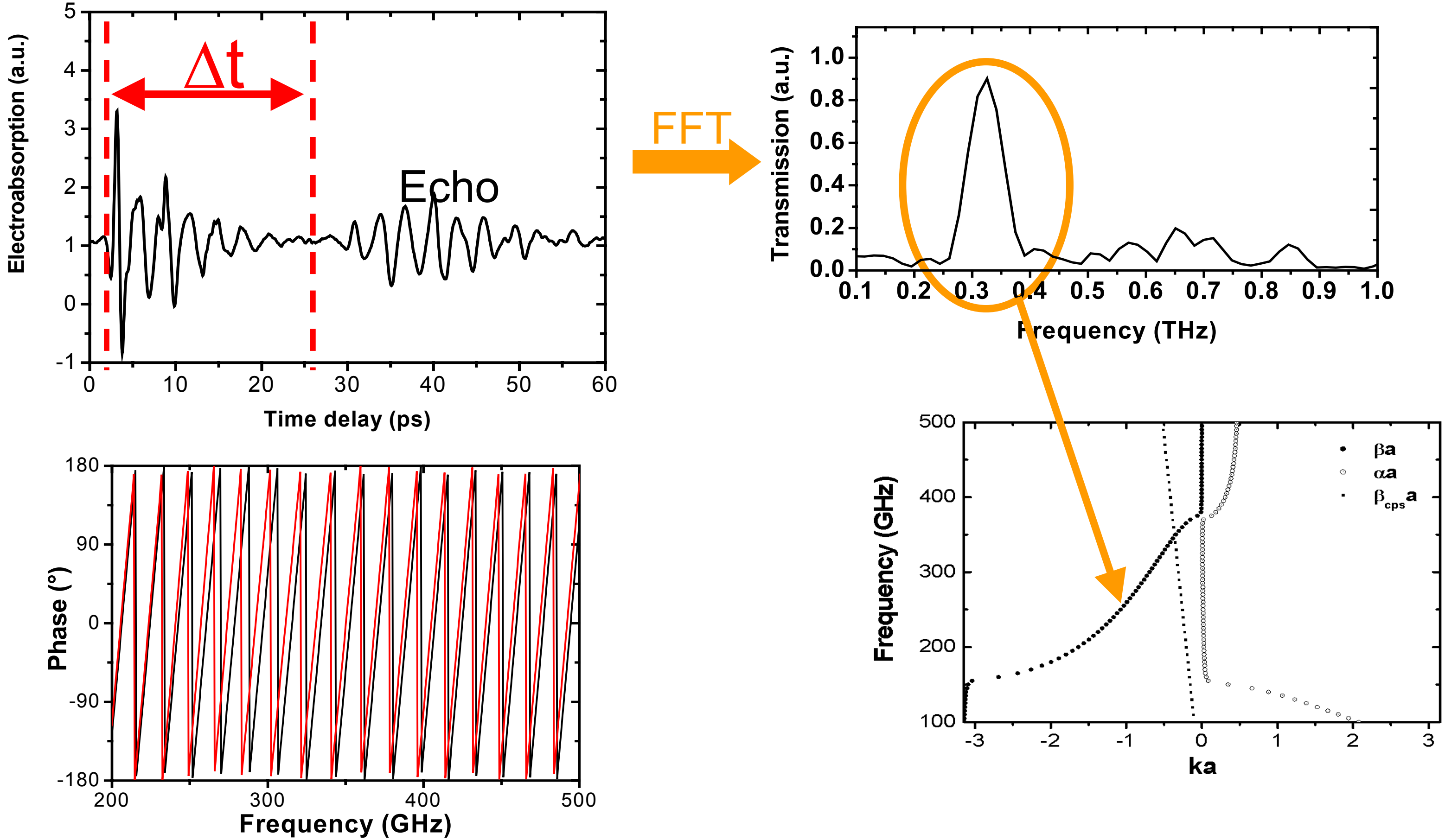


Pump-probe technique based on electrooptic sampling (L. Desplanque et al., *APL* **84**, p. 2049, 2004).



Metallic double negative media (13)

1-d examples : CL terahertz transmission line

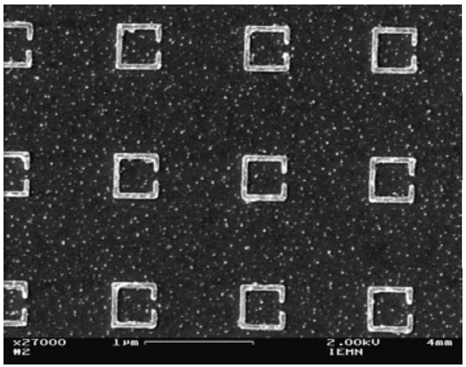


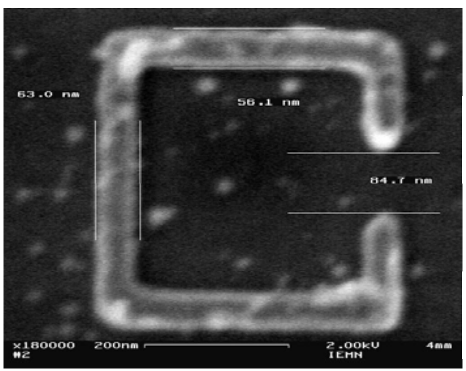
Phase comparison 21 versus 17 unit cells

