Optimization for Feedback Control

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Guidage

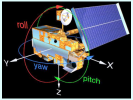
Survol

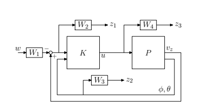
Solving challenging problems ...

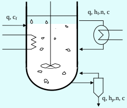
Controvert

Technicom

Robust Crymath

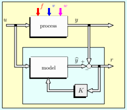




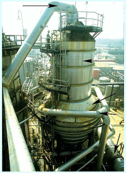


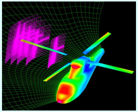
feedback

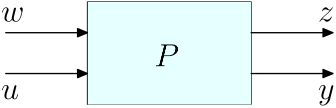
control



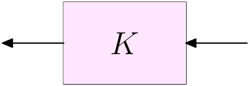
u = known input f = fault v = white noise
w = perturbation y = measured output
$\widehat{y}$ = model estimated output K = filter gain
r = residual generator ☐ detection filter



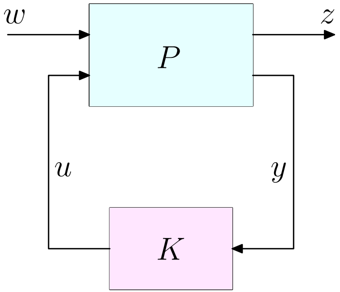




$$P:\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases}$$



$$K:\begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases}$$



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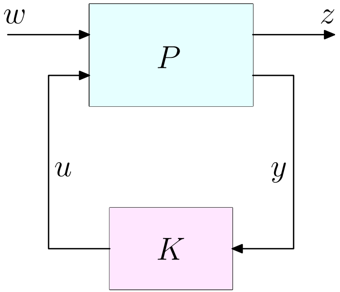
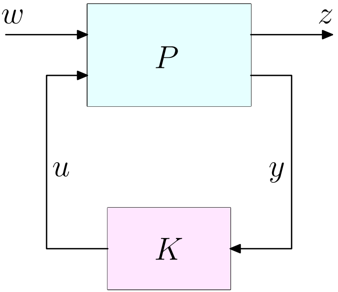


$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

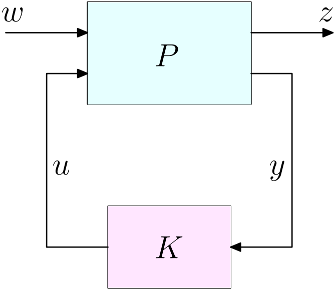
$$K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$



$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right]$$

$$K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

$$T_{w\to z}(P,K) = \left[\begin{array}{cc|c} A+B_2 D_K C_2 & B_2 C_K & B_1 + B_2 D_K D_{21} \\ B_K C_2 & A_K & B_K D_{21} \\ \hline C_1 + D_{12} D_K C_2 & D_{12} C_K & D_{11} + D_{12} D_K D_{21} \end{array}\right]$$



$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right]$$

$$K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

$$T_{w\to z}(P,K) = \left[\begin{array}{c|c} A(K) & B(K) \\ \hline C(K) & D(K) \end{array}\right]$$

K should :

- guarantee stability in closed loop
- assure good performance in closed-loop
- be robust with respect to model uncertainty
- be easy to embed

Stability

When is plant P stabilizable ?

When is plant P stabilizable by a practically useful controller K ?

When is plant P stabilizable by a practically useful controller K ?

$$P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right], \quad K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right], \quad A(K) \text{ stable}$$

- Can be decided by linear algebra if $\text{size}(A) = \text{size}(A_K)$ and A_K, B_K, C_K, D_K all free

  (Kalman 1960)

- Decision is NP-complete if K static, reduced-order, PID, decentralized

  (V. Blondel, J. Tsitsiklis, 1997)

What is a controller structure ?

$$K_{\rm pid} = \left[\begin{array}{cc|c} 0 & 0 & R_i \\ 0 & -\tau I_{m_2} & R_d \\ \hline I_{m_2} & I_{m_2} & D_K \end{array}\right] \qquad K_{\rm obs} = \left[\begin{array}{c|c} A - B_2 K_c - K_f C_2 & K_f \\ \hline -K_c & 0 \end{array}\right]$$

$$K_{\rm dec} = \left[\begin{array}{ccc|ccc} A^1 & & & B^1 & & \\ & \ddots & & & \ddots & \\ & & A^N & & & B^N \\ \hline C^1 & & & D^1 & & \\ & \ddots & & & \ddots & \\ & & C^N & & & D^N \end{array}\right] \qquad K_{\rm red} = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & A_K & B_K \\ \hline 0 & C_K & D_K \end{array}\right]$$

$$K(\theta) = \begin{array}{ll} \begin{array}{cc} \overbrace{\hphantom{A_K(\theta)\quad}}^{\substack{n_K \\ \text{order } K}} & \overbrace{\hphantom{B_K(\theta)\quad}}^{\substack{m_2 \\ \text{sensors}}} \end{array} & \\ \left[\begin{array}{c|c} A_K(\theta) & B_K(\theta) \\ \hline C_K(\theta) & D_K(\theta) \end{array}\right] & \begin{array}{l} \\ \left.\vphantom{D_K}\right\} \text{actuators} \qquad p_2 \end{array} \end{array}$$

$$n_K \le n_x \qquad \Longrightarrow \qquad N := n_x^2 + n_x m_2 + n_x p_2 + m_2 p_2 \ \text{ degrees of freedom}$$

