Computing the asymptotic behavior of low-dimensional dynamical systems

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Algorithmic Questions in Dynamical Systems
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Motivation

An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

— Pierre Simon Laplace, A Philosophical Essay on Probabilities

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What can we tell (compute) about the world? Is it sufficient to have high quality data and models? Are (digital) computers Laplace's demon?

Which model should we consider?

- Almost every (macroscopic) system which follows the classical (deterministic) laws of physics can be written in terms of differential equations.
- In particular many systems can be modeled as ordinary differential equations (ODEs).
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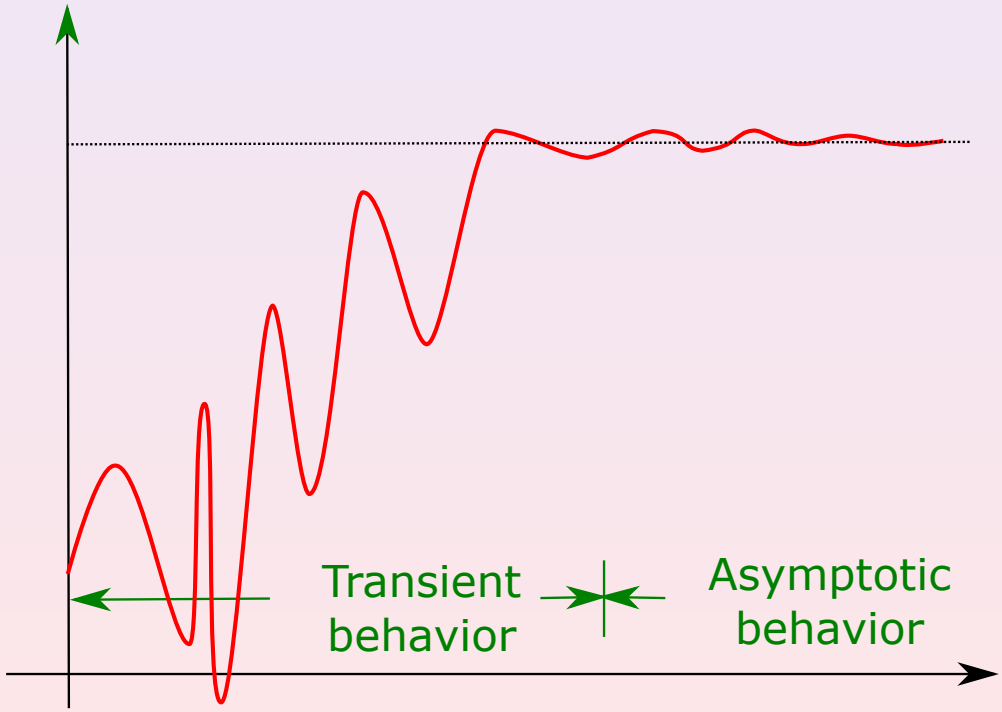
- What can be computed about ODEs using Turing-like models?
  - ▶ **This talk**
- What can be computed with models of computation defined with ODEs? ▶ **Olivier Bournez's talk**

Computation of the evolution of dynamical systems

Two common problems, with many applications in practice, are to know the behavior of a dynamical system:

1. At a given time t
2. Near infinity (asymptotic behavior)



For the first case we need to study the computability of the solutions of an ODE

$$y' = f(y)$$

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- But isn't this a trivial task?
- No. The standard theory (via Picard iterations) is only guaranteed to work in a compact, where a Lipschitz constant for f exists
- With this Lipschitz constant one is able to compute rigorous bounds on the error made by the approximation (the bound depends on the Lipschitz constant)
- But what to do in an open, potentially infinite domain, where no single Lipschitz constant is valid there?

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- The construction of this interval is as follows: a solution which is defined in a compact set is extended over and over again to a "bigger" compact set yielding, in the limit, a unique solution in the maximal interval of existence.
- Alas this procedure is not computable since this limit procedure is not computable.

Maximal interval of existence

Theorem (G., Zhong, Buescu '09)

The maximal interval of existence is not computable. We cannot even decide whether it is bounded. These results hold even if f is analytic.