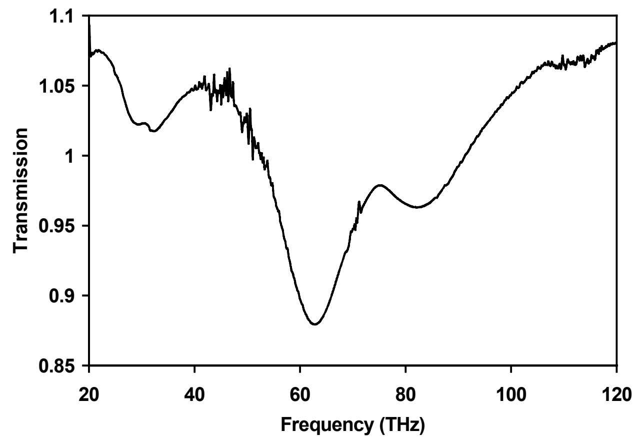
Metallic double negative media (14)

A 2-d example : NIR SRR array

- C letter at 60 THZ - FTIR Measurement
  - Drop in transmission → Magnetic activity







Metallic double negative media (15)

Toward homogeneization : band structure engineering

- Goal : definition of homogeneous parameters starting from ab-initio methods
  - from scattering parameters to dispersion curves : phase and group velocities

    (chain matrix approach)
  - from scattering parameters to « effective material parameters » : refraction index (n), surface impedance (z), permittivity (ε) and permeability (μ)

    (Weir extraction method)
  - from field maps to « effective material parameters »

    (Pendry's averaging scheme)

Metallic double negative media (16)

Toward homogeneization : band structure engineering

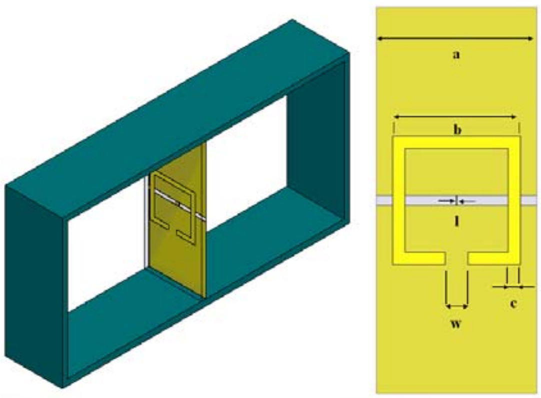
- Chain matrix approach
  - efficient for 1d-system as propagation lines where a characteristic impedance (Zc) and a propagation constant ($\gamma = \alpha + j\beta$) can be defined
  - using coefficients calculated (or measured) for one cell, properties of the periodic propagating medium can be obtained.

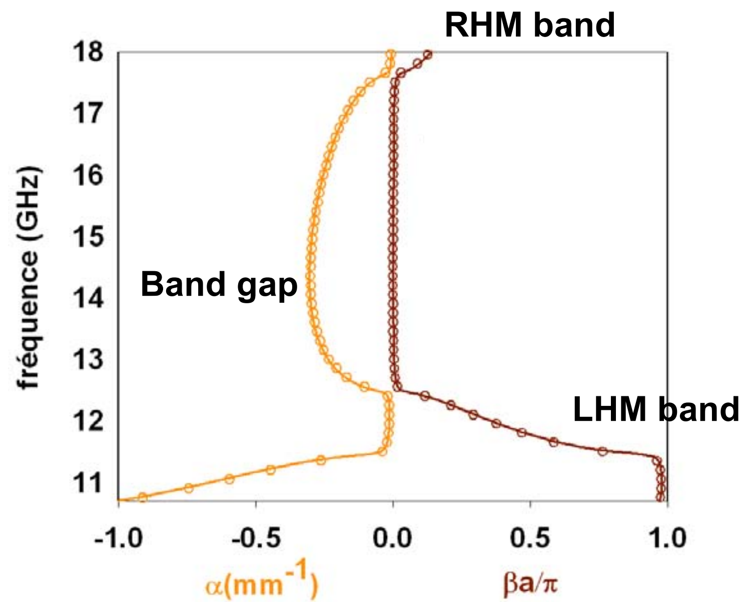
$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \Rightarrow M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} ch(\gamma a) & Z_c sh(\gamma a) \\ \frac{1}{Z_C} sh(\gamma a) & ch(\gamma a) \end{pmatrix} \Rightarrow \begin{cases} \alpha = \frac{1}{a} \ln \left| A \pm \sqrt{A^2 - 1} \right| \\ \beta a = \angle(A \pm \sqrt{A^2 - 1}) + 2k\pi, k \in Z \end{cases}$$

