AN ACCURATE MULTI-REGIME SPH SCHEME FOR BAROTROPIC FLOWS

Low-Mach behaviour

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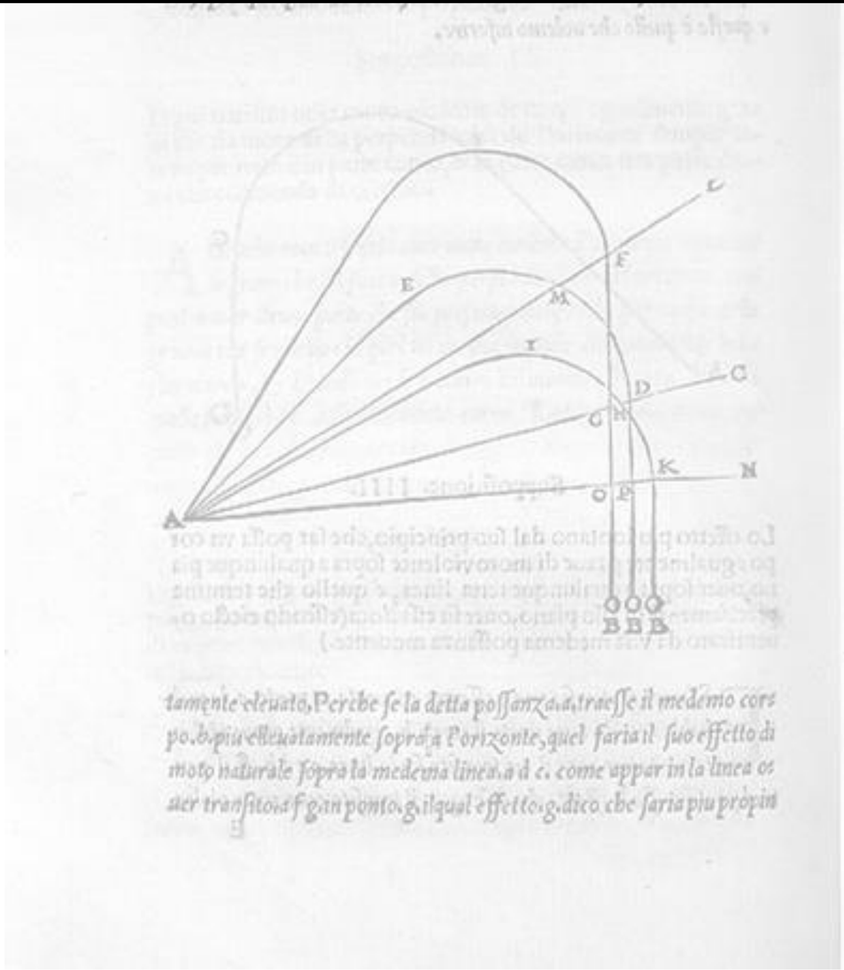


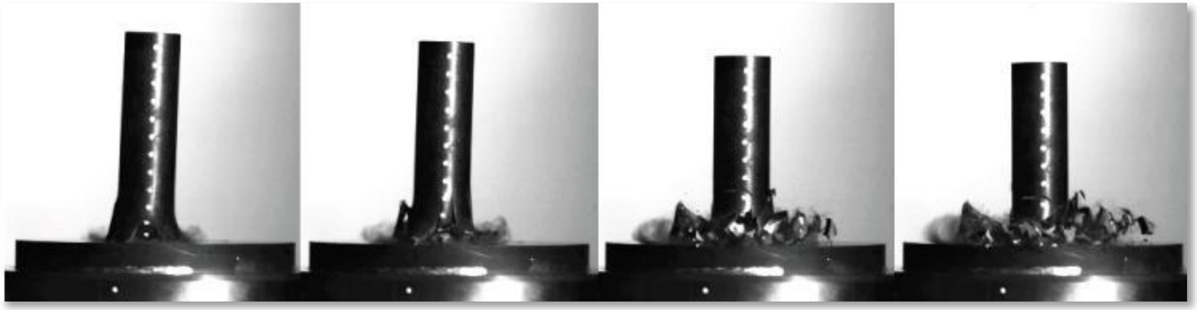
Tartaglia ballistic curves (edited in 1606)

Context

➢ Subject:

Numerical Mesh-less Methods *to solve* ***Fragmentation in Transient Dynamics****:*
Application to Aeronautics and Astronautics Structures

- ***Numerical Mesh-less Method***: Smoothed Particle Hydrodynamics
- ***Fragmentation in Transient Dynamics*** :
  - Material **cracking** process which could happen **simultaneously** at **multiple points**
  - Driven by events occurring at **high velocities** and during **short times**



Taylor Impact tests

Basics

SMOOTHED PARTICLE HYDRODYNAMICS

SPH basics | Literature

- Literature
  - Originally developed by Lucy [1] for **astrophysical problems** and by Gingold & Monaghan [2,3,4] for **hydrodynamic applications** (incompressible free surface flows)
  - Adapted by Benz [5,6] to the **solid dynamics**

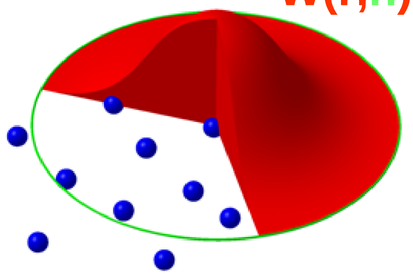
- Fluid Flows
  - Monaghan [4] proposed an **explicit** formulation based on a **Weakly-Compressible** assumption to handle incompressible flows (WCSPH)

[1] L. B. Lucy, A numerical approach to the testing of the fission hypothesis 82 (1977) 1013-1024.
[2] J. J. Monaghan, R. A. Gingold, Shock simulation by the particle method sph, Journal of Computational Physics 52 (1983) 374-389.
[3] J. J. Monaghan, Smoothed particle hydrodynamics 30 (1992) 543{574.
[4] J. Monaghan, Simulating free surface fows with sph, Journal of Computational Physics 110 (1994) 399{406.,
[5] W. Benz, Smooth Particle Hydrodynamics: A Review, Springer Netherlands, Dordrecht, 1990, pp. 269{288.,
[6] W. Benz, E. Asphaug, Impact simulations with fracture. i. method and tests, Icarus 107 (1994) 98{116.