Structure means:

$\dim(\theta) \ll N$, $\qquad$ $A_K(\theta), \ldots$ depend smoothly on θ

$A_K(\theta), \ldots$ as sparse as possible

Stability in closed loop :

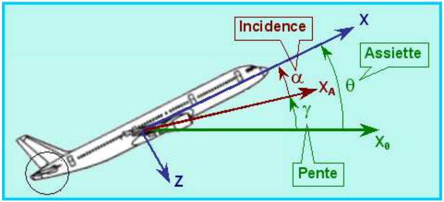
- Hard to decide for practically useful controller structures.
- Can be decided for full-order or observer-based controllers. Both of little use in practice.
- Stroke of luck : Technical systems are made to be stabilized. So often works.
- Practical method (Burke, Overton 2001) :

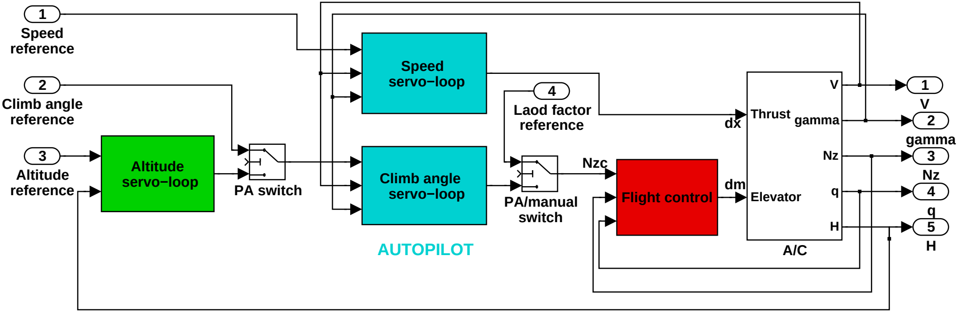
$$\min_K \alpha\,(A(K)) := \max\{\mathrm{Re}(\lambda) : \lambda \text{ eigenvalue of } A(K)\}$$

- What is hard is to stabilize *and* guarantee good performance.
- What about controllability, observability, detectability ?

Controller structure

Longitudinal Motion of an Aircraft

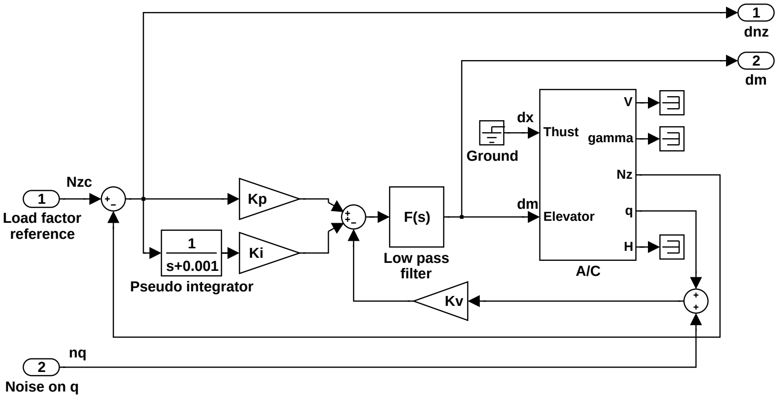




pitch	tangage	Nickwinkel	buruzka-angelu
roll	roulis	Rollwinkel	balantza-angelu
yaw	lacet	Gierwinkel	keinada

elevator	gouvernail de profondeur	Höhenruder	sakonerako jokaldi
aileron	aileron	Querruder	aleroi
rudder	gouvernail	(Seiten)ruder	lema

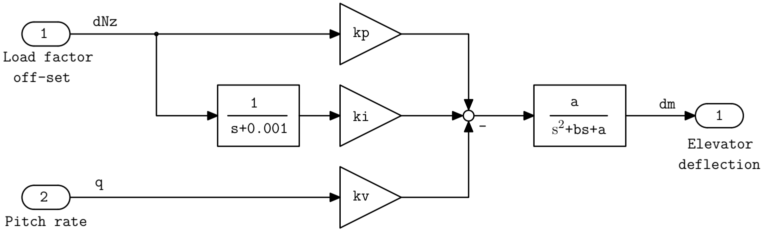
Flight Control Loop



$N_{zc} \rightarrow dN_z = N_{zc} - N_z$ (tracking error)
$n_q \rightarrow dm$ (avoid fatigue of elevator due to sensor noise)

Flight Control: High Angle of Attack Mode

Determine `Kp`, `Ki`, `Kv`, `a`, `b`



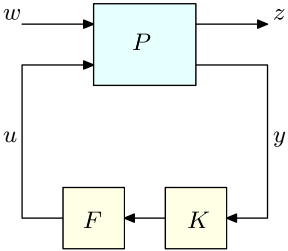
One can have it as non-linear function of $\theta = (a, b, k_p, k_i, k_v)$:

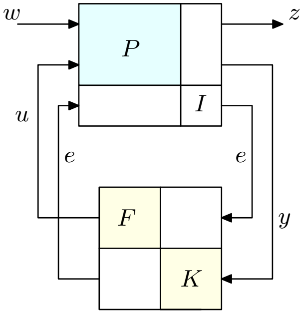
$$K(\theta) : \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \hline dm \end{array}\right] = \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ -a & -b & a & ak_p & -ak_v \\ 0 & 0 & -0.001 & k_i & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ \hline dN_z \\ q \end{array}\right]$$

or affine :

$$K(\theta) : \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \hline dm \\ e \end{array}\right] = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 \\ -a & -b & 0 & a & 0 & 0 \\ 0 & 0 & -0.001 & 0 & k_i & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & k_p & -k_v \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ \hline e \\ dN_z \\ q \end{array}\right]$$

$$P\colon \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \qquad \widetilde{P}\colon \left[\begin{array}{c|ccc} A & B_1 & 0 & B_2 \\ \hline C_1 & D_{11} & 0 & D_{12} \\ 0 & 0 & I & 0 \\ C_2 & D_{21} & 0 & D_{22} \end{array}\right]$$



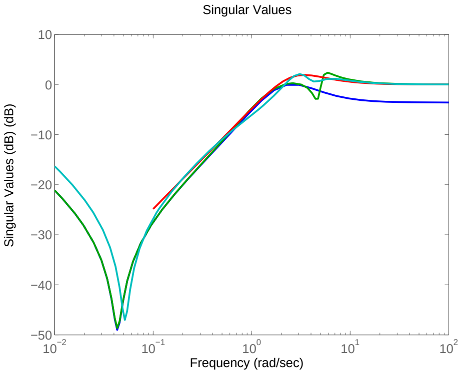


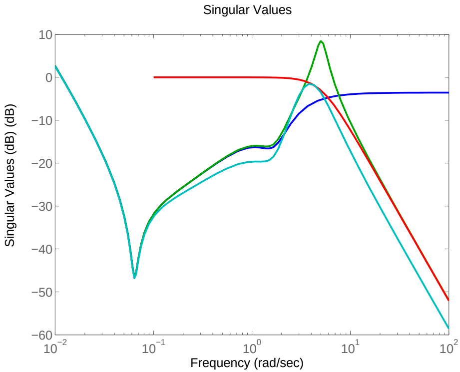
M. Gabarrou, D. Alazard, D. Noll. *Structured flight control law design using non-smooth optimization.* 18th IFAC ACA, 2010.

Flight control. (Mach = 0.7, Altitude = 5000 ft). 2 performance criteria, 5 variables.

Flight control + Autopilot. 6 performance criteria + 2 robustness criteria, 18 states, 11 variables. Combination of P,PI,PID and filters.

D. Alazard, P. Apkarian, A. Simões, D. Noll. *Lateral flight control design for a highly flexible aircraft using nonsmooth optimization.* Aerospace Science and Technology. 2010.





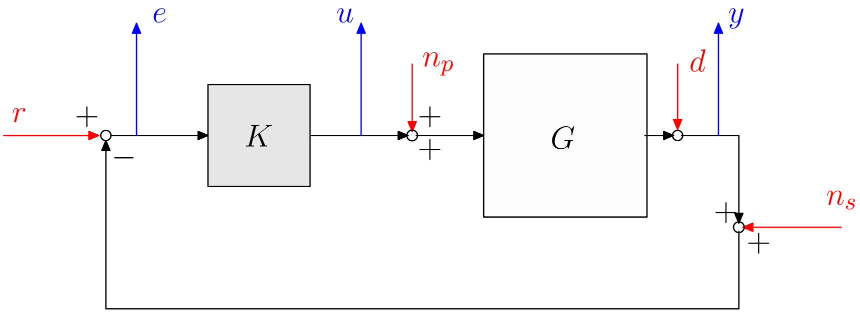
$$\| W_1^{-1} T_{N_{zc} \to dN_z} \|_\infty \leq 1$$

(good tracking)

$$\| W_2^{-1} T_{n_q \to dm} \|_\infty \leq 1$$

(avoid actuator fatigue)

Performance



r = reference input

n_p = process noise

n_s = sensor noise

d = disturbance

u = actuator signal

e = tracking error

y = sensor signal

H_∞-Loopshaping

Good tracking :

$$T_{r \to e} = (I + GK)^{-1}$$

$\implies$ want $G(j\omega)K(j\omega)$ large in low frequency ω.