Metallic double negative media (17)

Toward homogeneization : band structure engineering

- Chain matrix approach





$$\begin{cases} V_{\varphi} = \dfrac{\omega}{\beta} \\ V_{g} = \dfrac{\partial\omega}{\partial\beta} \end{cases}$$

V_{φ} and v_g : opposite sign for LHM band

Metallic double negative media (18)

Toward homogeneization : band structure engineering

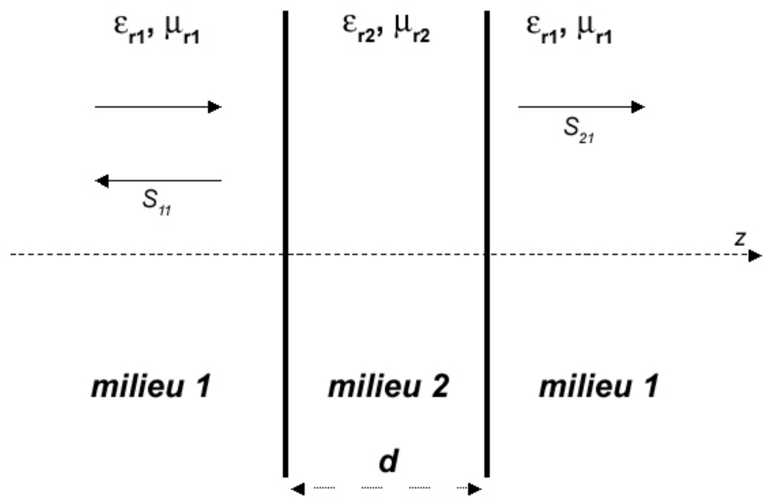
- Weir extraction method

  - Transmission and reflexion coefficients are related to intrinsic effective parameters by :

$$\Gamma = \frac{\sqrt{\mu_{r2}/\varepsilon_{r2}} - \sqrt{\mu_{r1}/\varepsilon_{r1}}}{\sqrt{\mu_{r2}/\varepsilon_{r2}} + \sqrt{\mu_{r1}/\varepsilon_{r1}}}$$

$$T = \exp(-j\frac{\omega}{c_0}\sqrt{\mu_{r2}\varepsilon_{r2}}d)$$

(ω frequency and c_0 light velocity in vacuum)



Metallic double negative media (19)

Toward homogeneization : band structure engineering

- Weir extraction method

$$S_{11} = \frac{(1 - T^2)\Gamma}{1 - T^2\Gamma^2}$$

$$S_{21} = \frac{(1 - T^2)T}{1 - T^2\Gamma^2}$$

$$\Gamma = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}} \pm \sqrt{\frac{(S_{11}^2 - S_{21}^2 + 1)^2}{4S_{11}^2} - 1}$$

$$T = \frac{S_{21}}{1 - S_{11}\Gamma}$$

$$\varepsilon_{r2} = j\sqrt{\frac{\varepsilon_{r1}}{\mu_{r1}}}\left(\frac{c}{\omega d}\right) \mathrm{Ln}(T) \left(\frac{1 - \Gamma}{1 + \Gamma}\right)$$

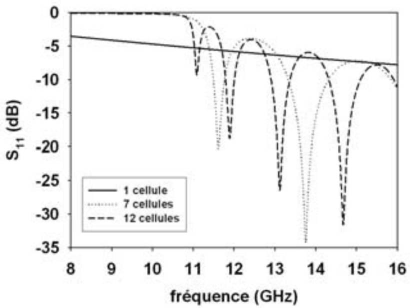
$$\mu_{r2} = j\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}\left(\frac{c}{\omega d}\right) \mathrm{Ln}(T) \left(\frac{1 + \Gamma}{1 - \Gamma}\right)$$

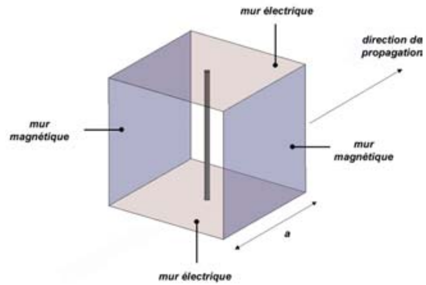
- Method adapted for TEM guided modes or free space plane wave
- Applicable to cascaded cells
- Ambiguity on solutions :
right choice guided by physical meaning of real and imaginary parts of ε and μ.