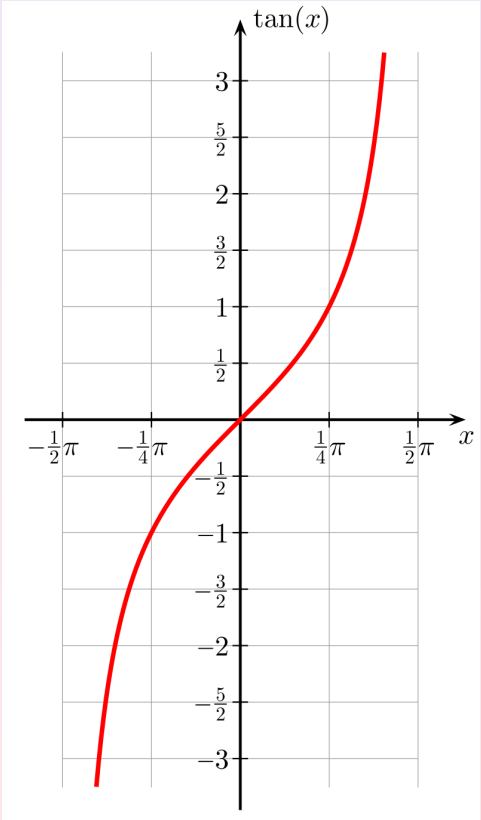
The solution of

$$\begin{cases} y' = \frac{1}{\cos^2 x} \\ y(0) = 0 \end{cases}$$

is

$$y(x) = tan(x)$$

The maximal interval of existence of the solution is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Computability of ODEs in the maximal interval

Theorem (Cauchy-Peano)

If f is continuous, then the initial-value problem $x' = f(t, x), x(t_0) = x_0$ has a solution on a neighborhood of x_0

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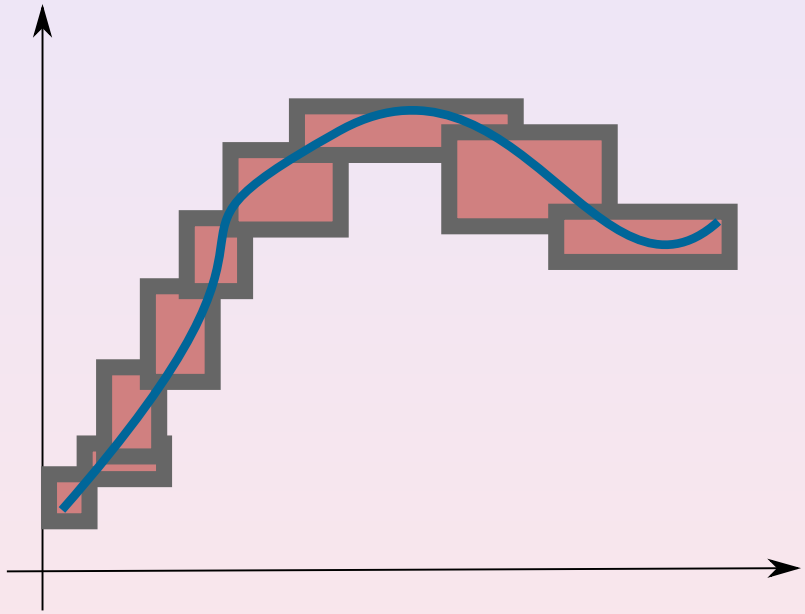
Theorem (Collins, G. '09)

If f is continuous, and y is the unique solution of $x' = f(t, x), x(t_0) = x_0$, then the operator which maps (f, t_0, x_0) to y is computable (you can compute $y(t)$ on the whole maximal interval)

Idea of the proof

Exhaustive: generate all possible (partial) coverings of the state space (and of the tangent space)

- We can check (recursively) if a covering covers the solution
- We show that coverings of arbitrary small diameters exist



- So just keep testing coverings until you find an appropriate one (Hence the "*ten thousand monkeys approach*" in the title of this paper. This is terribly inefficient, but enough for our purposes).

What about computational complexity?

Results for the compact case. The leftmost column has assumptions on f in $y' = f(y)$, $y(0) = 0$. The two rightmost columns indicate the complexity of the solution.

Assumptions	Upper bound	Lower bound
ODE with unique solution	Computable	Arbitrary high complexity
Lipschitz ODE	PSPACE	PSPACE
f is of class C^1	PSPACE	PSPACE
f is of class $C^k, k > 1$	PSPACE	CH
f is analytic	polynomial-time	polynomial-time

- These results were obtained by people such as [Miller '70], [Ko '83], [Müller, '87] [Kawamura, '10],[Kawamura et al., '14]

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- But this is incorrect!