SPH basics | Approximation & Conservation Law [7,8]

➢ Meshless Lagrangian Particle Method

- Computational domain discretized in **interpolation points** seen as **particles** interacting between each other and carrying material properties
- Interactions evaluated thanks to **approximation features**



➢ Smoothed Particle Approximation

- Set of **Moving Particles** $(x_i(t), \omega_i(t))_{i \in P}$
- **Regularizing Kernel** $W(r, h)$

2 spatial parameters: ***h*** *smoothing length* (half radius of the kernel support)
$\Delta \boldsymbol{x}$ *particle spacing*

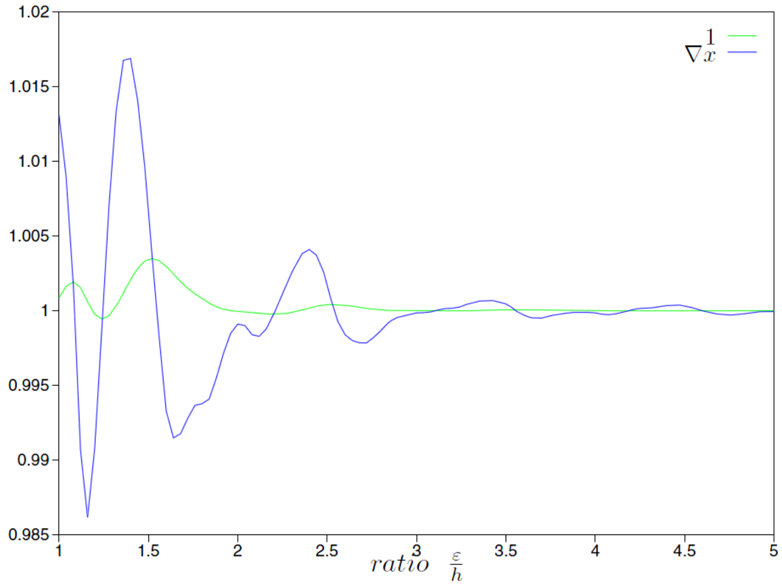
- Approximation of a function f: $\Pi^h(f)_i = \sum_{j \in P} \omega_j(t) f_j(t) W_{ij}(t)$
- Approximation of ∇f: $\nabla \Pi^h(f)_i = \sum_{j \in P} \omega_j(t) f_j(t) \nabla W_{ij}(t)$

With $$\begin{cases} W_{ij}(t) = W\left(x_i(t) - x_j(t), h\right) \\ \nabla W_{ij}(t) = grad_x W\left(x_i(t) - x_j(t), h\right) \end{cases}$$

[7] J. P. Vila, On particle weighted methods and SPH, M3AS, 1999

[8] N. Lanson, J.-P. Vila, SIAM Journal on Numerical Analysis 46 (4) (2008) 1912{1934.

SPH basics | Approximation & Conservation Law [7,8]



$\nabla \Pi^h f$ **does not exactly approximate** ∇f both on regular and non regular distributions

→ Depends on the ratio $\frac{\boldsymbol{h}}{\Delta \boldsymbol{x}}$

- Additional **derivative operators**

$$
\begin{cases}
D_h f_i = & \nabla \left(\Pi^h f\right)_i - f_i \nabla \left(\Pi^h 1\right)_i \\
= & \sum_{j \in P} \omega_j(t) \left(f_j(t) - f_i(t)\right) \nabla W_{ij}(t) \\
D_h^* f_i = & \nabla \left(\Pi^h f\right)_i + f_i \nabla \left(\Pi^h 1\right)_i \\
= & \sum_{j \in P} \omega_j(t) \left(f_j(t) + f_i(t)\right) \nabla W_{ij}(t)
\end{cases}
$$

- Consistency order increase using **Renormalized Kernels** $A_{ij} = B_{ij} \boldsymbol{\nabla} W_{ij}$ where B_{ij} is the symmetric renormalization matrix

SPH basics | Approximation & Conservation Law [7,8]

Approximation Properties

- Choosing kernels such that $W(x,h) = W(-x,h)$ and $\boldsymbol{\nabla} W_{ji} = -\boldsymbol{\nabla} W_{ij}$ ensures that $-D_h^*$ is the **adjoint operator** of D_h
- D_h **strongly approximates** ∇ in a sense that for a regular function φ

$$\sup_{i \in P} \|D_h \varphi_i - \nabla \varphi_i\| = 0 \text{ as } h \text{ and } \Delta x \to 0$$

Conservation Laws

- $v \in \mathbb{R}^d$ a regular velocity field
- L_v the transport operator
- $\Phi \in \mathbb{R}^p$ the conserved variables vector
- F the flux vector
- S the source term

$$t \in \mathbb{R}^+, \; x \in \mathbb{R}^d$$

$$L_v(\Phi) + \operatorname{div} F(x,t,\Phi) = S(x,t,\Phi)$$

$$L_v : \Phi \rightarrow \frac{\partial \Phi}{\partial t} + \sum_{l=1,d} \frac{\partial}{(\partial x^l)} (v^l . \Phi)$$

SPH basics | Discretization [7,8]

➢ Discretization

- Weak Formulation: $\forall \varphi \in \mathcal{C}_0^p\left(\mathbb{R}^d \mathrm{x} \mathbb{R}^{+,*}\right)$

$$\int_{\mathbb{R}^d \mathrm{x} \mathbb{R}^+} \left[\Phi . L_v^*(\varphi) + F(x,t,\Phi).\boldsymbol{\nabla}\varphi + S.\varphi\right] dxdt = 0$$

With $-L_v^*$ the **adjoint operator** of L_v such that: $L_v^* : \varphi \rightarrow \frac{\partial \varphi}{\partial t} + \sum_{l=1,d} v^l \frac{\partial \varphi}{\partial x^l}$

- Discrete Scalar Product : $(.,.)_h^t : (\varphi, \psi) \rightarrow \int_{\mathbb{R}^+} \left(\sum_i \omega_i \varphi_i \psi_i\right) dt$

- Discretized Weak Formulation:

$$\forall \varphi \in \mathcal{C}_0^p\left(\mathbb{R}^d \mathrm{x} \mathbb{R}^{+,*}\right), \left(\Phi, L_v^*(\varphi)\right)_h^t + \sum_{\alpha=1,\ldots,d} \left(F^\alpha(\Phi), \nabla^h \varphi\right)_h^t + \left(S + R_h(\Phi), \varphi\right)_h^t = 0$$

With $R_h(\Phi)$ the **residual** and ∇^h the **derivative operator** approximating ∇

- Choosing $\nabla^h = D_h$ gives the following **discrete scheme**

$$\frac{d}{dt}\left(\omega_i \Phi_i\right) + \omega_i \sum_{\alpha=1,\ldots,d} D_h^{\alpha,*}\left(F^\alpha\right)_i = \omega_i \left(S_i + R_h(\Phi)_i\right)$$

SPH basics | Discretization [7,8]

➢ Properties of the Discrete Scheme

✓ **Conservative** in a sense that $\frac{d}{dt}\left(\int_{\mathbb{R}^d} \Phi dx\right) = \int_{\mathbb{R}^d} S dx$