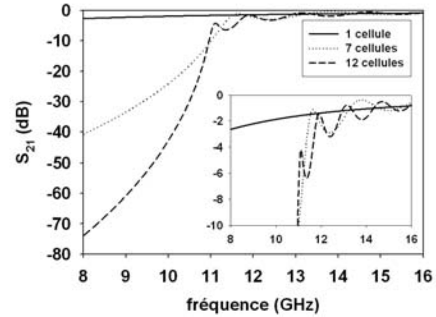
Metallic double negative media (20)

Toward homogeneization : band structure engineering

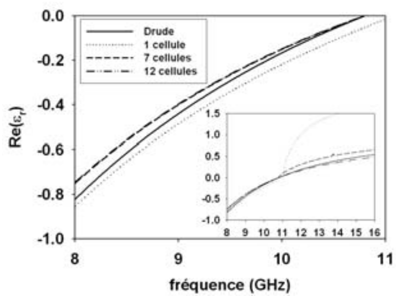
- Weir extraction method

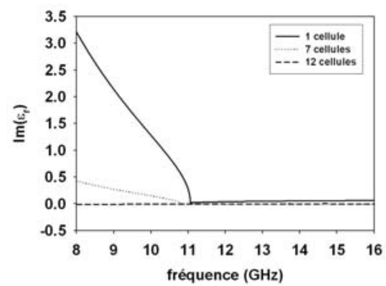




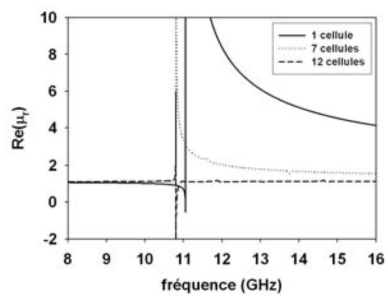


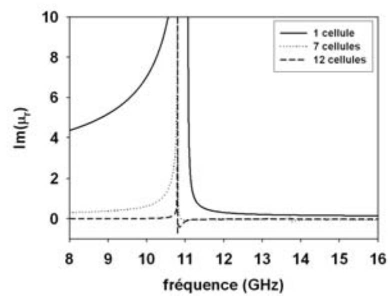
ε





μ

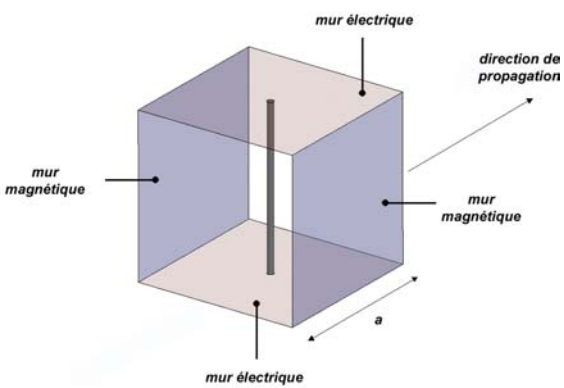




Metallic double negative media (21)

Toward homogeneization : band structure engineering

- Pendry's averaging scheme



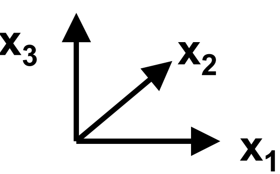
MAXWELL'S EQUATIONS

$$\vec{\nabla} x \vec{H} = +\frac{\partial \vec{D}}{\partial t} \qquad \int_c \vec{H}.\vec{dl} = +\frac{\partial}{\partial t}\int_S \vec{D}.\vec{dS}$$

$\rightarrow$ integral form :

$$\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \int_c \vec{E}.\vec{dl} = -\frac{\partial}{\partial t}\int_S \vec{B}.\vec{dS}$$

(with a loop c which encloses an area S)



Average values taken on a cell - for a cubic structure :

- averaging of E and H line integrals along edges of the cell

- averaging of B and D surface integrals over faces of the cell

Metallic double negative media (22)

Toward homogeneization : band structure engineering

- Pendry's averaging scheme

$$\langle E_i \rangle = a^{-1} \int \vec{E}.\overrightarrow{dx_i} \qquad \langle D_i \rangle = a^{-2} \varepsilon_0 \int \vec{E}.\overrightarrow{dS_i}$$

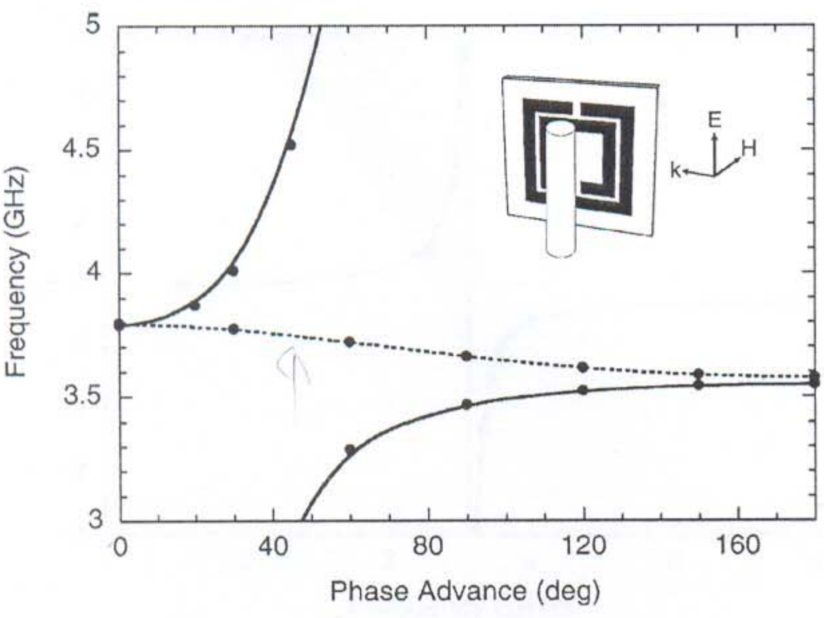
$$\langle H_i \rangle = \mu_0^{-1} a^{-1} \int \vec{B}.\overrightarrow{dx_i} \qquad \langle B_i \rangle = a^{-2} \int \vec{B}.\overrightarrow{dS_i}$$