Fatigue of actuator due to measurement noise :

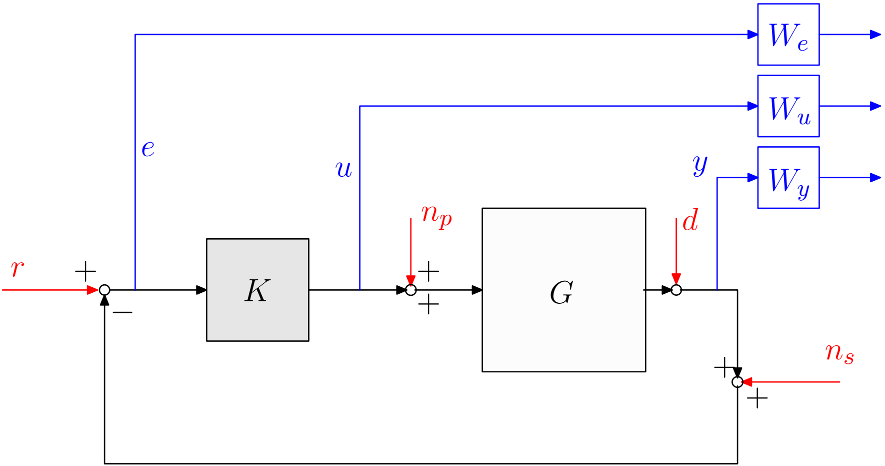
$$T_{n_s \to u} = -(I + KG)^{-1}K$$

$\implies$ want $K(j\omega)$ small in high frequency ω.

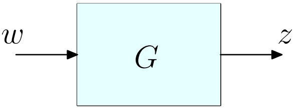
Influence of process noise on tracking :

$$T_{n_p \to e} = -(I + GK)^{-1}G$$

$\implies$ want GK large where G is large and in high frequency.


W_e
W_u
W_y
e
u
y
n_p
d
r
K
G
n_s
$+$
$-$

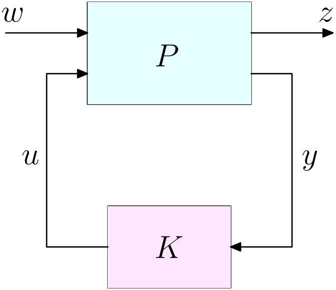
H_∞-norm



$$\begin{tikzcd}[column sep=huge, row sep=huge]
L_2[0,\infty) \arrow[r, shift left, "\text{Laplace}"] \arrow[d, "G"'] & \mathcal{H}_2 \arrow[l, shift left, "\text{Laplace}^{-1}"] \arrow[d, "G"] \\
L_2[0,\infty) \arrow[r, shift left, "\text{Laplace}"] & \mathcal{H}_2 \arrow[l, shift left, "\text{Laplace}^{-1}"]
\end{tikzcd}$$

$$\begin{aligned}
||G||_{\mathcal{H}_2\to\mathcal{H}_2} &= \sup_{||w||_2\le 1} ||Gw||_2 =: ||G||_\infty = H_\infty\text{-norm} \\
&= \max_{\omega\in[0,\infty]} \overline{\sigma}(G(j\omega))
\end{aligned}$$

Analogy with matrix norm $||A||_{2,2} = \sup_{||x||_2\le 1} ||Ax||_2 = \overline{\sigma}(A)$



minimize $||T_{w\to z}(P,K)||_\infty$

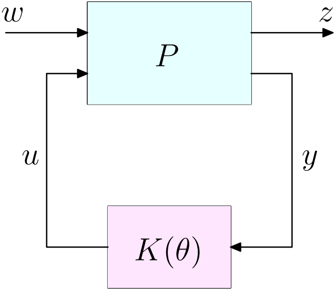
subject to

$$K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

stabilizing

J.C. Doyle, K. Glover, P.P. Khargonekar, B.A. Francis. *State-space solutions to standard H_2 and H_∞ control problems.* IEEE Trans. Autom. Control vol. 34, no. 8, 1989, pp. 831 - 847

$\Longrightarrow$ 2 algebraic Riccati equations (decoupled)



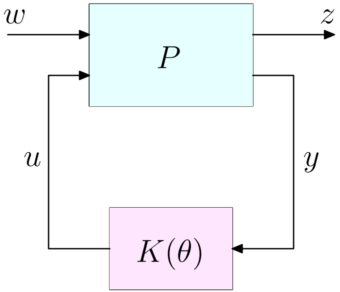
minimize $||T_{w\to z}(P, K(\theta))||_\infty$

subject to

$$K(\theta) = \left[\begin{array}{c|c} A_K(\theta) & B_K(\theta) \\ \hline C_K(\theta) & D_K(\theta) \end{array}\right]$$

stabilizing

P. Apkarian, D. Noll. *Nonsmooth H_∞-synthesis.*
IEEE Transactions on Automatic Control, vol. 51, no. 1, 2006, 71 - 86



minimize $||T_{w\to z}(P, K(\theta))||_\infty$

subject to

$$K(\theta) = \left[\begin{array}{c|c} A_K(\theta) & B_K(\theta) \\ \hline C_K(\theta) & D_K(\theta) \end{array}\right]$$

stabilizing

P. Apkarian, D. Noll. *Nonsmooth H_∞-synthesis.*
IEEE Transactions on Automatic Control, vol. 51, no. 1, 2006, 71 - 86

$\Longrightarrow$ `hinfstruct` – P. Apkarian, D. Noll, P. Gahinet (MathWorks).

hinfstruct

H_∞ tuning of fixed-structure controllers

Syntax

```
C = hinfstruct(Pdes,C0)
[C,gamma,info] = hinfstruct(Pdes,C0)
[...] = hinfstruct(Pdes,C0,options)
```

Description

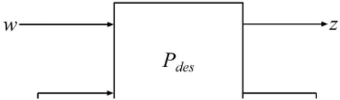
`C = hinfstruct(Pdes,C0)` tunes the parametric controller blocks `C0` t
model that includes the fixed elements of the control architecture. `Pdes`
parametric form. `C0` can be a single parametric model or a cell array of r

`[C,gamma,info] = hinfstruct(Pdes,C0)` provides `gamma` (the mini

`[...] = hinfstruct(Pdes,C0,options)` allows you to specify addit

Tips

- `hinfstruct` optimizes the free parameters of the tunable elements t
  Form . The following diagram illustrates Standard Form.



