An asymptotic preserving scheme for the quasi neutral Euler-Boltzmann system in the drift regime

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Outline

1. Physical introduction
   - Context and motivation
2. The Euler-Lorentz-Boltzmann system
3. A non linear finite volume schemes for the parallel dynamic
4. A linear iterative scheme to approach the non linear one
5. Numerical results
6. Conclusion and perspectives

Magnetic confinement fusion

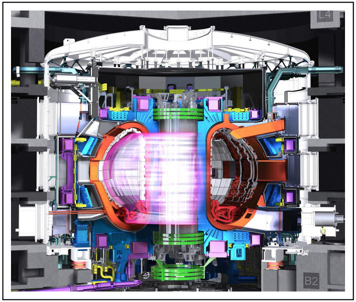

Figure: Tokamak ITER. Source : www.iter.org

- Imposed external magnetic field.
- Particles trajectories enroll along the magnetic field lines.
- An important Small parameter: $\sqrt{\varepsilon}$ (normalized ionic Larmor radius).

- Strongly magnetized (kinetic or fluid) plasma simulations ($\varepsilon \to 0$) may be very costly.
- Two approach for a model $\mathcal{P}_\varepsilon$:
    - Hilbert expansion of the solutions to $\mathcal{P}_\varepsilon$ yields new models consistent with $\mathcal{P}_0$ but increases the number of unknowns.
    - AP schemes : discretize the original model $\mathcal{P}_{\varepsilon,\Delta t,\Delta x}$ and design a scheme that is consistent with $\mathcal{P}_{0,\Delta t,\Delta x}$ and stable independently on ε.

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1 Physical introduction

2 The Euler-Lorentz-Boltzmann system
- The mathematical model
- The numerical difficulties
- The strategy

3 A non linear finite volume schemes for the parallel dynamic

4 A linear iterative scheme to approach the non linear one

5 Numerical results

6 Conclusion and perspectives

Quasi-neutral plasma with adiabatic electron response

Assumptions

- Quasi-neutral plasma.
- Boltzmannian electrons.

$d \in \{1,2,3\}$, $\Omega \subset \mathbb{R}^d$, unknown $(n, \mathbf{u}, w) : (t, \mathbf{x}) \in [0, T) \times \Omega \to \mathbb{R}^+ \times \mathbb{R}^3 \times \mathbb{R}^+$

$$
\begin{cases}
\partial_t n + \nabla \cdot (n\mathbf{u}) = 0 \text{ in } (0,T) \times \Omega, \\
\partial_t (nu) + \nabla \cdot (n\mathbf{u} \otimes \mathbf{u}) + \frac{1}{m} \nabla p = \frac{q}{m} \underbrace{(n\mathbf{E} + n\mathbf{u} \times \mathbf{B})}_{\text{Lorentz-Force}} \text{ in } (0,T) \times \Omega, \\
\partial_t (w) + \nabla \cdot (\mathbf{u}(w+p)) = qn\mathbf{u} \cdot \mathbf{E} \text{ in } (0,T) \times \Omega, \\
w = \frac{mn|\mathbf{u}|^2}{2} + \frac{1}{\gamma - 1} p, \\
E = -\nabla \phi \text{ where } \phi = \frac{k_b T_e}{q} \ln\left(\frac{n}{n_c}\right) \to \underbrace{n = n_c e^{\frac{q\phi}{k_b T_e}}}_{\text{Boltzmann}}
\end{cases}
$$

where $\gamma - 1 = \frac{2}{d}$, $m > 0$ is the ion mass, $q > 0$ is the electric charge, $n_c > 0$ is given, and $\mathbf{B}$ is a constant magnetic field.