According to the discrete scheme and considering antisymmetric residuals we get on the whole particle domain the discrete version of the expected property

$$\frac{d}{dt}\left(\sum_{i \in P} \omega_i \Phi_i\right) = \sum_{i \in P} \omega_i S_i$$

✓ **Weakly Consistent** with the conservation law according to a Lax Wendroff like theorem while

$$\forall \varphi \in \left[\mathcal{C}_0\left(\mathbb{R}^d \text{x} \mathbb{R}^{+,*}\right)\right]^p, lim_{h \text{ and } \Delta x \to 0}\left(R_h\left(\bar{\Phi}^{\triangle}\right), \varphi\right)_h^t = 0$$

Literature

RESEARCH AXIS I

Research Axis I

❑ **SPH-ALE Versus Finite Volume**: Vila M3AS 1999 [7]

➢ Combination of **Eulerian** and **Lagrangian** descriptions

- Eulerian : *Large deformations*
- Lagrangian : *Interface tracking*

→ **Eulerian description** recovered by choosing: $v_0 = 0$

→ **Lagrangian description** recovered by choosing: $v_0 = v$

➢ ALE Formulation for Euler Equations

$$L_{v_0}(\Phi) + div(F_E(\Phi) - \Phi \otimes v_0) = 0$$

- v_0 the **arbitrary** ALE velocity
- L_{v_0} the transport operator
- Φ the conserved variables vector
- F_E the Eulerian flux vector

$$F_E(\Phi) = \begin{pmatrix} \rho v_x & \rho v_y & \rho v_z \\ \rho v_x^2 + p & \rho v_y v_x & \rho v_z v_x \\ \rho v_x v_y & \rho v_y^2 + p & \rho v_z v_y \\ \rho v_x v_z & \rho v_y v_z & \rho v_z^2 + p \end{pmatrix}$$

$$\Phi = \left(\rho, \rho v_x, \rho v_y, \rho v_z\right)^T$$

[7] J. P. Vila, On particle weighted methods and SPH, M3AS, 1999 ,

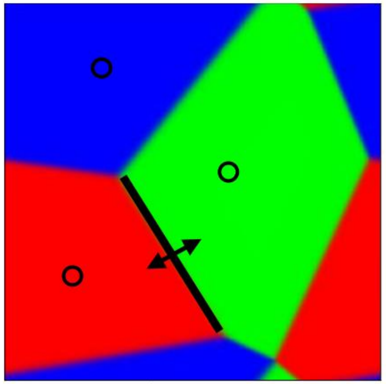
Research Axis I

➢ Finite Volume Analogy [7,8]

- Interaction between 2 particles *i* and *j*
- Consider the following **Riemann Problem** at the interface $x_{ij} = \frac{1}{2}(x_i + x_j)$

$$\begin{cases} \frac{\partial}{\partial t}(\Phi) + \frac{\partial}{\partial x}((F_E(\Phi).n_{ij} - v^0(x_{ij},t).n_{ij}\Phi)) = 0 \\ \Phi(x,0) = \begin{cases} \Phi_i \text{ if } x < x_{ij} \\ \Phi_j \text{ if } x > x_{ij} \end{cases} \end{cases}$$

With $n_{ij} = \frac{A_{ij}}{||A_{ij}||}$



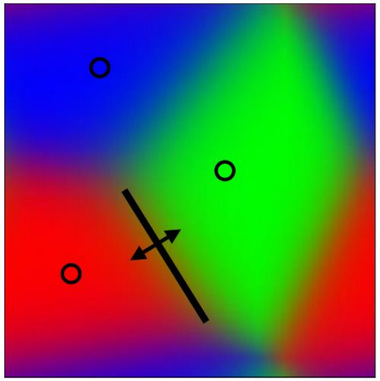
Unstructured / Moving−Mesh Methods

- Reasonable choice for the **flux** $g_E(n_{ij}, \Phi_i, \Phi_j)$ is

$$\begin{cases} \lambda_{ij}^0 = v^0(x_{ij},t).n_{ij}, \\ \Phi_{ij}(\lambda_{ij}^0) = \Phi_E(\lambda_{ij}^0; \Phi_i, \Phi_j), \\ G_E(\Phi_i, \Phi_j) = F_E(\Phi_{ij}(\lambda_{ij}^0)) - v^0(x_{ij},t) \otimes \Phi_{ij}(\lambda_{ij}^0), \\ g_E(n_{ij}, \Phi_i, \Phi_j) = G_E(\Phi_i, \Phi_j).n_{ij}. \end{cases}$$

Giving

$$\begin{cases} \frac{d}{dt}(x_i) = v^0(x_i,t) \\ \frac{d}{dt}(w_i) = w_i \mathrm{div}(v^0(x_i,t)) \\ \frac{d}{dt}(w_i\Phi_i) + w_i \sum_{j \in P} w_j 2G_E(\Phi_i, \Phi_j)\vec{\nabla}_i W_{ij} = 0 \end{cases}$$



New Meshless Methods Here (MFV, MFM)

Research Axis I

- Regularizing Technics

  - Move particles with smooth velocity as in Monaghan's **XSPH** [3,9]
  - **Smart choice** of v_0 increases both **stability** and **robustness** by preventing the formation of anisotropic spatial particle distribution

- Riemann Solvers

  - **Global solution** built on the combination of solutions to **local Riemann Problems**
  - **MUSCL HLLC** Riemann Solver used as a reference [10]

✓ Increases the **accuracy**
❖ Increases the **solving complexity**
❖ Limitations in **Low-Mach regimes**

[9] J. Monaghan, On the problem of penetration in particle methods, JCP 82 (1) (1989) 1-15
[3] J. Monaghan, Smoothed Particle Hydrodynamics 30 (1992) 543-574
[10] E. F. Toro, Riemann Solvers and numerical methods for fluid dynamics: a practical introduction, Springer, 1997

Research Axis I

❑ Finite Volume Low-Mach Scheme:

➢ Stabilizing **velocity term** proportional to the **Pressure gradient**

- **Grenier, Vila, Villedieu [11]** : *Two-fluid free surface flows*

→ ***SEMI IMPLICIT*** formulation

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + div\big(\rho \otimes (\boldsymbol{v} - \gamma h \nabla P)\big) = 0 \\ \dfrac{\partial \rho \boldsymbol{v}}{\partial t} + div\big(\rho \boldsymbol{v} \otimes (\boldsymbol{v} - \gamma h \nabla P)\big) + \nabla P = 0 \end{cases}$$

- **h** is the mesh size (representative of the volume)
- let $\boldsymbol{v_\gamma} = \boldsymbol{\gamma h \nabla P}$ the stabilizing velocity term