Example

The solution of

$$\begin{cases} y_1'(t) = y_1(t) \\ y_2'(t) = y_1(t)y_2(t) \\ \quad \cdots \\ y_n'(t) = y_1(t) \cdots y_n(t) \end{cases} \qquad \begin{cases} y_1(0) = 1 \\ y_2(0) = 1 \\ \quad \cdots \\ y_n(0) = 1 \end{cases}$$

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is polynomial-time computable on any (time) bounded set. However its solution is

$$y_1(t) = e^t \qquad y_2(t) = e^{e^t - 1} \qquad y_n(t) = e^{e^{\cdot^{\cdot^{\cdot^{e^{e^t}-1}}}}-1}$$

which is obviously not polynomial-time computable on its maximal interval of definition ($\mathbb{R}$).

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▶ Therefore we have to use parametrized complexity (complexity is measured against one or more extra parameters).

Proposition

If (α, β) is the maximal interval of existence of the solution y of an ODE $y' = f(t, y)$ and $\beta < +\infty$ then $y(t)$ gets unbounded as $t \to \beta$

Solution: measure complexity against the length of the solution curve y between $(0, y(0))$ and $(t, y(t))$

Theorem (G., Pouly '16)

There is a numerical method SolvePIVP *such that, for any* $t \in \mathbb{R}$, $\varepsilon > 0$, *if* y *satisfies* $y' = p(y)$, $y(t_0) = y_0$, *it halts and returns a value* $x = \mathrm{SolvePIVP}(t_0, y_0, p, t, \varepsilon)$ *such that:*

- $\|x - y(t)\| \leqslant \varepsilon$
- *the (bit) complexity of the algorithm is bounded by*

$$\mathrm{poly}(k, \mathrm{Len}(t_0, t), \log \|y_0\|, \log \Sigma p, -\log \varepsilon)^d$$

where k *is the maximum degree of the components of* p, d *is the number of components of* p, Σp *is the sum of the absolute values of the coefficients of* p, *and* $\mathrm{Len}(t_0, t)$ *is a bound on the length of the curve* $y(\cdot)$ *from the point* $(t_0, y(t_0))$ *to the point* $(t, y(t))$.

Idea behind the proof

- Use a variable order method
- Use the hypothesis that the function defining the ODE is constituted by polynomials to get majorants
- Use an argument based on Cauchy majorants to establish a lower bound on the local radius of convergence
- Choose the step length $|t_{i+1} - t_i|$ to be a constant fraction of the estimated radius of convergence
- Choose an order of the method ω_i in the interval $[t_{i+1} - t_i]$ based on the computed majorants
- Start with a bound $l = 1$ for the length of the curve and double it in each run of the method if it does not succeed (this can be algorithmically detected).

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  - In general this is a very hard problem!
- Related problems can be found in applications (e.g. verification, control theory):
  - Given an initial point x_0, will the trajectory starting from x_0 eventually reach some "unsafe region" (Reachability)?
  - How many attractors ("steady states") a system has? Can we characterize these attractors? Can we compute their basins of attractions – the set of points on which the trajectory will converge towards a given attractor?

Are attractors (limit sets) computable?

Roughly, attractors are invariant sets to which nearby trajectories converge. Some types of attractors:

- Fixed points
- Periodic orbits (cycles)
- Strange attractors (Smale's horseshoe, Lorenz attractor, etc.): attractors with a fractal structure

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This is an interesting problem, but...