Dimensionless model

Still denote $(n, \mathbf{u}, w)$ the dimensionless unknown.

$$\begin{cases} \partial_t n + \nabla \cdot (n\mathbf{u}) = 0 \text{ in } (0,T) \times \Omega, \\ \partial_t (n u) + \nabla \cdot (n\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\varepsilon} \nabla p = \frac{1}{\varepsilon}(n\mathbf{E} + n\mathbf{u} \times \mathbf{b}) \text{ in } (0,T) \times \Omega, \\ \partial_t (w) + \nabla \cdot (\mathbf{u}(w+p)) = n\mathbf{u} \cdot \mathbf{E} \text{ in } (0,T) \times \Omega, \\ w = \frac{\varepsilon n |\mathbf{u}|^2}{2} + \frac{1}{\gamma - 1} p, \\ E = -\nabla \phi \text{ where } \phi = T_e \ln\left(\frac{n}{n_c}\right) \end{cases}$$

$\sqrt{\varepsilon} > 0$ is the ionic Larmor radius and also the ions Mach number, $|\mathbf{b}| = 1$ and $T_e > 0$ is a normalized electronic temperature. Remark that,

$$n\mathbf{u} \cdot E = -T_e n\mathbf{u} \cdot \nabla \ln(n) = -T_e \nabla \cdot (n\mathbf{u} \ln(n)) - T_e \partial_t (n(\ln(n) - 1)).$$

$$n\mathbf{E} = T_e \nabla n,$$

$$\text{Notation : } (.)_{||} = (.) \cdot b, \quad (.)_{\perp} = (Id - \mathbf{b} \otimes \mathbf{b})(.),$$

$$\forall q \in \mathbb{R}^3, q = q_{||}\mathbf{b} + q_{\perp},$$

Three numerical difficulties

D1 : Capture the drift limit

Letting $\varepsilon \to 0$ in the momentum eq yields the balance:

$$\nabla_\perp p = n\mathbf{E}_\perp + nu_\perp \times \mathbf{b} \Rightarrow nu_\perp = \underbrace{nE_\perp \times \mathbf{B}}_{\text{Electric drift}} - \underbrace{\nabla_\perp p \times \mathbf{b}}_{\text{Diamagnetic drift}},$$

$$\nabla_{\|} p = n\mathbf{E}_{\|} \to \mathbf{u}_{\|} \quad ?$$

D2 : Preserve the energy and the positivity

Assume $\mathbf{u} \cdot \nu = 0$ on $\partial\Omega$ then $\forall t > 0$,

$$\frac{d}{dt}\int_\Omega \frac{\varepsilon n\mathbf{u}^2}{2} + \frac{1}{\gamma - 1}p + T_e n(\ln(n) - 1)dx = 0,$$

$n(t), w(t)$ and $p(t)$ are positive provided they are at initial time.

Three numerical difficulties : the third

Assume $d = 1$ and look at the dynamic in the parallel direction to the magnetic field. The E-L system can be re-cast into:

$$\begin{cases} \partial_t U + \partial_x f(U) = 0, \\ U = (n, nu, w + T_e n(\ln(n) - 1))^t, \\ f(U) = (nu, nu^2 + \frac{1}{\varepsilon} p, u(w + p + T_e n \ln(n))), \end{cases}$$

The system is strictly hyperbolic : the jacobian of f has three distinct eigen values $\lambda_- = u - \sqrt{\frac{\frac{\gamma p}{n} + T_e}{\varepsilon}}, \lambda_0 = u, \lambda_+ = u + \sqrt{\frac{\gamma \frac{p}{n} + T_e}{\varepsilon}}$.

D3 : Infinite acoustic waves speed

As $\varepsilon \to 0$, the parallel acoustic wave speed becomes infinite. For explicit discretization it yields a CFL stability condition restricted $\sqrt{\varepsilon}$.

What do we exactly need to implicit ?

Linearize the previous hyperbolic system around a constant state (n^0, u^0, p^0, w^0) and consider a semi-discretization in time:

$$\begin{cases} \forall k \in \{0, ..., \lfloor \frac{T}{\Delta t} \rfloor\}, \\ \frac{\tilde{n}^{k+1} - \tilde{n}^k}{\Delta t} + u^0 \partial x \tilde{n}^k + n^0 \partial_x \tilde{u}^{k*} = 0, \\ \frac{\tilde{u}^{k+1} - \tilde{u}^k}{\Delta t} + u^0 \partial_x \tilde{u}^k + \frac{1}{\varepsilon n^0} \partial_x (\tilde{p}^{k+1} + T_e \tilde{n}^{k+1}) = 0, \\ \frac{\tilde{p}^{k+1} - \tilde{p}^k}{\Delta t} + u^0 \partial_x \tilde{p}^k + \gamma p^0 \partial_x \tilde{u}^{k*} = 0, \\ \tilde{w}^k = \varepsilon n^0 \tilde{u}^k + \frac{\varepsilon u^0 \tilde{n}^k}{2} + \frac{\tilde{p}^k}{\gamma - 1} \end{cases}$$

$k* = k$ yields an explicit stiff term $\frac{1}{\varepsilon} \partial_{xx} (p^k + T_e n^k)$, $k* = k + 1$ yields an implicit stiff term $\frac{1}{\varepsilon} \partial_{xx} (p^{k+1} + T_e n^{k+1})$.