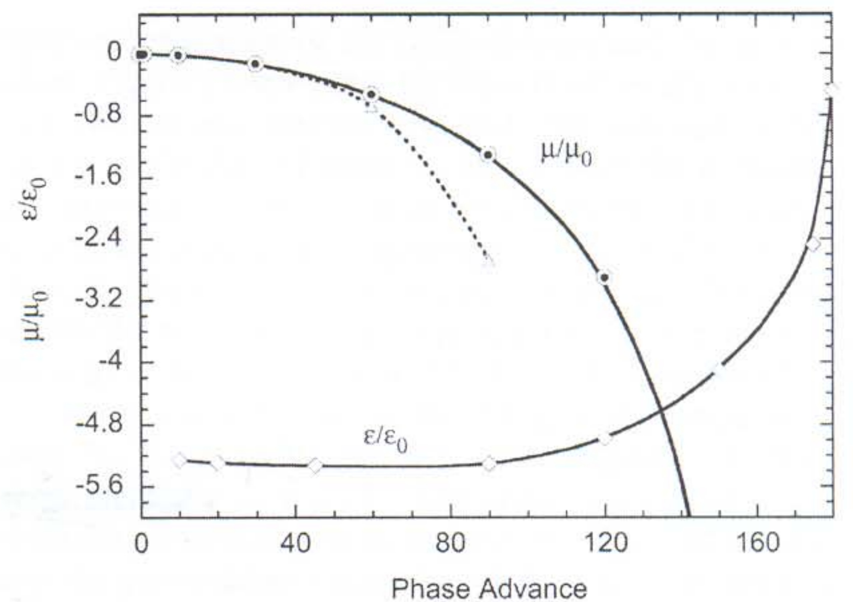
Effective material parameters given by : $\mu_{eff}^{i,j} \equiv \dfrac{\langle B_i \rangle}{\langle H_j \rangle}, \ \varepsilon_{eff}^{i,j} \equiv \dfrac{\langle D_i \rangle}{\langle E_j \rangle}$

Metallic double negative media (23)

Toward homogeneization : band structure engineering

- Pendry's averaging scheme – Results



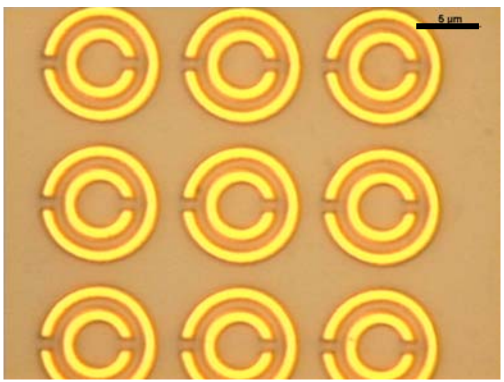


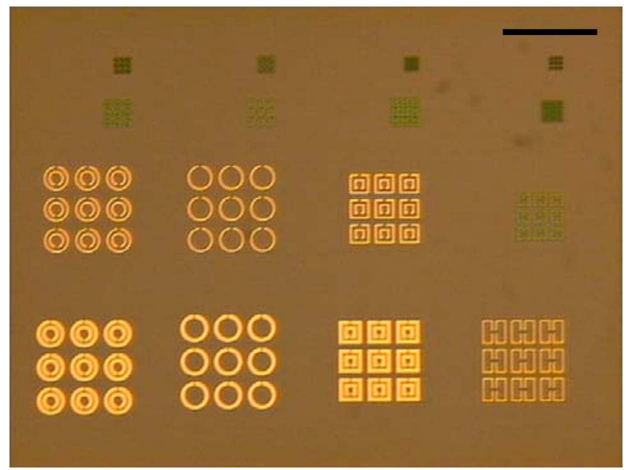
from D.R. Smith et al, APL 77(14), 2000

Metallic double negative media (24)

Transition

- Limitations of metallic structures for optical frequencies
  - saturation of « magnetic » resonance for SRR’s based structures induced by electron inertial beyond infrared wavelengths
  - intrinsic dimensions reach nanofabrication limits