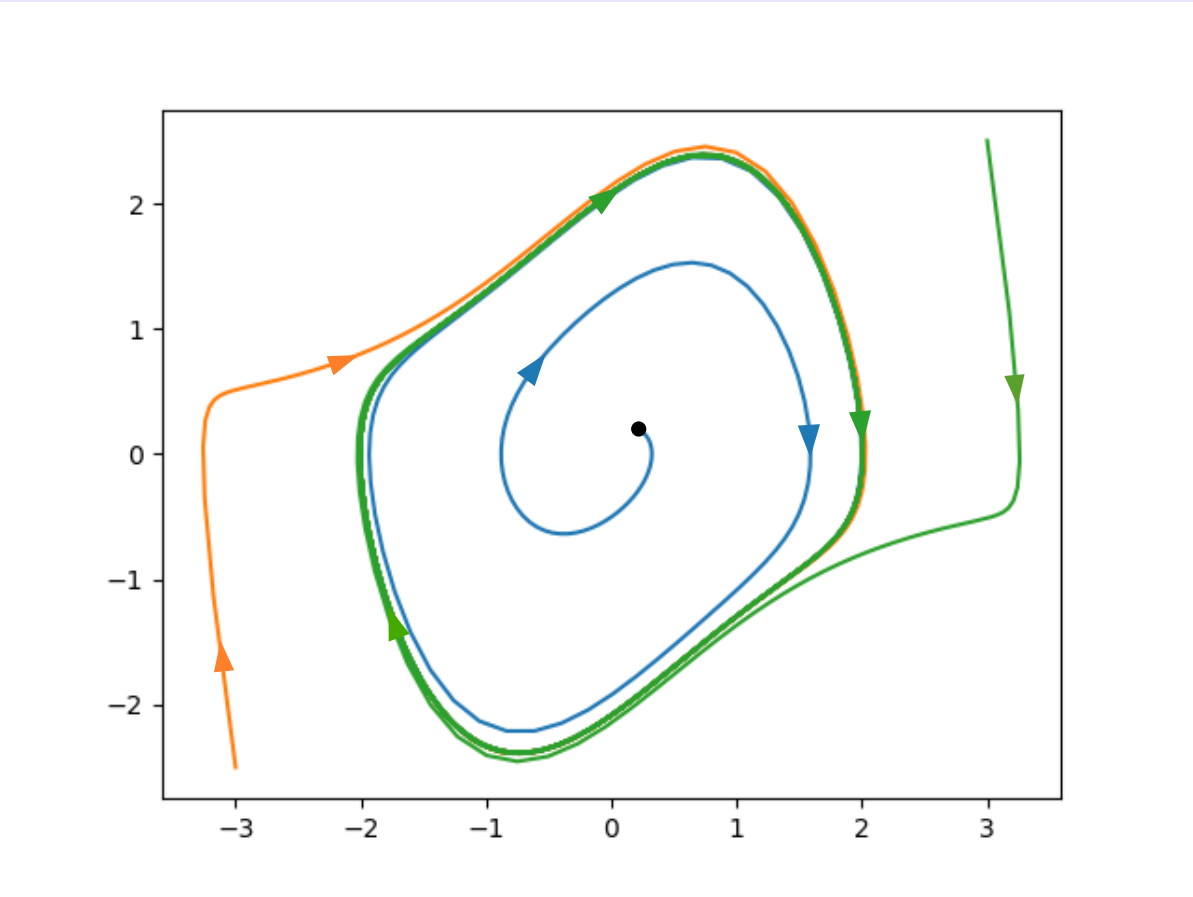
- Except for some very particular classes of systems, it is unknown in general how $NW(f)$ looks like
- We do know what happens on the (compact) two-dimensional case
- The are some theories (still on their early infancy) which try to address the three-dimensional case
- But for dimensions ≥ 4: ????

The 2-dimensional case

Theorem (Peixoto '59)

On the two-dimensional disk $D = [0,1]^2$, the set of structurally stable systems forms an open and dense set over the class of C^1 dynamical systems defined over D (i.e. structurally stable systems are generic on $C^1(D)$). Moreover, any structurally stable system $y' = f(y)$ over D has the following properties:

- *$NW(f)$ consists only of a finite number of periodic orbits and fixed points*
- *All periodic orbits and equilibria are hyperbolic*
- *There are no saddle connections*



Example: Van der Pol system $\begin{cases} x' = y \\ y' = \mu(1-x^2)y - x \end{cases}$, where $\mu = 0.7$

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- Idea: use bifurcation phenomena.

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- Consider the function g defined by

$$g(k,i) = \begin{cases} 0 & \text{if } TM_k \text{ stops in } \leq i \text{ steps with input } k \\ 1 & \text{otherwise.} \end{cases}$$

  Then $\sum_{i=1}^{\infty} \frac{g(k,i)}{2^i}$ is a computable number.
- The computable system given in polar coordinates

$$\begin{cases} r' = \left(r - \sum_{i=1}^{\infty} \frac{g(k,i)}{2^i}\right)(r-1) \\ \theta' = 1 \end{cases}$$

  has two periodic orbits if the Turing machine TM_k halts on input k, and has only one periodic orbit otherwise.