Summary

We need to implicit the gradient of total pressure and the gradient of velocity in the continuity and pressure equation.

The strategy

- D1 The parallel momentum equation does not degenerate if we ensure numerically $\nabla_{\|}(p - T_e n) = O(\varepsilon)$ for some norm.
- D2 Reformulate the equations and work with the non conservative variable $(n, \mathbf{u}, p)$, so as to ensure the positivity of the ionic temperature.
- D3 Implicit the acoustic and gradient of velocity $\rightarrow$ use an equation on the pressure.

Non conservative form brings another difficulty D4

For a smooth solution $(n, \mathbf{u}, w)$ the E-L system is equivalent to its non conservative from of unknown $(n, \mathbf{u}, p)$

$$\begin{cases} \partial_t n + \nabla \cdot (n\mathbf{u}) = 0 \text{ in } \Omega \times (0, T), \\ \partial_t(nu) + \nabla \cdot (n\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\varepsilon} \nabla p = \frac{1}{\varepsilon}(n\mathbf{E} + n\mathbf{u} \times \mathbf{b}) \text{ in } \Omega \times (0, T), \\ \partial_t(p) + \nabla \cdot (\mathbf{u}p) + (\gamma - 1)p\nabla \cdot u = 0 \text{ in } \Omega \times (0, T), \\ p = nT, \\ E = -\nabla\phi \text{ where } \phi = T_e \ln\left(\frac{n}{n_c}\right) \end{cases}$$

For a solution with discontinuities it is not equivalent ! Measure should appear at the r.h.s of the pressure eq.

D4 : How to compute correct shock speeds

Toy example : 1d Burger equation, $u = u_l \mathbf{1}_{x<\sigma t} + u_r \mathbf{1}_{x \geq \sigma t}$ R-H condition gives

$$\partial_t u + \partial_x \frac{u^2}{2} = 0 \to \sigma = \frac{u_r + u_l}{2},$$

$$\partial_t(u^2) + \partial_x \frac{2u^3}{3} = 0 \to \sigma = \frac{2}{3}(u_r + u_l).$$

The tool to circumvent D4 and get the conservative properties

Use an idea of R.Herbin, W.Kheriji and J-C Latché :

Staggered grids

Use staggered grids : scalar quantities are discretized on a primal mesh, while the velocity is discretized on a dual mesh to ensure the duality formula

$$\int_{\Omega} p \nabla \cdot u dx = - \int_{\Omega} \nabla p \cdot u dx.$$

The continuity equation

The continuity equation $\partial_t n + \nabla \cdot (n\mathbf{u}) = 0$ plays a crucial role at the continuous level to go from conservative equations to non conservative ones. We must ensure that it is valid on both the primal and dual mesh !

Recover a consistent discretization of the energy equation

We shall ad corrective source term in the pressure equation so as to recover a consistent discretization of the energy equation $\rightarrow$ Recover the R-H relations.

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The simplified parallel dynamic

$[0,1]_{per} = \mathbb{R}/\mathbb{Z}$. Consider the model of unknown $(n,u,p) : [0,T) \times [0,1]_{per} \to \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R}^+$

$$\begin{cases} \partial_t n + \partial_x (n\mathbf{u}) = 0, \\ \partial_t (nu) + \partial_x (n\mathbf{u} \otimes \mathbf{u}) + \frac{1}{\varepsilon} \partial_x p = -T_e \frac{\partial_x n}{\varepsilon}, \\ \partial_t (p) + \partial_x (up) + (\gamma - 1) p \partial_x u = 0, \\ + \text{ initial condition} \end{cases}$$

