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Stability and asymptotic behavior of numerical schemes in bounded domains

Description. The aim of this Master 2 internship is to study numerical schemes for wave propagation phenomena in bounded domains. Such phenomena arise in various physical contexts and are frequently modeled by partial differential equations of evolutionary type. Some examples include the transport equation, the wave equation or the Schrödinger equation. Performing efficient and accurate numerical simulations of these phenomena heavily relies on a good understanding of the influence of the numerical boundary conditions that are imposed on the boundary of the computational domain. These numerical boundary conditions may have a physical interpretation or may simply arise from the artificial truncation of the spatial domain that has been performed for implementing the simulation.

Depending on the considered equation, the numerical scheme or the geometry of the physical domain, some waves (be they either physical or parasite) may reflect on the boundary. The aim of this internship is to perform both a quantitative and qualitative analysis of such reflections in the model situation of the transport equation on an interval. This framework already encompasses many technical difficulties and includes the most relevant aspects of the theory. The student will first get familiar with the case of the half-line and the characterization of stable numerical schemes by means of the so-called uniform Kreiss-Lopatinskii condition. Under this condition, an accurate description of the asymptotic behavior of solutions to the numerical scheme may be achieved by studying the Green's function. The student will then focus on the case of an interval where the very recently developed stability analysis follows earlier works by Osher. The description of the asymptotic behavior of solutions as well as the possible multiple reflections that may lead to large time instabilities will constitute a natural research direction which the student may follow either during the internship, if time allows, or during a subsequent PhD thesis. Extending the analysis to dispersive phenomena such as described by the Schrödinger equation is another possible research direction for a PhD subject, not mentioning multidimensional problems that are still widely open.

The student will be strongly encouraged to implement simple, or more elaborate, numerical schemes in order to illustrate and/or predict the results proved during the internship.

Prerequisite : for this internship, it is desirable to know the theory of integration (L^p spaces, convolution...), the standard results from Fourier analysis (Fourier series and Fourier transform, Parseval-Bessel equality, Plancherel's theorem...) and some elements of complex analysis (holomorphic and meromorphic functions, calculus of residues...).

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Location : the student will work within the Institut de Mathématiques de Toulouse on the Université Paul Sabatier campus (Toulouse, France).

Duration : 4/5 months from March to June/July 2024.

Gratification : this internship may give rise to payment of a monthly bonus.

References : here are some references that will serve during this internship :

— A. Benoit, Stability of finite difference schemes approximation for hyperbolic boundary value problems in an interval, *Mathematics of Computation*, Vol. 91, 2022.

— J.-F. Coulombel, Stability of finite difference schemes for hyperbolic initial boundary value problems, dans *Nonlinear Hyperbolic PDEs, Dispersive and Transport Equations*, American Institute of Mathematical Sciences, 2013.

— B. Gustafsson, H.-O. Kreiss and J. Oliger, *Time dependent problems and difference methods*, John Wiley & Sons, 1995.

— M. Inglard, F. Lagoutière and H. H. Rugh, Ghost solutions with centered schemes for one-dimensional transport equations with Neumann boundary conditions, *Annales de la Faculté des Sciences de Toulouse*, Tome 29, 2020.