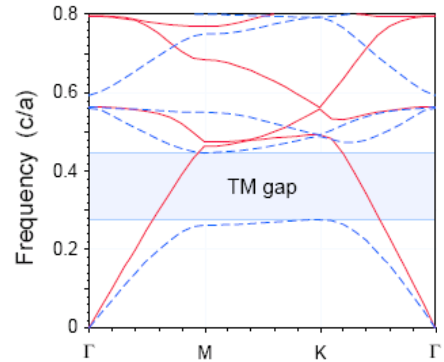
- Alternative solution : *dielectric photonic crystals*

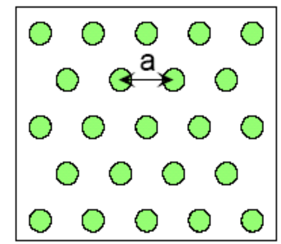
Dielectric artificial structures (1)

Photonic crystals

- Two-dimensional photonic crystals : patterning of semiconductors for applications in the optical domain
  - for λ = 1.55 µm

  → period : few hundreds of nm

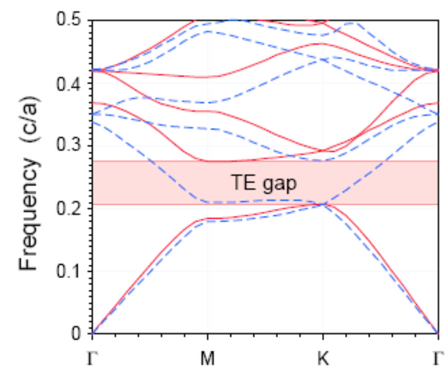


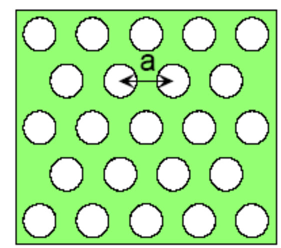


pillar or hole arrays patterned in a dielectric host substrate

↓

band structure with pass- and stop bands with RH or LH behaviours

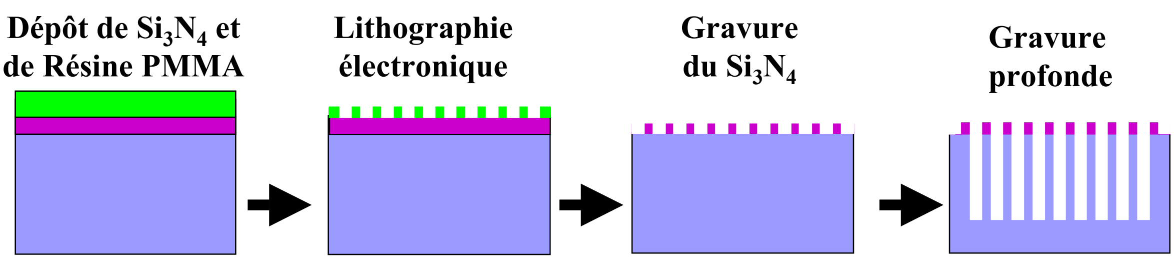




Dielectric artificial structures (2)

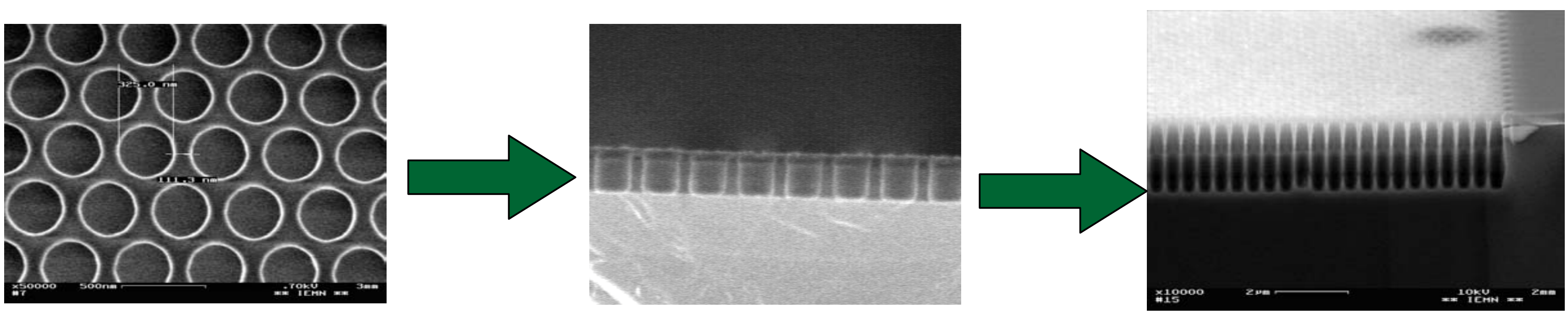
Photonic crystals

- Aspects technologiques…



Diamètre: ~300nm

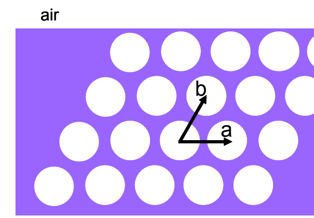
Profondeur: ~ 2um



Dielectric artificial structures (3)