The discretization

Let $N \in \mathbb{N}^*, \Delta x := \frac{1}{N+1}$, $\Delta t > 0$, $x_i := i\Delta x$, $t^k := k\Delta t$. For all $i \in \mathbb{Z}$ the primal cell is $C_i := [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}})$, the dual cells of C_i are $C_{i-\frac{1}{2}} := [x_{i-1}, x_i)$ and $C_{i+\frac{1}{2}} := [x_i, x_{i+1})$.

$$\text{Primal mesh } \mathcal{T} := \bigcup_{i=0}^{N+1} C_i, \qquad \text{Dual mesh } \mathcal{T}^* = \bigcup_{i=0}^{N+1} C_{i-\frac{1}{2}}.$$

We approach the solution at time t^k by :

$$\begin{cases} n(t^k, x) \approx n^k_{\Delta x}(x) := \sum_{i=0}^{N+1} n_i^k \mathbf{1}_{C_i}(x), \\ p(t^k, x) \approx p^k_{\Delta x}(x) := \sum_{i=0}^{N+1} p_i^k \mathbf{1}_{C_i}(x), \\ u(t^k, x) \approx u^k_{\Delta x}(x) := \sum_{i=0}^{N+1} u_{i-\frac{1}{2}} \mathbf{1}_{C_{i-\frac{1}{2}}}(x). \end{cases}$$

Duality formula

$$\int_{[0,1]_{per}} p^k_{\Delta x}(x) \partial_x u^k_{\Delta x}(x) = - \int_{[0,1]_{per}} \partial_x p^k_{\Delta x}(x) u^k_{\Delta x}(x) dx,$$

$$\partial_x u_{\Delta x}(x) = \sum_{i=0}^{N} \frac{u_{i+\frac{1}{2}} - u_{i-\frac{1}{2}}}{\Delta x} \mathbf{1}_{C_i}(x), \ \partial_x p_{\Delta x}(x) = \sum_{i=0}^{N} \frac{p_{i+1} - p_i}{\Delta x} \mathbf{1}_{C_{i+\frac{1}{2}}}(x).$$

A non linear implicit scheme

Integrate the continuity and the internal energy eq on C_i and the momentum eq on $C_{i-\frac{1}{2}}$ and defines the approximation :

$$\begin{cases} \forall k \in \mathbb{N}, \forall i \in \{0, \dots N\}, \\ \frac{\Delta x}{\Delta t}(n_i^{k+1} - n_i^k) + F_{i+\frac{1}{2}}^{k+1} - F_{i-\frac{1}{2}}^{k+1} = 0, \\ \frac{\Delta x}{\Delta t}(p_i^{k+1} - p_i^k) + (up)_{i+\frac{1}{2}}^{k+1} - (up)_{i-\frac{1}{2}}^{k+1} + (\gamma - 1)(p_i^{k+1})^+ \delta_i(u^{k+1}) = S_i^{k+1}, \\ \frac{\Delta x}{\Delta t}(n_{i-\frac{1}{2}}^{k+1} u_{i-\frac{1}{2}}^{k+1} - n_{i-\frac{1}{2}}^k u_{i-\frac{1}{2}}^k) + F_i^{k+1} u_i^{k+1} - F_{i-1}^{k+1} u_{i-1}^{k+1} + \frac{1}{\varepsilon}\delta_{i-\frac{1}{2}}(p^{k+1} + T_e n^{k+1}) = 0, \\ + \text{ periodic b.c} \end{cases}$$

$$F_{i+\frac{1}{2}}^{k+1} := n_i^{k+1}(u_{i+\frac{1}{2}}^{k+1})^+ - n_{i+1}^{k+1}(u_{i+\frac{1}{2}}^{k+1})^-, \quad F_i^{k+1} := \frac{F_{i+\frac{1}{2}}^{k+1} + F_{i-\frac{1}{2}}^{k+1}}{2}, \delta_i(\cdot) = (\cdot)_{i+\frac{1}{2}} - (\cdot)_{i-\frac{1}{2}}.$$

Upwind w.r.t the sign of the velocity:

$$(up)_{i+\frac{1}{2}} := \begin{cases} p_i \text{ if } u_{i+\frac{1}{2}} \geq 0, \\ p_{i+1} \text{ else} \end{cases} \qquad u_i := \begin{cases} u_{i-\frac{1}{2}} \text{ if } F_i \geq 0, \\ u_{i+\frac{1}{2}} \text{ else.} \end{cases}$$

- $S_i^{k+1} \geq 0$ is designed to compensate the residual term that comes from a kinetic energy balance ! We want to be consistent with the energy equation.
- How to recover the discrete energy balance ?