Some facts on `hinfstruct`

- Problems are non-convex. Local minima are computed.
- Problems are non-smooth. Non-convex bundle method used.
- Best suited to design small controllers for large systems.
- Problem dimension depends on free parameters in $K(\theta)$.
- Function evaluation is iterative and depends on size(P)+size(K).
- Avoids use of Lyapunov variables in optimization.
- Replaces AREs or LMIs.
- Problems can be set-up directly from `simulink`.

Synthesis of reduced-order controllers using `hinfstruct`

	$n_x/n_K/\text{vbl}$	cpu			$n_x/n_K/\text{vbl}$	cpu
AC10	55/10/144	288		JE1	30/8/143	110
AC13	28/7/110	28		JE2	21/8/121	87
AC14	40/10/182	8		JE3	24/8/154	24
HE6	20/8/168	79		IH	21/0/110	57
HE7	20/8/168	85		IH	21/2/156	71
CM2	60/11/156	78		IH	21/8/342	157
CM3	120/14/240	306		CSE1	20/8/180	6
CM4	240/17/342	2272		CSE2	60/10/480	48
TL	256/2/16	3601		NN11	16/6/99	34
CRY1	250/2/16	691		CRY2	250/2/9	411

$$\text{vbl} = n_K^2 + n_K(\text{inputs} + \text{outputs}) + \text{inputs} \cdot \text{outputs}$$
$$n_x = \text{order of plant},\ n_K = \text{order of } K$$

Power oscillation damping controller for Brazilian North and South power subsystem interconnection (Apkarian, Simões, Noll 2008)

90 states (already reduced)
3 performance interconnections.
$\dim(y) = 1$, $\dim(u) = 1$
Controller order 8.
$\dim(K) = 81$

If solved as a BMI (bilinear matrix inequality) :

Have to add $3 \cdot \dfrac{90 \cdot 91}{2} = 12285$ for Lyapunov function.

12366 variables :

P. Apkarian, D. Noll, O. Prot. *A trust region spectral bundle method for nonconvex eigenvalue optimization*. SIAM Journal on Optimization, 19, no. 1, 2008, pp. 281 – 306.

- LQG-control : 2 algebraic Riccati equations (decoupled).
  (1 of them analogue of Hamilton-Jacobi equation for state-feedback for infinite horizon)
  (Kalman 1960)
- H_2-optimal control : 2 algebraic Riccati equations (decoupled).
- H_∞-control : 2 algebraic Riccati equations (decoupled)
  (Doyle, Glover, Khargonekar, Francis 1989)

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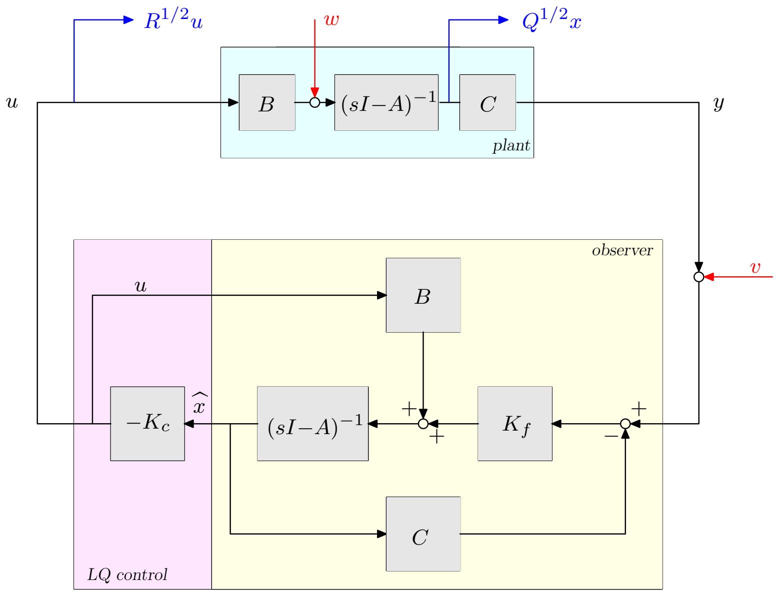
- H_∞-control : 2 algebraic Riccati equations (decoupled)
  (Doyle, Glover, Khargonekar, Francis 1989)

- Disclaimer : Even for reduced-order $K = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & A_K & B_K \\ \hline 0 & C_K & D_K \end{array}\right]$:

$\Longrightarrow$ 4 coupled algebraic Riccati equations



$R^{1/2}u$
w
$Q^{1/2}x$
u
B
$(sI-A)^{-1}$
C
y
plant
observer
v
u
B
$-K_c$
$\widehat{x}$
$(sI-A)^{-1}$
K_f
C
LQ control

Some observations :

- Practical problems lead to structure. Theory prefers unstructured.
- The vicious circle : Practical controllers have to be "small". AREs and LMIs only give "large". Leads to overly reduced systems in practice.
- Approaches based on AREs of little use. LQG as stand-alone obsolete.
- In practical problems always one or several criteria to optimize.
- Non-linear system not available as a rule ; only (A, B, C, D).