- **Couderc, Duran, Vila [12]** : *Multilayer Shallow Water Model with Density stratification*

→ ***EXPLICIT*** formulation

[11] N. Grenier, J. P. Vila, P. Villedieu, An accurate low-Mach scheme for a compressible two-fluid model applied to free surface flows, JCP 252 :1-19, 2013,
[12] F. Couderc, A. Duran, J. P. Vila, An explicit asymptotic preserving low Froude scheme for the multilayer shallow water model with density stratification, JCP, 2017

$$
\varepsilon_T = \varepsilon_k + \varepsilon_p = f_0\left(\int_{\widehat{V}} \widehat{W}\, ds^3 - 1\right) + h\frac{\partial f}{\partial x_j}\bigg|_{\underline{x_0}} \int_{\widehat{V}} s_j \widehat{W}\, ds^3 + \frac{h^2}{2}\frac{\partial^2 f}{\partial x_j x_k}\bigg|_{\underline{x_0}}
$$

$$
\cdot \int_{\widehat{V}} s_j s_k \widehat{W}\, ds^3 + h\frac{\partial f}{\partial x_j}\bigg|_{\underline{x_0}} \sum_h \widehat{W}_h \delta_{hj} (\omega_h)^{\frac{4}{3}} l_{hj} - h\frac{\partial f}{\partial x_j}\bigg|_{\underline{x_0}} \sum_h \frac{\partial \widehat{W}_h}{\partial s_k}\bigg|_{\underline{x_0}}
$$

$$
\cdot \left[\delta_{jk}\left(\frac{(\omega_h)^{\frac{1}{3}} l_{hj}}{12}\right) - \delta_{hk}\overline{s_{hj}}\right](\omega_h)^{\frac{4}{3}} l_{hk} - \frac{f_0}{2}
$$

$$
\times \sum_h \cdot \left\{\frac{\partial^2 \widehat{W}_h}{\partial s_j s_k}\bigg|_{\underline{x_0}} \left(\delta_{jk}\frac{l_{hj}}{12} + \delta_{hj}\delta_{hk} l_{hk}\right)(\omega_h)^{\frac{1}{3}} - 2\frac{\partial \widehat{W}_h}{\partial s_j}\bigg|_{\underline{x_0}} \delta_{hj}\right\}(\omega_h)^{\frac{4}{3}} l_{hj}
$$

$$
- \frac{h^2}{2}\frac{\partial^2 f}{\partial x_j x_k}\bigg|_{\underline{x_0}} \sum_h \widehat{W}_h \left\{\delta_{jk}\frac{l_{hj}^2 (\omega_h)^{\frac{2}{3}}}{12} - \left(\delta_{hj}\delta_{hk} l_{hj} l_{hk} (\omega_h)^{\frac{2}{3}}\right.\right.
$$

$$
\left.\left. + \overline{s}_{hj}\delta_{hj} l_{hj} (\omega_h)^{\frac{1}{3}} + \overline{s}_{hk}\delta_{hk} l_{hk} (\omega_h)^{\frac{1}{3}}\right)\right\}(\omega_h) + \ldots
$$

$$
\cong \varepsilon_{k1} + \varepsilon_{k2} + \varepsilon_{k3} + \varepsilon_{p1} + \varepsilon_{p2} + \varepsilon_{p3} + \varepsilon_{p4}
$$

Theory & Validation

IMPROVED SMOOTHED PARTICLE HYDRODYNAMICS

γ-SPH-ALE | Theory

➢ Discretizing Approach

- Conservation Law $$L_{v_0}(\Phi) + \mathrm{div}(F_E(\Phi) - \Phi \bigotimes v_0) = S$$

- Equation Set $$\begin{cases} \frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_0\left(\boldsymbol{x}_i, t\right) \\ \frac{d\omega_i}{dt} = \omega_i \mathrm{div}\left(\boldsymbol{v}_0\left(\boldsymbol{x}_i, t\right)\right) \\ \frac{d}{dt}\left(\omega_i \Phi_i\right) + \omega_i \sum_{\alpha=1,..,d} \nabla^h \left(F_E^\alpha\left(\Phi\right) - v_0^\alpha . \Phi\right)_i = 0 \end{cases}$$

→ The idea is to **choose ∇^h** such that the LHS term of the following equation can be recovered by generating **conservative** and **consistent residuals**

$$\frac{d}{dt}\left(\omega_i \Phi_i\right) + \omega_i \sum_{\alpha=1,..,d} D_h^{\alpha,*}\left(F^\alpha\right)_i = \omega_i\left(S_i + R_h(\Phi)_i\right)$$

γ-SPH-ALE | Theory

➤ <u>Non linear stability analysis</u>: Similarly to **Grenier, Vila, Villedieu [11]** , **Lavalle, Vila et al. [13] and [12]**

→ The scheme has to be **conservative**, **robust**, **stable** and **consistent**

- Conservation

We can show that

$$\frac{d}{dt}\left(\sum_{i\in P}\omega_i\Phi_i\right)=0$$

- Robustness

Enforcing the condition

$$\Delta t \leq \min_{n,i}\left(\Delta t_{\rho}^{i,n}\right)$$

Insures that

$$\forall t,\ \rho_i\left(t\right)>0$$

While $\rho_i^0>0$

[11] N. Grenier, J. P. Vila, P. Villedieu, An accurate low-Mach scheme for a compressible two-fluid model applied to free surface flows, JCP 252 :1-19, 2013,

[13] G. Lavalle, J.-P. Vila, G. Blanchard, C. Laurent, F. Charru, A numerical reduced model for thin liquid flms sheared by a gas ow, J. Comput. Phys. 301 (2015) 119-140.

γ-SPH-ALE | Theory

- Stability

→ We want a **control** on the total energy noted $\mathcal{E}$. The idea is to complete the following process:

1. An **energy balance** is performed on the scheme, and the *production terms* coming from the *kinetic* and *potential energy* are exhibited

2. These terms are *estimated* and *gathered* to provide a **global estimation of the energy production**

3. A *negativity condition* on one part of the *production estimate* provides **stability conditions** on γ and α

4. The remaining production terms are evaluated under such stability conditions and provide a **finite energy bound** noted $\mathcal{E}_T$

γ-SPH-ALE | Theory

As a result of this estimation process, under the following stability conditions

$$\Delta t \leq \min_n \left(\Delta t_\alpha^n, \Delta t_\gamma^n\right) \text{ and } \begin{cases} \gamma_{min} \leq \gamma \leq \gamma_{max} \\ \alpha_{min} \leq \alpha \leq \alpha_{max} \end{cases}$$

We can show that

$$\forall 0 \leq t \leq T, \exists \mathcal{E}_T \geq 0,\ \mathcal{E}(t) \leq \mathcal{E}_T$$

Ensuring the **bounded behavior** of the scheme total energy and gives the expected **stability property**.