The non linear implicit scheme : kinetic energy balance

One has the following kinetic energy balance :

Kinetic energy balance

$\forall k \in \mathbb{N}, \forall i \in \{0, ..., N\}$,

$$\frac{\varepsilon \Delta x}{2\Delta t}\left(n^{k+1}_{i-\frac{1}{2}}(u^{k+1}_{i-\frac{1}{2}})^2 - n^{k}_{i-\frac{1}{2}}(u^{k}_{i-\frac{1}{2}})^2\right) + \frac{\varepsilon F^k_i (u^k_i)^2}{2} - \frac{\varepsilon F^k_{i-1}(u^k_{i-1})^2}{2}$$
$$+ \delta_{i-\frac{1}{2}}(p^{k+1} + T_e n^{k+1})u^{k+1}_{i-\frac{1}{2}} = -R^{k+1}_{i-\frac{1}{2}}, \text{ where } R^{k+1}_{i-\frac{1}{2}} \geq 0.$$

Define S^{k+1}_i to compensate the contribution in the cell C_i of $R^{k+1}_{i-\frac{1}{2}}$ and $R^{k+1}_{i+\frac{1}{2}}$ so as $\sum_{i=1}^N S^{k+1}_i - R^{k+1}_{i-\frac{1}{2}} = 0$. To get this balance we use the important property :

$$\frac{\Delta x}{\Delta t}(n^{k+1}_i - n^k_i) + F^{k+1}_{i+\frac{1}{2}} - F^{k+1}_{i-\frac{1}{2}} = 0 \Rightarrow \frac{\Delta x}{\Delta t}(n^{k+1}_{i-\frac{1}{2}} - n^k_{i-\frac{1}{2}}) + F^{k+1}_i - F^{k+1}_{i-1} = 0.$$

A non linear implicit scheme : discrete potential balance

One has also the potential energy balance :

Potential energy balance

$\forall k \in \mathbb{N}, \forall i \in \{0, ..., N\}$,

$$\frac{\Delta x}{\Delta t}\left(n_i^{k+1}(\ln(n_i^{k+1}) - 1) - n_i^k(\ln(n_i^k) - 1)\right) + F_{i+\frac{1}{2}}^{k+1}\ln(n_{i+1}^{k+1}) - F_{i-\frac{1}{2}}^{k+1}\ln(n_i^{k+1})$$
$$- u_{i+\frac{1}{2}}^{k+1}\delta_{i+\frac{1}{2}}(n^{k+1}) = -D_i^{k+1} \leq 0$$

The proof uses Taylor Expansion + the fact that the flux is upwind with respect the velocity.

A non linear implicit scheme : energy dissipation

Define the discrete energy
$\mathcal{E}^k = \Delta x \sum_{i=0}^N \frac{\varepsilon}{2} n^k_{i-\frac{1}{2}} (u^k_{i-\frac{1}{2}})^2 + \frac{1}{\gamma-1} p_i^k + T_e n_i^k (\ln(n_i^k) - 1)$.Then for all $k \in \mathbb{N}$ one has :

$$\mathcal{E}^{k+1} - \mathcal{E}^k = -T_e \Delta x \sum_{i=0}^N D_i^{k+1}.$$

Thus, $-T_e \leq \mathcal{E}^k \leq \mathcal{E}^0$. One has eventually:

- Energy dissipation + unconditional positivity of density and pressure + conservation of the total mass.
- Uniform in ε estimates for the pressure and density. Estimate for the velocity is still an open question.
- Existence proof based on the Brouwer fixed point theorem: the key point energy decay + control of the density (lower bound).