Robustness

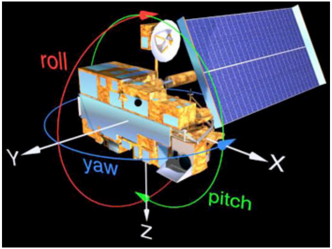
A robust baby.

With a house full of boys you have to have robust furniture.

The robust flavor of fresh brewed coffee.

How can a refined girl like her be drawn to such a robust man ?

Control of roll-axis of geostationary flexible satellite (solar pannels)
1 rigid mode, 3 flexible modes.
(Gauvrit, Alazard 1999)



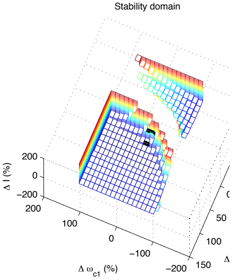
$$I\ddot{\phi} + \sum_{i=1}^{3}(\lambda_i^2 - 1)\,\ddot{\eta}_i = u$$

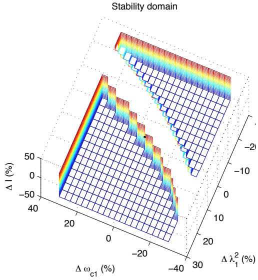
$$\ddot{\eta}_i + 2\lambda_i\zeta_c\dot{\eta}_i + \omega_i^2\eta_i, \quad i = 1, 2, 3$$

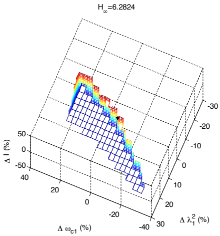
ϕ = intertial angular position of satellite, $\eta = (\eta_1, \eta_2, \eta_3)$ = free mode state vector, u = control torque, I = inertia, ω_i = free pulsations, $\lambda_i\zeta_c$ = free damping coefficients, $\lambda_i = \omega_i/\omega_{c_i}$ free cantilever pulsation ratios, ζ_c = cantilever damping, ω_{c_i} = cantilever pulsations.

Output = ϕ, control = u, 8 states + 2 states from filter.

6 uncertain parameters due to cantilever arm ; 3 severe

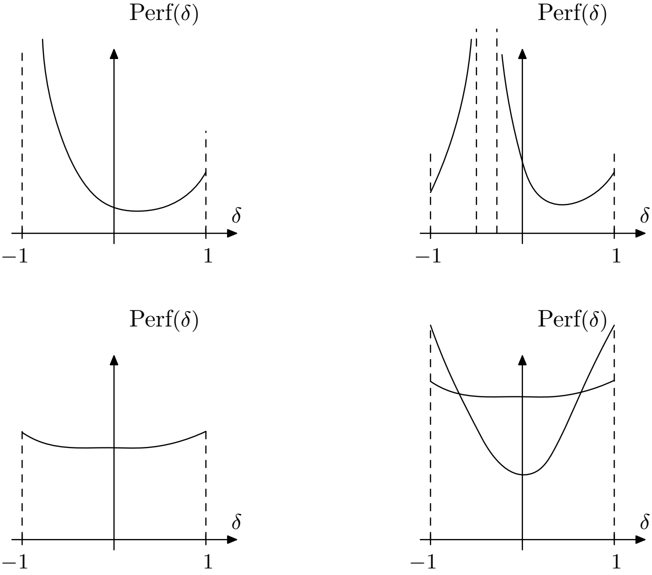


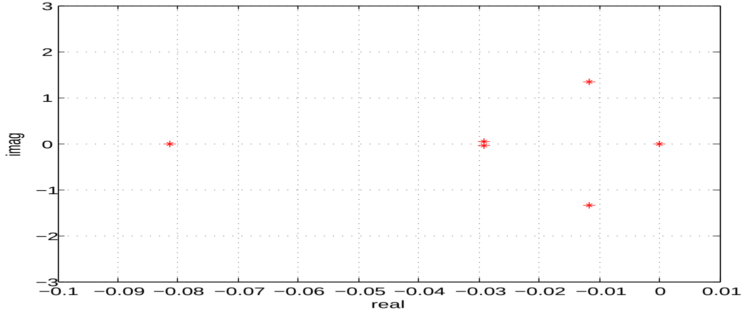




$$\delta I = \frac{I - I^{\mathrm{nom}}}{I^{\mathrm{nom}}} \qquad \delta\omega_{c_1} = \frac{\omega_{c_1} - \omega_{c_1}^{\mathrm{nom}}}{\omega_{c_1}^{\mathrm{nom}}} \qquad \delta\lambda_1^2 = \frac{\lambda_1^2 - \lambda_1^{2,\mathrm{nom}}}{\lambda_1^{2,\mathrm{nom}}}$$

$$Q_0 = \{\delta = (\delta I, \delta\omega_{c_1}, \delta\lambda_1^2) : -1 \leq \delta_i \leq 1\}, \qquad Q = qQ_0$$


Perf(δ)
δ
-1
1
Perf(δ)
δ
-1
1
Perf(δ)
δ
-1
1
Perf(δ)
δ
-1
1



Poles of $A(K)$ = closed-loop matrix

8 open-loop poles, 6 poles from K, 2 filter = 16 poles. Only 6 visible.