- Regardless the **expression of v_0** but depends on a **geometrical constant C**
- **Not optimal** du to Cauchy Schwarz' inequality and 1st order time integrator

→ In a **Weakly-Compressible** and **Quasi-Lagrangian** framework we have:

$$\begin{cases} \Delta t = \frac{0.15}{C} \frac{h}{c_0} \\ \gamma \in [0.68, 0.98] \\ \alpha \in [0.5, 1.16] \end{cases}$$

γ-SPH-ALE | Theory

- Consistency [7]

→ **Weak Consistency** achieved thanks to a Lax-Wendroff like theorem

(1) $\forall \varphi \in \left[\mathcal{C}_0\left(\mathbb{R}^d \mathrm{x} \mathbb{R}^{+,*}\right)\right]^p \, sup_{i \in P} \left\| \nabla^h \varphi_i - \nabla \varphi_i \right\| \rightarrow 0 \text{ as h and } \Delta x \rightarrow 0$

(2) $\forall \varphi \in \left[\mathcal{C}_0\left(\mathbb{R}^d \mathrm{x} \mathbb{R}^{+,*}\right)\right]^p, lim_{h \text{ and } \Delta x \rightarrow 0} \left(R_h\left(\bar{\Phi}^{\triangle}\right), \varphi\right)_h^t = 0$

→ We have the **current** discretization:

$$\frac{d}{dt}(\omega_i \Phi_i) + \omega_i \sum_{\alpha=1,\ldots,d} \nabla_h^{\alpha,*}(F^\alpha)_i = 0$$

No convergence properties

→ Which can be write has:

Convergence previously introduced

$$\frac{d}{dt}(\omega_i \Phi_i) + \omega_i \sum_{\alpha=1,\ldots,d} D_h^{\alpha,*}(F^\alpha)_i = \omega_i R_h(\Phi)_i$$

→ It remains to enforce **(2)**

[7] J. P. Vila, On particle weighted methods and SPH, M3AS, 1999 ,

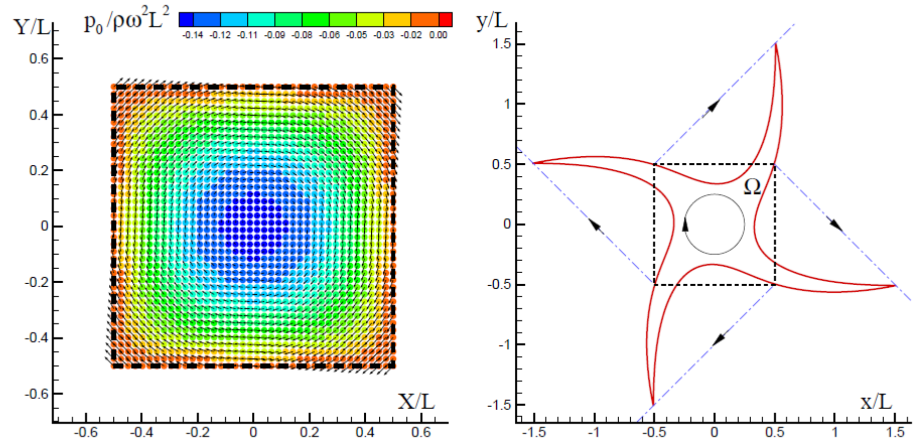
γ-SPH-ALE | Validation

❑ Rotating Square Patch of fluid: Colagrossi [14]

➢ Reference Work

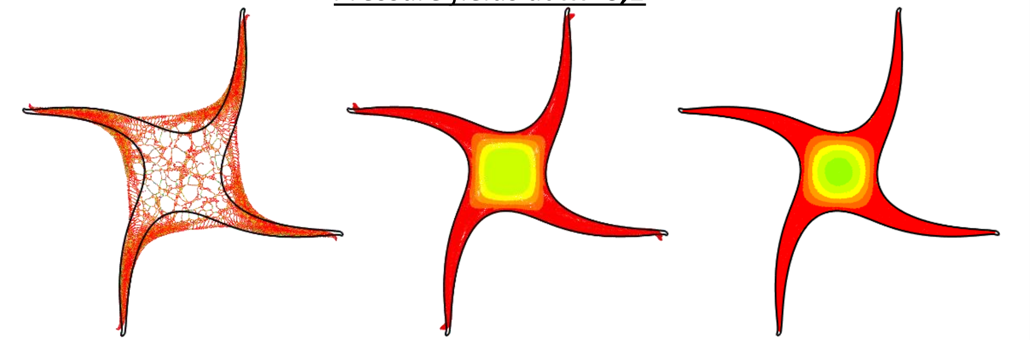
$$\begin{cases} v_x = +\omega y \\ v_y = -\omega x \end{cases}$$

- Initial Velocity & Pressure fields
- Weakly-Compressible

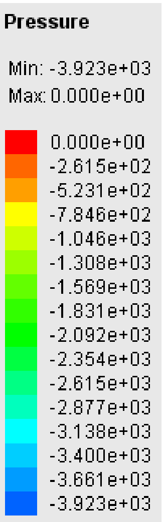


➢ In Practice

*Pressure fields at **M=0,1***



Classical SPH *ALE SPH Riemann* *γ-SPH-ALE*

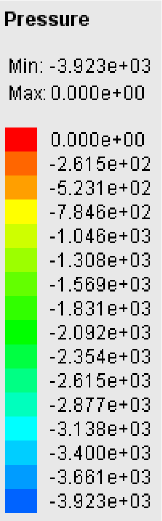


[14] A. Colagrossi, A meshless lagrangian method for free-surface and interface fows with fragmentation, These, Universita di Roma.