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A linear iterative scheme to solve the non linear one

- Energy control : $-T_e \leq \mathcal{E}^k \leq \mathcal{E}^0$.
- Unconditionnal positivity of pressure and density.
- Unconditionnal linear L^2 stability around constant state.
- Need to solve it but not anyhow ! We want to avoid the CFL number being restricted by $\sqrt{\varepsilon}$!

Given the solution at step $k \in \mathbb{N}$, define the iterative scheme (two step recursive): $(n_i^{-1,0} = n_i^k, u_i^{-1,0} = u_i^k, p_i^{-1,0} = p_i^k)$ and for all $r \in \mathbb{N}$

$$\begin{cases} \frac{\Delta x}{\Delta t}(n_i^{r+1,k} - n_i^k) + F_{i+\frac{1}{2}}^{r+1,k} - F_{i-\frac{1}{2}}^{r+1,k} = 0 \\ \dfrac{\Delta x}{\Delta t}(p_i^{r+1,k} - p_i^k) + (up)_{i+\frac{1}{2}}^{r,k} - (up)_{i-\frac{1}{2}}^{r,k} \\ + (\gamma - 1)p_i^{r,k}\delta_i(u^{r+1,k}) = S_i^{r,k} \\ \dfrac{\Delta x}{\Delta t}(n_{i-\frac{1}{2}}^{r,k} u_{i-\frac{1}{2}}^{r+1,k} - n_{i-\frac{1}{2}}^k u_{i-\frac{1}{2}}^k) + F_i^{r,k} u_i^{r,k} - F_{i-1}^{r,k} u_{i-1}^{r,k} \\ + \dfrac{1}{\varepsilon}\delta_{i-\frac{1}{2}}(p^{r+1,k} + T_e n^{r+1,k}) = 0, \end{cases}$$

The trick is in the definition of the flux of mass :

$$F_{i+\frac{1}{2}}^{r+1,k} := n_{i+\frac{1}{2}}^{r,k} u_{i+\frac{1}{2}}^{r+1,k} - \frac{(n_{i+1}^{r,k} - n_i^{r,k})}{2} |u_{i+\frac{1}{2}}^{r,k}|.$$

Reduction to an elliptic system

Linear elliptic system on pressure and density

$$(L_\varepsilon^{r+1,k}) : \begin{cases} \forall i \in \{0, ..., N\}, \\ n_i^{r+1,k} - \frac{(\Delta t)^2}{\varepsilon(\Delta x)^2} \Delta_i (p^{r+1,k} + T_e n^{r+1,k}) = n_{\varepsilon,i}^k + \bar{n}_i^{r,k}, \\ p_i^{r+1,k} - (\gamma - 1)(p_i^{r,k})^+ \frac{(\Delta t)^2}{\varepsilon(\Delta x)^2} \Delta_i^{n^{-1}} (p^{r+1,k} + T_e n^{r+1,k}) = p_{\varepsilon,i}^k + \bar{p}_i^{r,k}, \\ \text{ with } \bar{n}_i^{r,k} \bar{p}_i^{r,k} \text{ some residual terms .} \\ + \text{ periodic b.c} \end{cases}$$

- The velocity update becomes explicit.
- The well-posedness of $(L_\varepsilon^{r+1,k})$ follows from its elliptic structure.
- It enjoys a maximum principle $\rightarrow$ positivity under a CFL that does not depend on ε.
- Linear L^2 stability analysis shows that the CFL depends on u^r but not on ε.

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Shock speed computation $T_e = 0$

Riemann problem : $(n^0, u^0, p^0)^t = (1, 0, 10^3)^t \mathbf{1}_{x<0.5}(x) + (1, 0, 10^{-3})\mathbf{1}_{x\geq 0.5}(x)$, $t_{final} = 0.012, \Delta x = 10^{-3}, \varepsilon = 1$. The iterative scheme stops when the relative L^2 error between two iterations is lower than 10^{-8}.