Photonic crystals

- Negative refraction : *search of maximal isotropy along with a reversal of band concavity.*

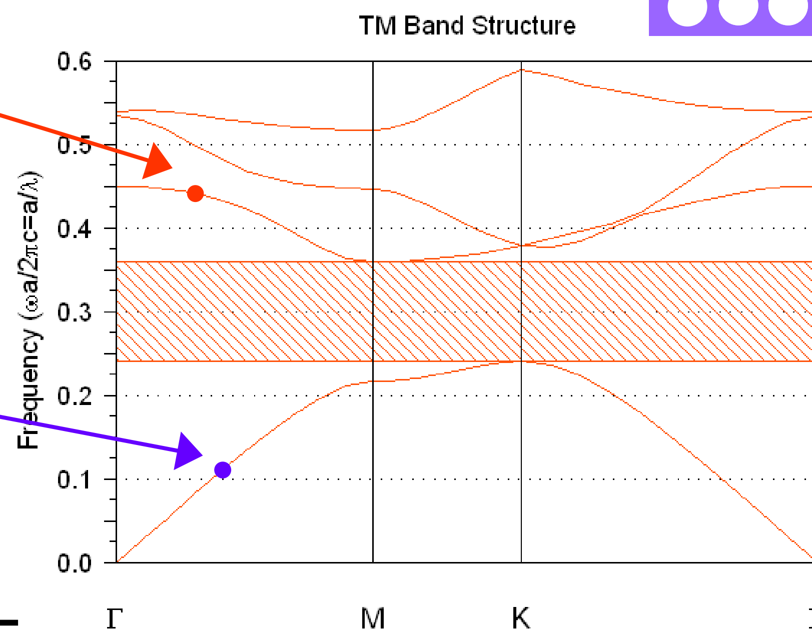


→ In second band : **LH band**

- isotropic band reversal : negative refraction index
- k and Vg are in opposite direction but colinear near Γ

→ In first band : **RH band**

- in general a positive refraction index is defined
- k and Vg are in the same direction



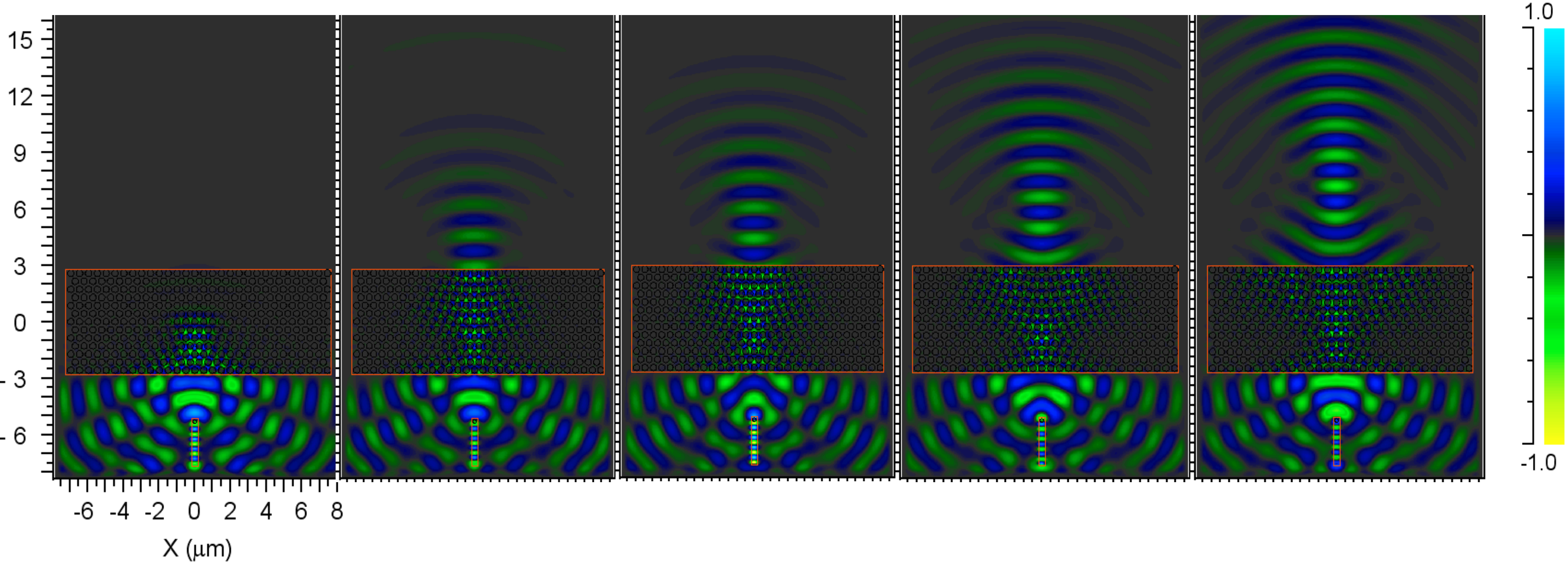
BUT FOR SOME INCIDENCE ANGLE

Near K-point (for triangular lattice) : negative refraction can occur

Dielectric artificial structures (4)

