Hybrid physics-AI model reductions with large dimensional parameter. Application to flood plain dynamics

Scientific context

We, as applied mathematicians from INSA - Mathematics Institute of Toulouse IMT, develop both direct and inverse numerical models of geophysical fluid flows, particularly stream flows for dynamic flood plain simulations. These parametrized PDE-based models need to be fused with large datasets derived from satellite images and in-situ measurements using Data Assimilation (DA) methods. While purely physics-based approaches like Variational DA are robust, they are also very CPU-time and memory consuming. Moreover, newly acquired data should be assimilated on the fly.

To build reduced models, we adopted offline-online strategies. The approach relies on the use of classical Proper Orthogonal Decompositions (POD) combined with Deep Neural Networks (DNN). DNN are attractive in this context due to their universal approximation property and their ability to be evaluated in real-time during the online phase. Such an approach is now quite well understood, including for non-linear hyperbolic systems like the present 2D shallow water system, but only for small dimensional cases, say $O(10)$ unknown parameters, as shown in [3] and the references therein.
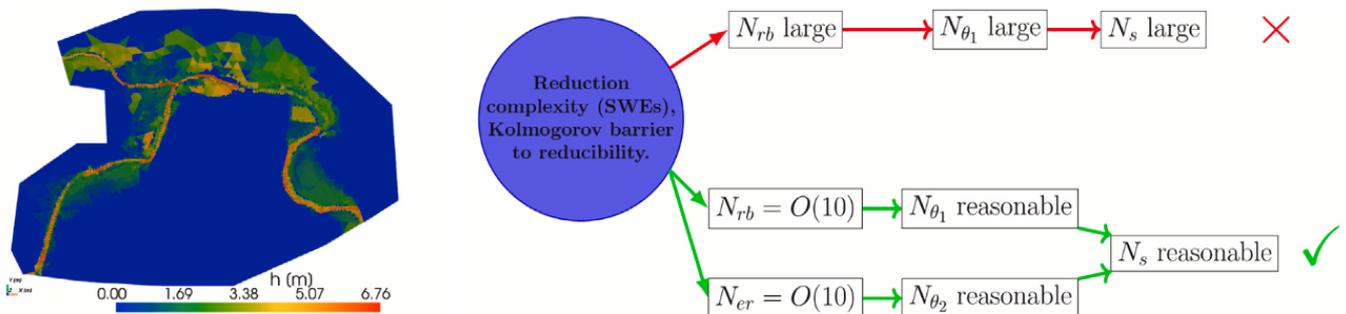

Fig. 1. (Left) Real-world test case, [3]: a flood plain dynamics in southern France. Plot: the water depth $h(x)$ (in m). (Right) Reducibility of the model equations (here the 2D Shallow-Water system in variables $(h, q_x, q_y)(x, t)$). *Images extracted from previous work [3].*

Objectives

The aim of this internship is to extend the approach developed in [3] as follows:

1. Adding more physics for the DNN training to reduce the number of snapshots required for training, which is crucial in high dimensions. This consists to incorporating the PDE based physical model as a weak constraint in the optimization process as done in PINNs.

2. Investigating upstream reduction of the input parameters (in the case of large dimensional). To achieve this, techniques as those based e.g. on the use of semi-supervised VAEs [4] will be considered.

The research computational code is written in Python. It is well controlled by the research team and documented (see DassFlow 2D on `https://mathhydronum.insa-toulouse.fr/codes_presentation/`).

The supervisor team is co-PI or member of multidisciplinary research projects (AI ANITI chaire, NASA/CNES mission Science Team, ANR projects, ESA project etc) in collaboration with numerous academic and industrial partners (INRAe Aix-en-Provence, Hydro-Matters etc).

Required skills.
Computational sciences, numerical schemes for PDEs, machine learning, programming.

Interested in this internship ?
Send CV + motivation letter + transcripts to J. Monnier & R. Bouclier.
(`jerome.monnier@insa-toulouse.fr`, `https://www.math.univ-toulouse.fr/ jmonnie`)
(`robin.bouclier@insa-toulouse.fr`, `https://www.math.univ-toulouse.fr/ rbouclie`)
The call is opened until an applicant is hired.

References
[1] Chen, W., Wang, Q., Hesthaven, J. S., & Zhang, C. (2021). Physics-informed machine learning for reduced-order modeling of nonlinear problems. *Journal of computational physics*, 446, 110666.

[2] Boulenc, H., Bouclier, R., Garambois, P. A., & Monnier, J. (2024). Spatially-Distributed Parameter Identification by Physics-Informed Neural Networks illustrated on the shallow-water equations. *Journal of computational physics*, To appear.

[3] Allabou, M., Bouclier, R., Garambois, P. A.,& Monnier, J. (2024). Reduction of the shallow water system by an error aware POD-neural network method: Application to floodplain dynamics. *Computer Methods in Applied Mechanics and Engineering,* 428, 117094.

[4] Ismail Fawaz, H., Forestier, G., Weber, J., Idoumghar, L.,& Muller, P. A. (2019). Deep learning for time series classification: a review. *Data mining and knowledge discovery,* 33(4), 917-963.