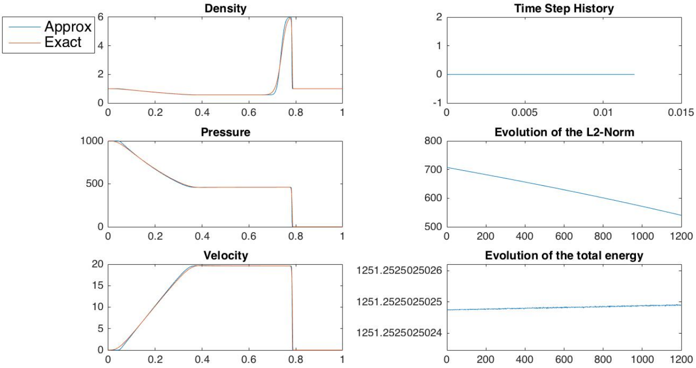


Figure: Implicit scheme $\Delta t = \Delta x/50$

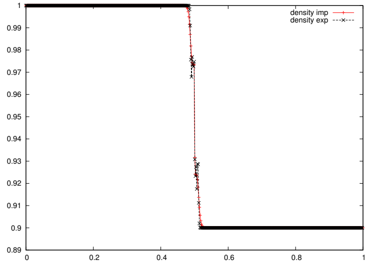
A plasma expansion problem: $T_e = 1$ and asymptotic $\varepsilon \to 0$.

Riemann problem:
$(n_\varepsilon^0, u_\varepsilon^0, p_\varepsilon^0)^t = (1,0,1)^t \mathbf{1}_{x<0.5}(x) + (1-\varepsilon, 0, 1-\varepsilon)\mathbf{1}_{x\geq 0.5}(x),\ t_{final} = 0.002$. The physical scaling is $\partial_x(p + T_e n) = \mathcal{O}(\varepsilon)$. $\Delta x = \frac{1}{2^9+1}$. The iterative scheme stops when the relative L^2 error between two iterations is lower than 10^{-8}. Comparison with an explicit version of the scheme.

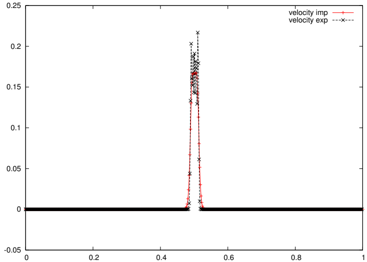
ε	CFL EXP	IT IMP	IT EXP	R	CPU IMP(s)	CPU EXP(s)	R
10^{-1}	10^{-1}	78	11	7.09	77.4158	0.002266	34164
10^{-2}	10^{-2}	78	103	0.75	77.5185	0.01706	4542
10^{-4}	10^{-3}	67	1026	0.06	64.2173	0.1879	341.7
10^{-8}	10^{-5}	24	102600	0.0002	22.8352	17.6862	1.2
10^{-10}	10^{-6}	12	1026000	0.00001	12.2736	185.55	0.06

Table: Number of total iterations and computational time as function of ε with $\Delta x = \frac{1}{2^9+1}$, CFL IMP $= 10^{-1}$.

A plasma expansion problem: $T_e = 1$ and $\varepsilon = 10^{-1}$.

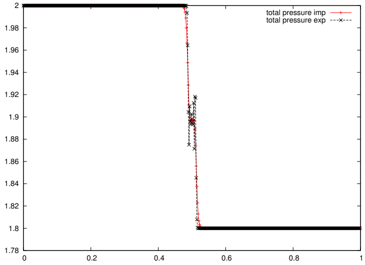


(a) Approximate density in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-1}$.

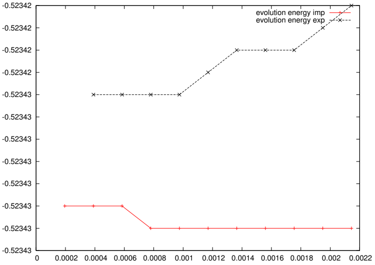


(b) Approximate velocity in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-1}$

A plasma expansion problem: $T_e = 1$ and $\varepsilon = 10^{-1}$.

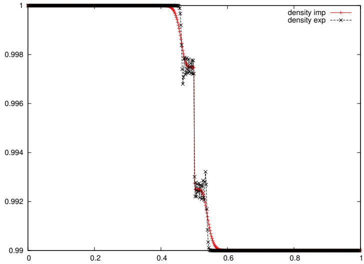


(c) Approximate total pressure in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-1}$.