Movie : Poles of $A(K, \delta)$ as $\delta \in qQ_0$. Watch in time q.

$A(K)$ = closed-loop system matrix when $K = K(\theta)$

$A(K,\delta)$ = closed-loop system matrix when $K = K(\theta)$ and uncertain parameter is $\delta = (\delta_1, \delta_2, \delta_3) \in Q$.

How robust ? Structured stability radius :

$$r_{\text{struct}}\,(A(K)) = \inf\{\|\delta\|_\infty : \delta = (\delta_1, \delta_2, \delta_3) \text{ and } A(K,\delta) \text{ unstable }\}$$

Compare to well-known stability radius :

$$\begin{aligned} r\,(A(K)) &= \inf\{\|E\| : A(K) + E \text{ unstable}\} \\ &= \inf\{\sigma_{\max}(E) : A(K) + E \text{ unstable}\} \end{aligned}$$

$$A(K,\delta) = A(K) + \overbrace{B(K)\Delta(I - D(K)\Delta)^{-1}C(K)}^{\color{red}{E=E(K,\delta)}}$$

$$\Delta = \left[\begin{array}{c|cc|cc} \delta_1 & & & & \\ \hline & \delta_2 & & & \\ & & \delta_2 & & \\ \hline & & & \delta_3 & \\ & & & & \delta_3 \end{array}\right]$$

$$r_{\mathrm{struct}}\,(A(K)) = \inf\{\sigma_{\max}(\Delta) : A(K,\delta) \text{ unstable }\}$$

Checking robust stability of $A(K)$ over $Q = qQ_0$ means checking

$$r_{\mathrm{struct}}\,(A(K)) \overset{?}{>} q$$

$$A(K,\delta) = A(K) + \overbrace{B(K)\Delta(I - D(K)\Delta)^{-1}C(K)}^{E=E(K,\delta)}$$

$$\Delta = \left[\begin{array}{c|cc|cc} \delta_1 & & & & \\ \hline & \delta_2 & & & \\ & & \delta_2 & & \\ \hline & & & \delta_3 & \\ & & & & \delta_3 \end{array}\right]$$

$$r_{\text{struct}}\,(A(K)) = \inf\{\sigma_{\max}(\Delta) : A(K,\delta) \text{ unstable }\}$$

Checking robust stability of $A(K)$ over $Q = qQ_0$ means checking

$$r_{\text{struct}}\,(A(K)) \overset{?}{>} q$$

Disclaimer. Computing r_{struct} NP-complete (Poljak, Rohn '93, Demmel '92). Solving robustness problem therefore $\mathrm{NP}^{\mathrm{NP}}$-complete.

True robust spectrum :

$\Lambda_{\text{struct}} = \{\lambda : \lambda \text{ eigenvalue of } A(K, \delta) \text{ for some } \delta \in qQ_0\}$

Pseudo-spectrum :

$\Lambda_{\text{unstruct}} = \{\lambda : \lambda \text{ eigenvalue of } A(K) + E \text{ for some } \|E\| \leq \rho\}$

Semi-structured spectrum

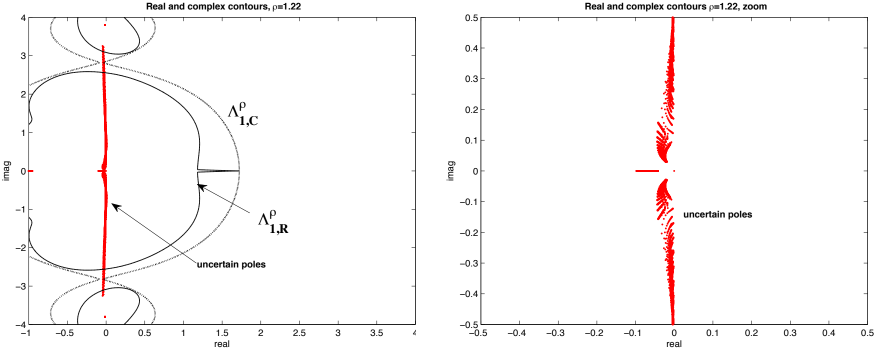
$\Lambda_{\text{semi-struct}} =$
$\{\lambda : \lambda \text{ eigenvalue of } A(K) + B(K)F(I - D(K)F)^{-1}C(K) \text{ for } \|F\| \leq q\}$

Want K such that $\Lambda_{\text{struct}} \subset \mathbb{C}^-$.

Why not force ?