Work on progress with N. Zhong

The limit set $NW(f)$ is computable for structurally stable systems defined on the two-dimensional disk $D = [0,1]^2$.

- The set consisting of all fixed points/periodic orbits of f can be upper semi-computed by discretizing the space into small squares and by defining a transition function over a finite set. We can then compute the invariant sets of this discretization.
- Structural stability ensures that approximating a system by finite but arbitrarily accurate approximations will still provide meaningful results.
- We can then measure the "diameter" of the approximation to get a bound on its accuracy

A small detour: Hilbert's 16th problem

Hilbert's 16th Problem

Determine an upper bound on the maximum number of periodic orbits that a nth degree polynomial ODE

$$\begin{cases} x' = p_n(x, y) \\ y' = q_n(x, y) \end{cases}$$

in the plane can have (the original formulation also asks for the relative position of the periodic orbits).

Problem

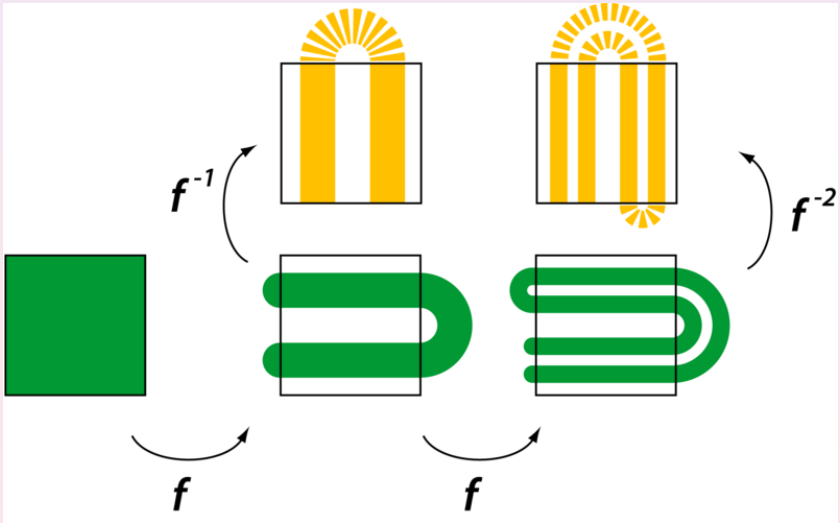
Can we uniformly compute the maximum number periodic orbits that a nth degree polynomial ODE can have?

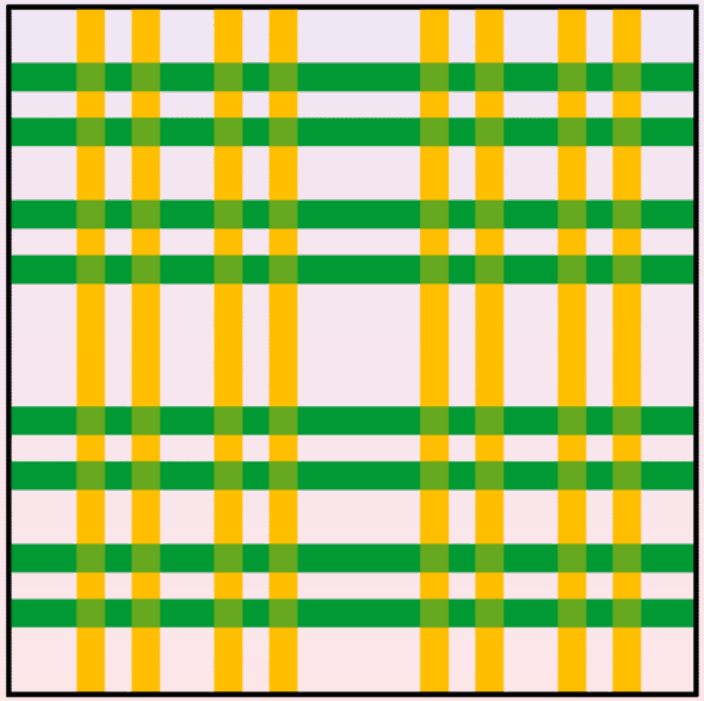
- Non-computability does not imply that no solution exists for Hilbert's 16th problem.

What about strange attractors?

Theorem (G., Zhong, Buescu '12)

The Smale Horseshoe is a computable (recursive) closed set.





