Sujet de stage M2:

Undecidability in perturbed dynamical systems

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Dynamical systems is a very natural and widely used model to capture systems evolving over time. Generally dynamics are computable, in the following informal sense : given a configuration, it is possible to compute the next configuration of the configuration after some finite time. In many of the contexts the properties of true interest are however about their long-term evolution. However, if bounded-time simulations are usually possible, the computational outlook for the "infinite" time horizon problems is grim : the long-term behavior of many interesting systems is uncomputable.

A notable feature, shared by prior works on computational intractability in dynamical systems [Moo90, BY07, AB01], is that the non-computability phenomenon is not robust : uncomputability disappears once one introduces even a small amount of noise into the system. Thus, if one believes that natural systems are inherently noisy, one would not be able to observe such non-computability phenomena.

A natural approach is to study the invariant measures of perturbed systems. In [BGR12], the authors show that while simple dynamical systems defined by transformations of the interval have uncomputable invariant measures, perturbing such systems makes the invariant measures efficiently computable. Thus, noise that makes the short term behavior of the system harder to predict, may make its long term statistical behavior computationally tractable.

This type of results is not known for dynamical systems on discrete space as cellular automata. In this case it is possible that the noise cannot break uncomputability since some computation can be implemented in noisy cellular automata [Gác01]. The purpose of this stage is to see where the technics developed in [BGR12] cannot hold and try to construct dynamical systems with uncomputable asymptotic behaviors even if they are perturbed.

Références

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