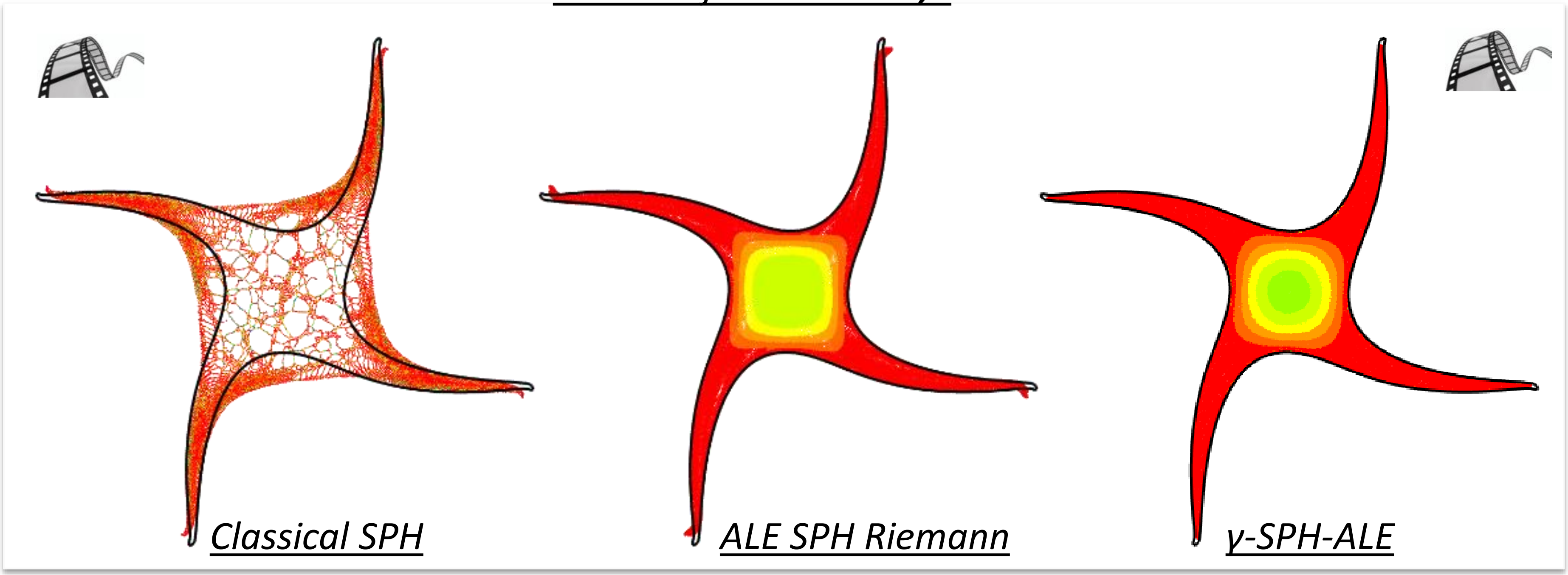
γ-SPH-ALE | Validation

- **Rotating Square Patch of fluid:**
  - In Practice

    → Comparison with a FE solution



*Pressure fields at **M=0,1***



γ-SPH-ALE | Validation

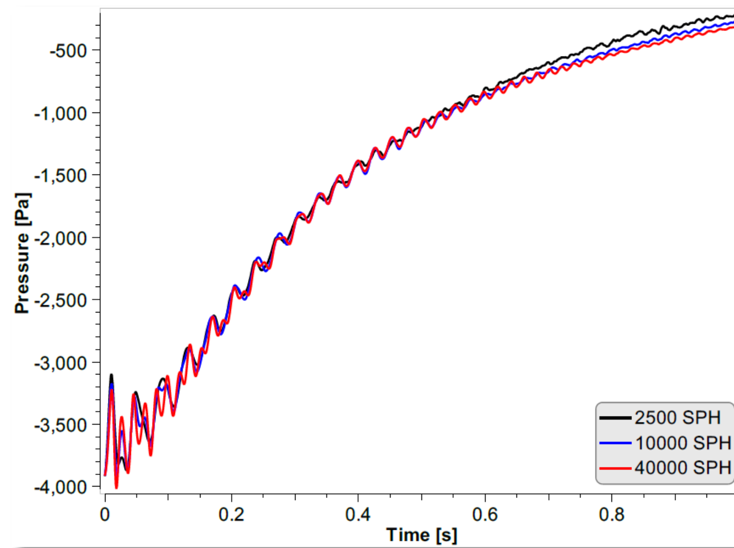
➢ Convergence

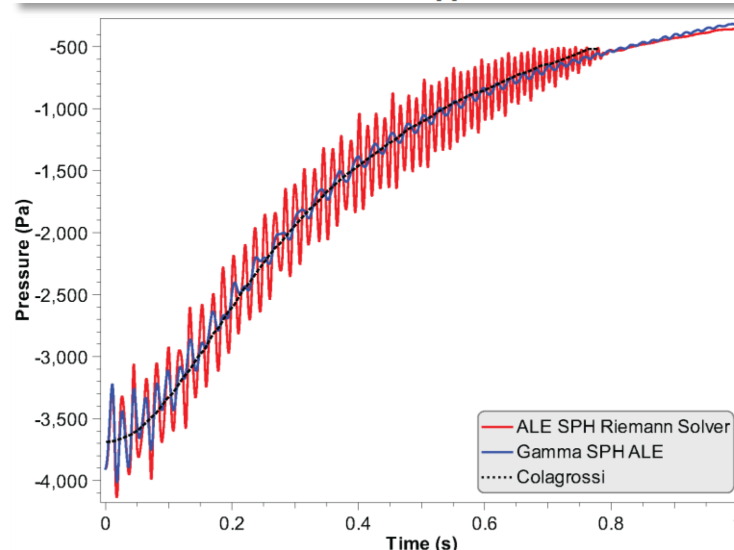
- Pressure at the center of the patch for 3 different **initial particle spacing** Δx
- **γ-SPH-ALE** : **Damping** of the oscillations
- ALE SPH Riemann Solver **DOES NOT** converge

➢ Acoustic

- Comparison with the ALE SPH Riemann Solver
- Remaining oscillations corresponding to the acoustic part of the flow: **Weakly-Compressible**