Idea of the proof

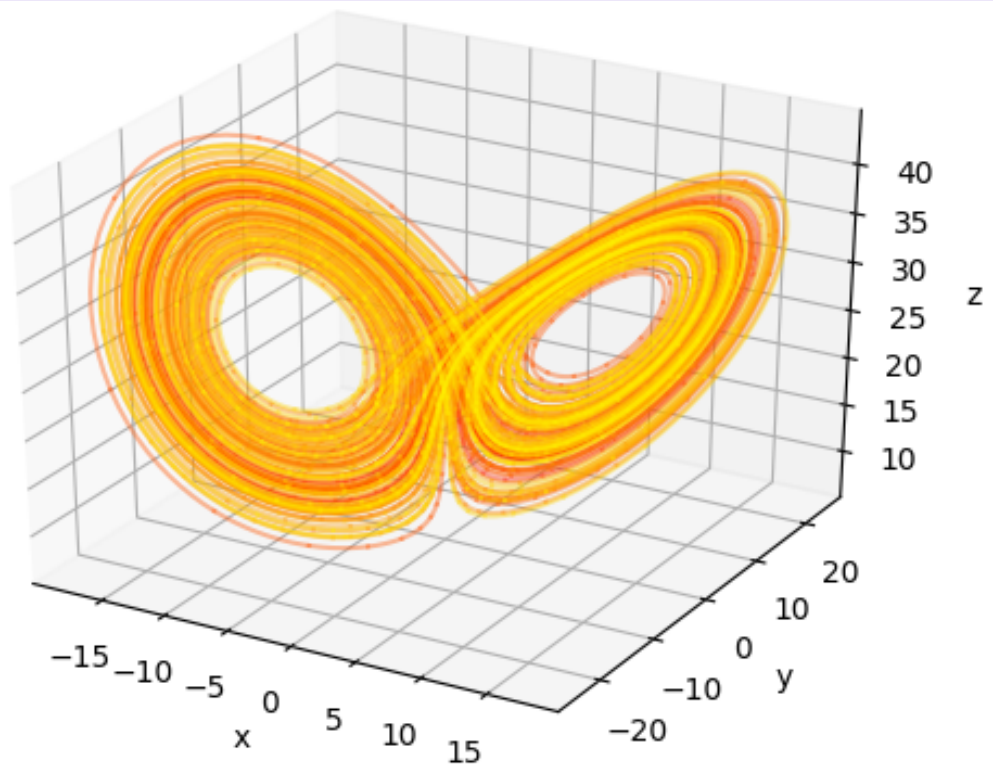
We show that the complement of Smale's horseshoe is computable by using the following fact (Zhong, 1996): An open subset $U \subseteq I$ is computable if and only if there is a computable sequence of rational open rectangles (having rational corner points) in I, $\{J_k\}_{k=0}^{\infty}$, such that

(a) $J_k \subset U$ for all $k \in \mathbb{N}$,

(b) the closure of J_k, $\bar{J}_k$, is contained in U for all $k \in \mathbb{N}$, and

(c) there is a recursive function $e : \mathbb{N} \to \mathbb{N}$ such that the Hausdorff distance $d(I \setminus \cup_{k=0}^{e(n)} J_k, I \setminus U) \leq 2^{-n}$ for all $n \in \mathbb{N}$.

The Lorenz attractor



$$\begin{cases} x' = \sigma(y - x) \\ y' = x(\rho - z) - y \\ z' = xy - \beta z \end{cases}$$

