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M2 Internship Proposal

Title : Wave propagation properties in hierarchical neural networks with predictive coding feedback dynamics

Internship summary. The overall objective of this internship is to investigate the qualitative properties of a class of hierarchical neural networks which incorporate predictive coding feedback dynamics. Such types of neural networks with predictive coding naturally appear in the modeling of cortical areas [1, 4], but also in some new developments in the context of deep neural networks [2]. Recently, a mathematical study [3] has systematically investigated the dynamics of such network and determined in which direction, and at what speed neural activity propagates in the network as a function of the hyper-parameters of the model. The model studied in [3] is intrinsically linear and the outstanding objective of this internship will be to analyze the full nonlinear version of the model.

In a first step, the student will get familiar with the class of models under consideration and work on the continuous in time nonlinear version of the neural network studied in [3] in the so-called *identity* regime which fully decouples the system and lead to the study of a scalar functional differential equation on the lattice $\mathbb{Z}$ (in the limit of a network with infinitely many layers). The objective will be in particular to determine the long time behavior of the system as a function of the hyper-parameters and the nonlinearity. From this first step, several possible directions will be considered, such as :

- analysis of the initiation of propagation in the network as a function of the initial condition ;
- analysis of the semi-infinite case with a given input layer ;
- extension of the model beyond the *identity* regime, with fully connected matrices of interactions.

The student will be strongly encouraged to implement simple, or more elaborate, numerical schemes in order to illustrate and/or predict the results proved during the internship.

Prerequisites : some basic PDE courses, some basic skills in scientific computing, theory of integration (L^p spaces, convolution...), the standard results from Fourier analysis (Fourier series and Fourier transform, Parseval-Bessel equality, Plancherel's theorem...), a strong interest in an interdisciplinary project.

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Location : the internship will take place at the Institut de Mathématiques de Toulouse, but regular meetings will be planed at the Centre de Recherche Cerveau et Cognition.

Dates : 5 months from March to July 2024

NB : an internship stipend is possible and this internship may lead to a PhD thesis.

Références

[1] A. Alamia and R. VanRullen. Alpha oscillations and traveling waves : Signatures of predictive coding ? *PLoS Biology*, 17(10) :e3000487, 2019.

[2] B. Choksi, M. Mozafari, C. Biggs O'May, B. Ador, A. Alamia, and R. VanRullen. Predify : Augmenting deep neural networks with brain-inspired predictive coding dynamics. *Advances in Neural Information Processing Systems*, (2021), 34, 14069-14083.

[3] G. Faye, G. Fouilhé and R. VanRullen. Mathematical derivation of wave propagation properties in hierarchical neural networks with predictive coding feedback dynamics. *Bulletin of Mathematical Biology*,vol 85, no 80 (2023) , pp. 1-75.

[4] R.P. Rao and D. H. Ballard. Predictive coding in the visual cortex : a functional interpretation of some extra-classical receptive-field effects. *Nature neuroscience* (1999) vol 2 (1), pp 79-87.