Pressure at the center of the Patch

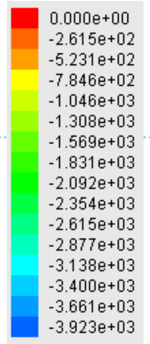




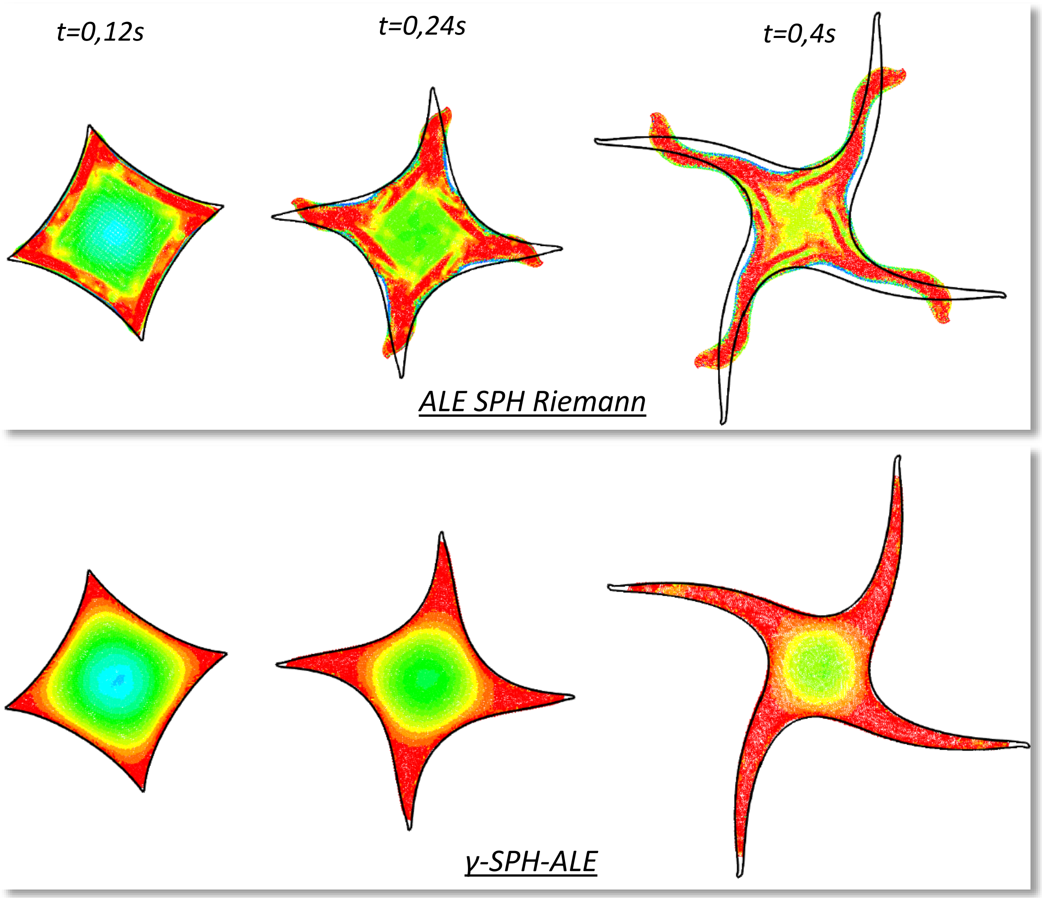
γ-SPH-ALE | Validation

➢ Low-Mach Behavior

- Riemann Solver **too dissipative** if the Mach is decreased in spite of particle refinement
- γ-SPH-ALE **stays in agreements** with the FE solution



*Pressure fields at **M=0,01***



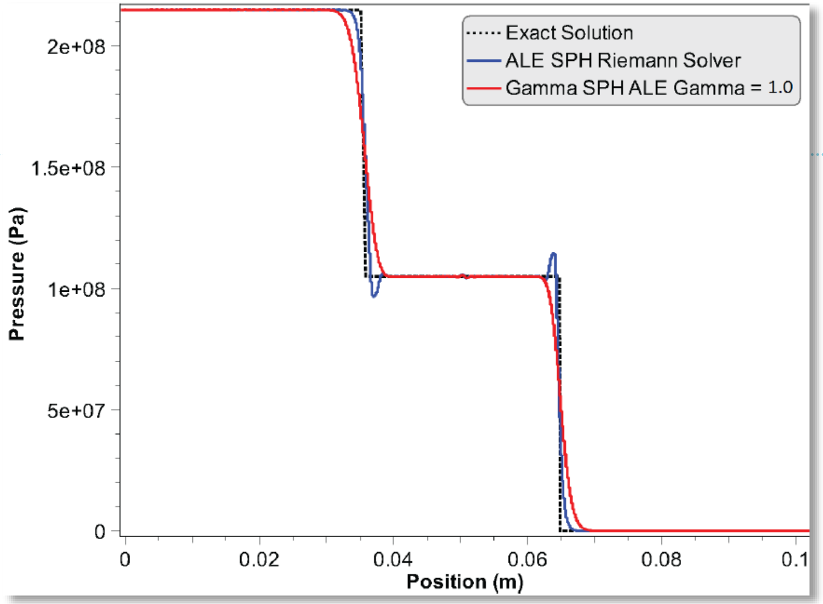
γ-SPH-ALE | Validation

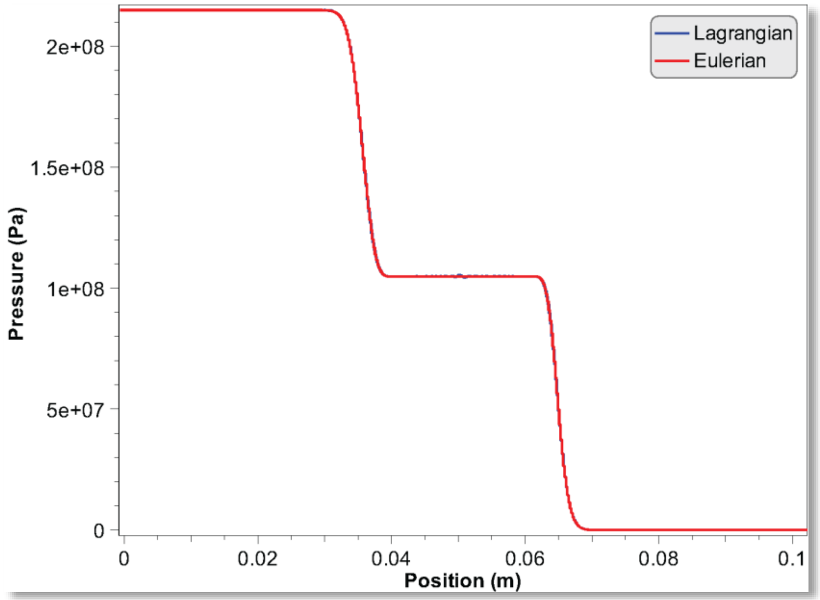
❑ 2D Isentropic shock tube:

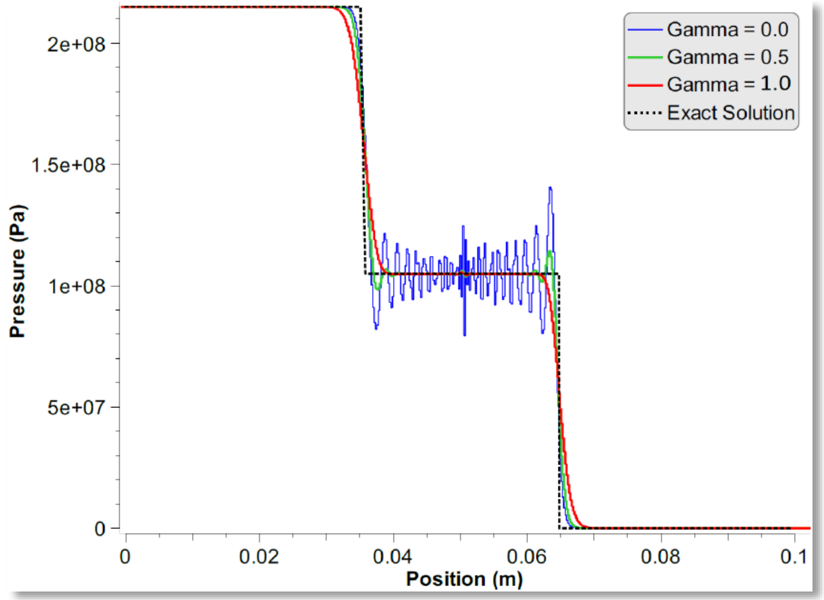
See **Marongiu [15]** and **Leduc [16]** for similar tests to evaluate SPH Riemann Solver