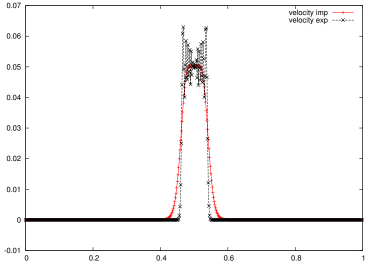


(d) Evolution in time of the energy computed with the iterative linear implicit scheme (black) and the explicit scheme (red) for $\varepsilon = 10^{-1}$.

A plasma expansion problem: $T_e = 1$ and $\varepsilon = 10^{-2}$

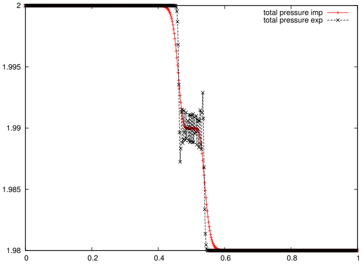


(e) Approximate density in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-2}$.

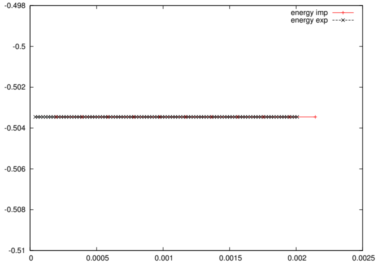


(f) Approximate velocity in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-2}$

A plasma expansion problem: $T_e = 1$ and $\varepsilon = 10^{-2}$

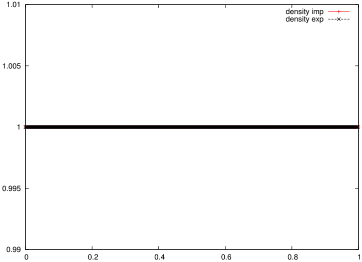


(g) Approximate total pressure in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-2}$.

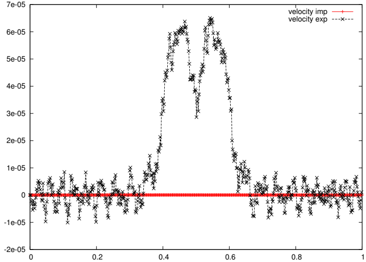


(h) Evolution in time of the energy computed with the iterative linear implicit scheme (black) and the explicit scheme (red) for $\varepsilon = 10^{-2}$.

A plasma expansion problem: $T_e = 1$ and $\varepsilon = 10^{-8}$

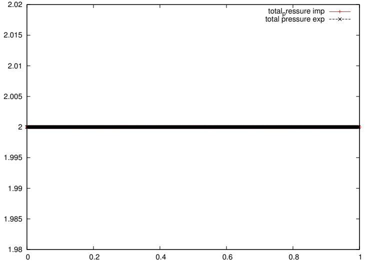


(i) Approximate density in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-8}$.

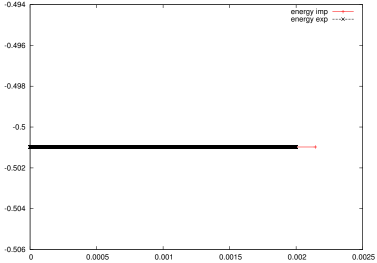


(j) Approximate velocity in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-8}$

A plasma expansion problem: $T_e = 1$ and $\varepsilon = 10^{-8}$



(k) Approximate total pressure in space computed with the iterative linear implicit scheme (black) and the explicit scheme (red) at time $T = 0.002$ for $\varepsilon = 10^{-8}$.



(l) Evolution in time of the energy computed with the iterative linear implicit scheme (black) and the explicit scheme (red) for $\varepsilon = 10^{-8}$.

Outline

1. Physical introduction
2. The Euler-Lorentz-Boltzmann system
3. A non linear finite volume schemes for the parallel dynamic
4. A linear iterative scheme to approach the non linear one
5. Numerical results
6. Conclusion and perspectives

On going work

- Non linear scheme and iterative scheme are stable (Positivity + Energy dissipation) independently on ε.
- Numerical results shows the AP property of the scheme, though uniform in ε estimate for the velocity is missing.
- Discrete entropy inequality ?
- Extension of the scheme to the diffusion limit for the electrons, capture the Boltzman regime.
- On going work : Implementation for the three dimensional Euler-Lorentz model with magnetic field.

Thank you for paying attention.