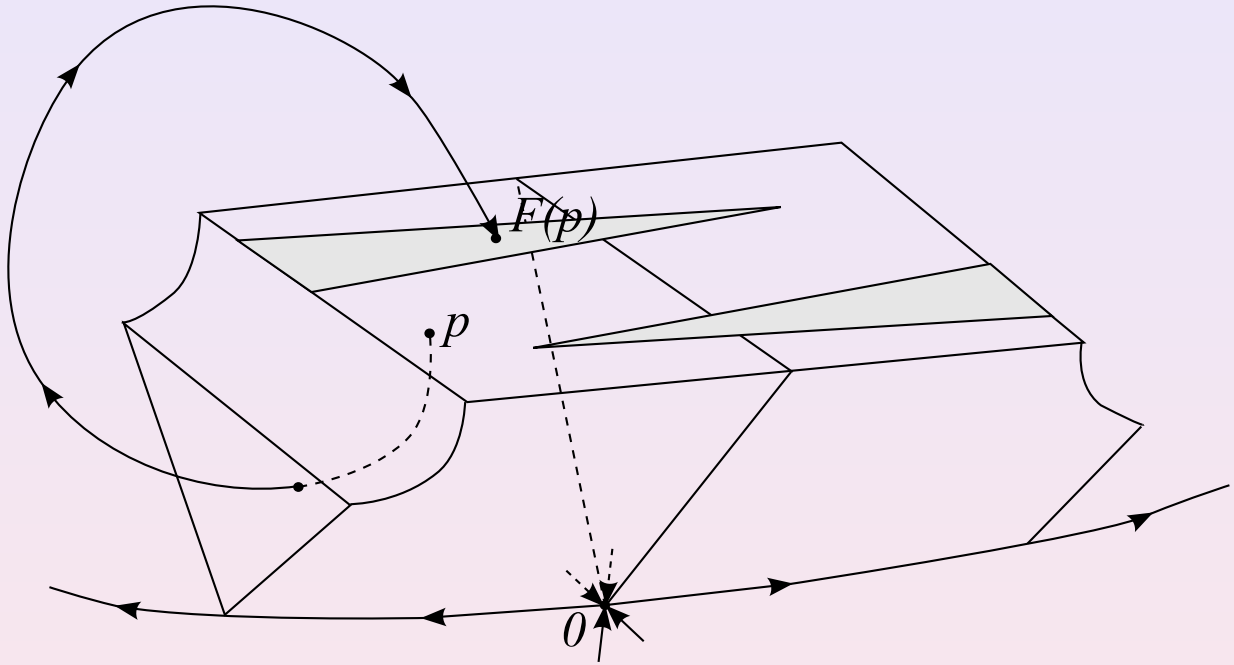
Classical values for parameters: $\sigma = 10, \rho = 28, \beta = 8/3$

The butterfly effect

- However the Lorenz system has sensitive dependence on initial conditions – i.e. it exhibits the "butterfly effect": the flutter of a butterfly's wing can ultimately cause a typhoon halfway around the world
- This term was coined by E. Lorenz since the Lorenz system represents a simplified mathematical model for atmospheric convection.

Some preliminaries

- In the late 1970s several authors suggested the use of geometrical Lorenz models to better understand the Lorenz attractor.
- Such models assume the qualitative behavior which was numerically observed on the Lorenz system.
- It was soon shown that geometrical Lorenz models have a strange attractor with properties compatible to those observed via numerical experiments.
- W. Tucker essentially showed in 2002 (using rigorous numerics and normal form theory) that the Lorenz system behaves like a geometric Lorenz model, thus supporting a strange attractor.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Theorem (G., Rojas, Zhong '18)

Let ϕ be the a (C^2) flow of some Lorenz geometric system. Then:

1. *The global attractor $\mathcal{A}$ of a geometric Lorenz flow ϕ is computable from a (C^2) name of ϕ.*
2. *The geometric Lorenz flow admits a physical measure which is computable from a (C^2) name of ϕ.*

What about the Lorenz attractor

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- Not yet!
- The problem has to do with the fact that we need a constructive version of Tucker's proof.
- In particular it is not enough to show that a foliation exists for the Lorenz attractor.
- We still have to show that a computable foliation of the Lorenz attractor exists which can be computably mapped into the standard foliation.
- Probably some combination of theory about foliations and rigorous numerics is needed to prove that result (to be done!)

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▶ But what if the system is analytic?

Recall that in analytic functions, local behavior determines global behavior $\Rightarrow$ no C^k gluing allowed, even if $k = +\infty$

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- Trajectories converge in a well-behaved manner towards the fixed point (hyperbolicity: trajectories converge exponentially fast to the fixed point)
- The system is analytic (no gluing tricks allowed)

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Then the resulting basin of attraction may not be computable

Idea behind the proof

- Simulate a Turing machine with an analytic map (use interpolation techniques, and allow a certain error in the simulation—the map can still simulate a Turing machine even if the initial point and/or the dynamics are constantly perturbed. Use special techniques to keep the error under control)

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- Then deciding which initial points will converge to the previous hyperbolic fixed point is equivalent to solving the Halting Problem

Thank you!