- Initial state:

$$\begin{cases} \rho_L = 1100 \\ v_L = 0 \end{cases} \qquad \begin{cases} \rho_R = 1000 \\ v_R = 0 \end{cases}$$







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Theory & Validation

MULTIPHASE SPH

Research Axis II

❑ **Multi-Fluid SPH**: [2,7,8,9]

➢ Volume Fraction Formulation

- An **equilibrium** between **all phases** for each SPH particle
- α_k the **volume fraction** of fluid k
- φ_m a **mixture variable**

$$1 = \sum_k \alpha_k$$

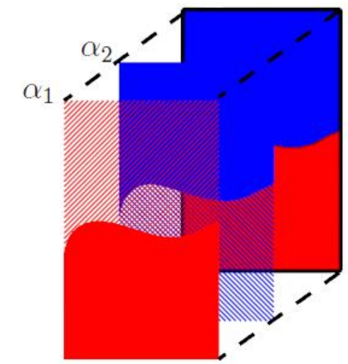
$$\varphi_m = \sum_k \alpha_k \varphi_k$$

➢ Bi-Fluid particular case

- α the **volume fraction** of fluid 1 giving: $\alpha_1 = \alpha$, $\alpha_2 = 1 - \alpha$
- Same equation set using the **mixture variables**: ω, ρ, v
- **Evolution** of $\tilde{\rho}_1 = \alpha\rho_1$ giving $\tilde{\rho}_2 = \rho - \tilde{\rho}_1$
- **Pressure equilibrium** giving α

$$\frac{\partial \tilde{\rho}_1}{\partial t} + div(\tilde{\rho}_1 \boldsymbol{v}) = 0$$

$$p_1\left(\frac{\tilde{\rho}_1}{\alpha}\right) = p_2\left(\frac{\tilde{\rho}_2}{1-\alpha}\right)$$



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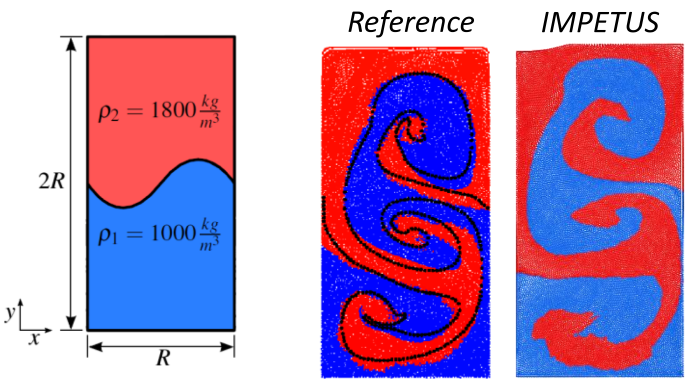
γ-SPH-ALE | Validation

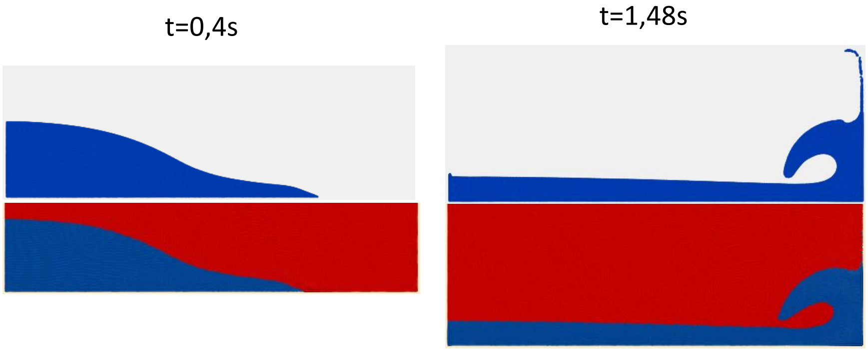
❑ Bi-Fluid γ-SPH-ALE :

- Rayleigh-Taylor Instability

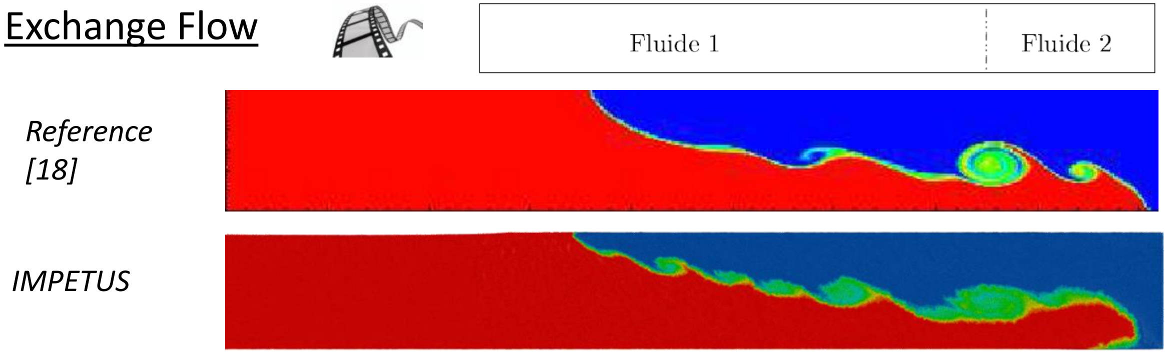


- 2D Dam Break

→ Comparison between γ-SPH-ALE **Monofluid** (top) and **bi-Fluid** (bottom)



- Exchange Flow



Conclusion & Prospects

- Implementation of a **mathematical and mechanical framework** handling the **dynamic fragmentation via meshless methods**

- Hydrodynamics Context
  - ✓ New meshless scheme **γ-SPH-ALE**
    - ALE formulation
    - FV Low-Mach scheme
  - ✓ **Non linear** stability analysis
    - Calibrated stabilizing parameters
    - CFL conditions
  - ✓ **Multiphase** formulation
    - Two Phase

- Solid Dynamics Context
  - ✓ **Stability** when dealing with solid materials
    - Purely Lagrangian
    - HVI cases
    - Monolithic code

Prospects

- **Multiphase** formulation
  - Under Water Explosions
  - Buried Mine Blast
- **Fragmentation** process
  - Fracture treatment
  - Warhead fragmentation