Toward a perfect lens in optics…

- Simulation results
  - 2D calculation using FDTD techniques

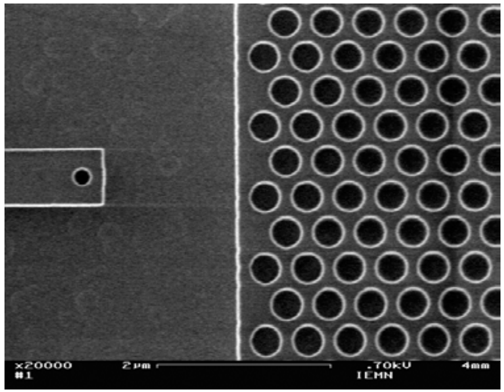


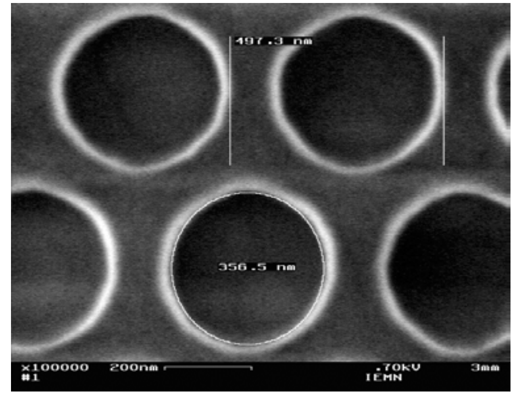
  - Both band structure or ray tracing gives n ~- 0.95

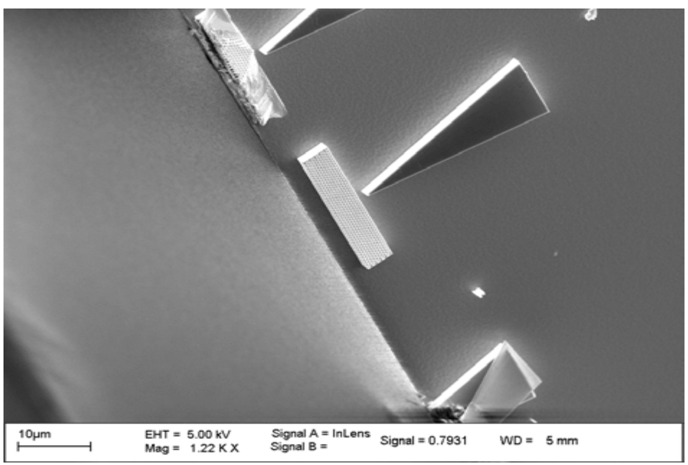
Dielectric artificial structures (5)

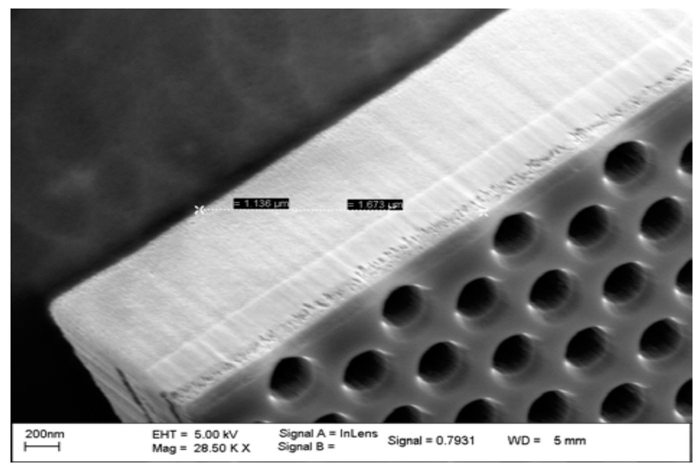
Toward a perfect lens in optics…

- Prototype









Conclusions and Prospects…

- Negative refraction and backward waves…
  - an old subject re-visited owing to technological facilities
  - still needs a deep understanding of « electromagnetic » properties of constitutive elements of metamaterials
    - magnetic activity in resonators
    - coupling between magnetic and electric artificial networks…
  - from sub-wavelength stucturation to mean effective parameters
    - mean field analysis still lacking
  - dielectric approaches : photonic crystals… (Mie resonances : $\mu<0$ with high ε lattices)
- Super-lensing and Evanescent wave amplification
  - what is the real limit of resolution for a flat lens?
  - real materials present losses…. what will remain ?

- **Is there any new mathematics in these approaches ?**
  - *physicists learn to accept non-intuitive physically acceptable solutions from Maxwell's equations…*

- ***Nevertheless…***
  Preliminary experiments (in microwaves) are very exciting !!!