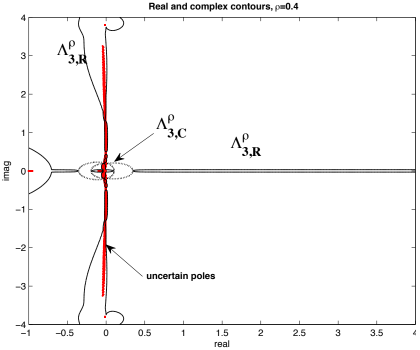
$\Lambda_{\text{struct}} \subset \Lambda_{\text{unstruct}} \overset{!}{\subset} \mathbb{C}^-$

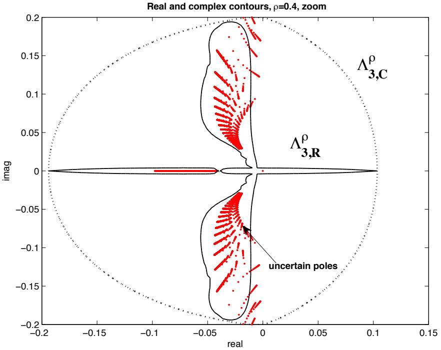
$\Lambda_{\text{struct}} \subset \Lambda_{\text{semi-struct}} \overset{!}{\subset} \mathbb{C}^-$



Pseudo-spectra (Trefethen 1999)

Optimize matrix stability (Burke, Lewis, Overton 2002)





Semi-structured stability radius (Hinrichsen, Pritchard 1989)

Semi-structured distance to instability (Fan, Tits, Doyle 1991)

1. Solve nominal structured H_∞ problem

$$\begin{array}{ll} \text{minimize} & \mathrm{Perf}(K) = \| T_{w\to z}(K)\|_\infty \\ \text{subject to} & K = K(\theta) \end{array}$$

   Optimal value $p_{\mathrm{nom}} = \mathrm{Perf}(K(\theta^*)) =$ nominal performance.

2. If $K(\theta^*)$ is not robustly stable, then solve

$$\begin{array}{ll} \text{maximize} & r_{\mathrm{semi-struct}}\,(A(K), B(K), C(K), D(K)) \\ \text{subject to} & \mathrm{Perf}(K) \le (1+\alpha)p_{\mathrm{nom}} \\ & K = K(\theta) \end{array}$$

Accept α% loss of performance to buy some robustness

Semi-structured stability radius is a H_∞-norm

$$r_{\mathrm{semi-struct}}(A,B,C,D)^{-1} = \|C(sI-A)^{-1}B + D\|_\infty$$

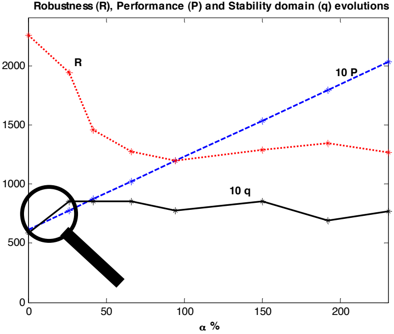
Hinrichsen, Pritchard 1989

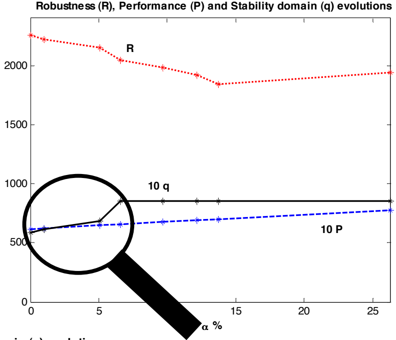
Qiu, Bernhardsson, Rantzer, Davidson, Young, Doyle 1995

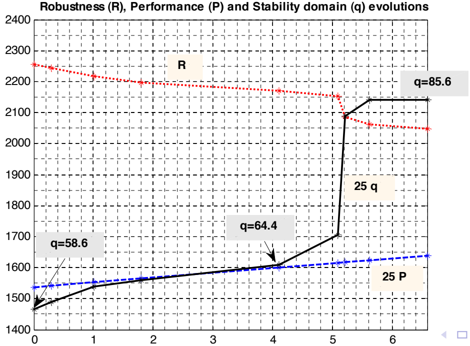
$$\begin{array}{ll} \text{minimize} & \mathrm{Rob}(K) = r_{\mathrm{semi-struct}}\left(A(K), B(K), C(K), D(K)\right)^{-1} \\ \text{subject to} & \mathrm{Perf}(K) \leq (1+\alpha)p_{\mathrm{nom}} \\ & K = K(\theta) \end{array}$$

therefore : constrained H_∞/H_∞-program ($\Longrightarrow$ `hinfstruct`)

Apkarian, Hosseini-Ravanbod, Noll 2010


Robustness (R), Performance (P) and Stability domain (q) evolutions
R
10 P
10 q
2000
1500
1000
500
0
0
50
100
150
200
α %


Robustness (R), Performance (P) and Stability domain (q) evolutions
R
10 q
10 P
2000
1500
1000
500
0
0
5
15
20
25
α %


Robustness (R), Performance (P) and Stability domain (q) evolutions
R
q=85.6
25 q
q=64.4
q=58.6
25 P
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
0
1
2
3
4
5
6