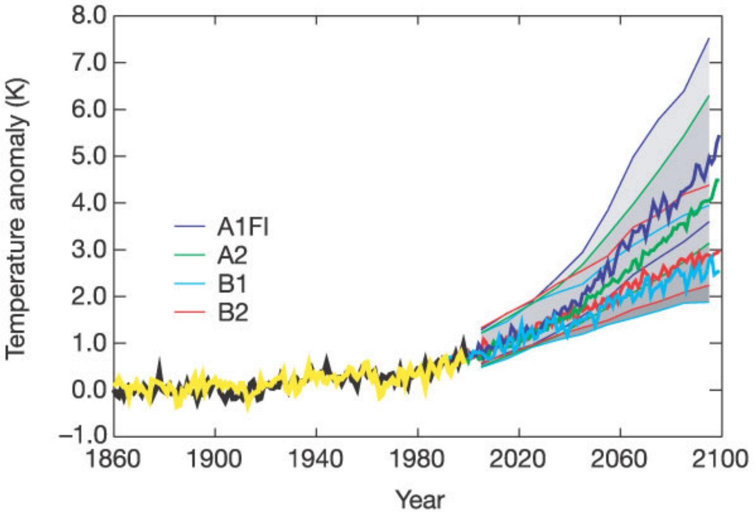
How does pollen dispersal affect species range shift and adaptation under climate change?

Robin Aguilée, Gaël Raoul, François Rousset and Ophélie Ronce

Université Paul Sabatier, Janvier 2016

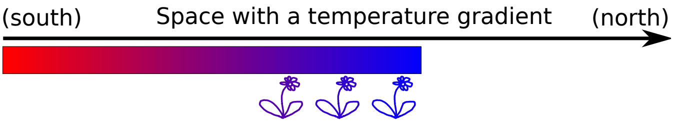


Fast climate change forecasted



(from Stott & Kettleborough, 2002)

How to **escape** from **extinction** under climate change?



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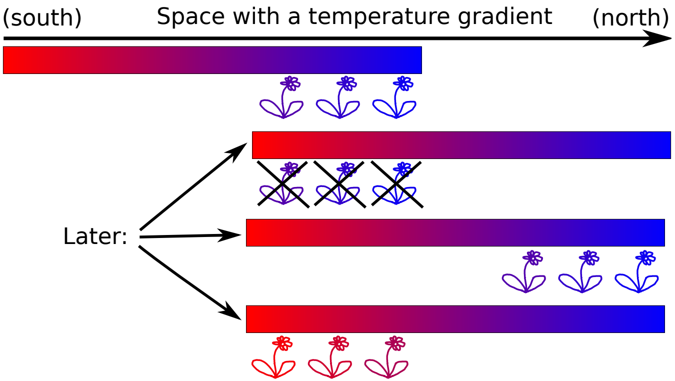

1. Spatial range shift via dispersal
2. Climatic niche shift via *in situ* adaptation

Parmesan (2006), Lavergne et al. (2010), Hoffmann & Sgrò (2011), Bellard et al. (2012)

Effects of **dispersal** and **feedbacks**

Holt & Gaines (1992), Holt (1996a,b), Holt (1997), Ronce & Kirkpatrick (2001), Holt et al. (2003)

Dispersal

Demography

Adaptation

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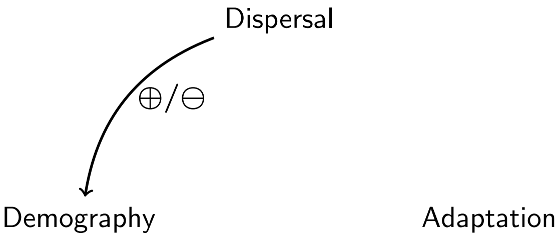

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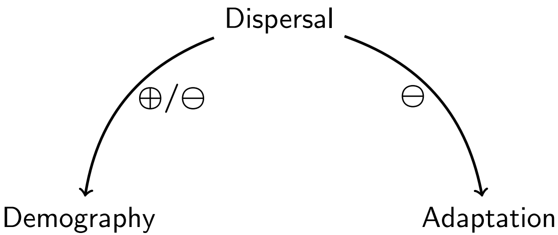

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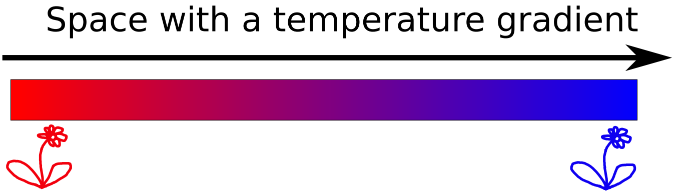

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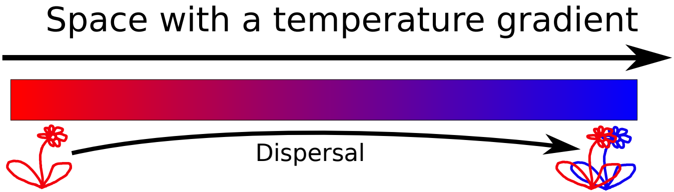

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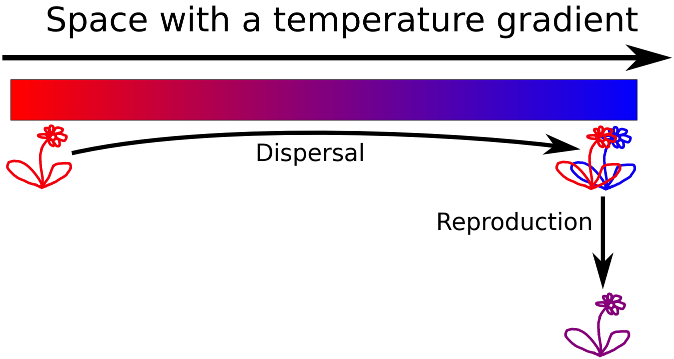

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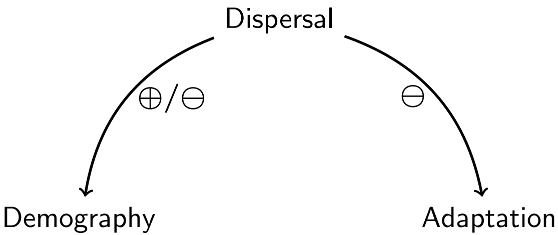

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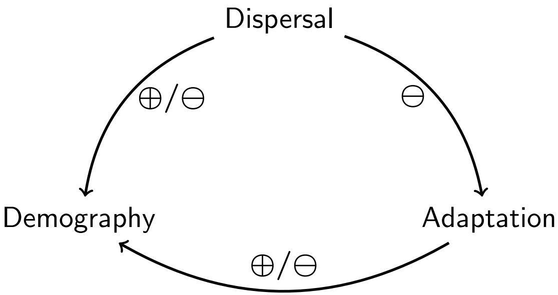

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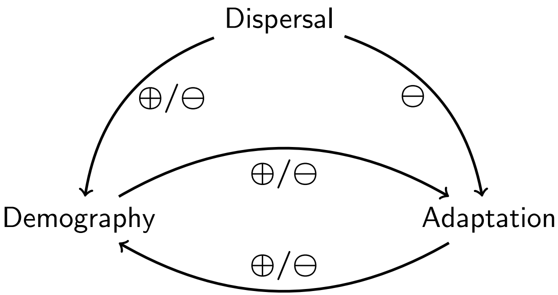
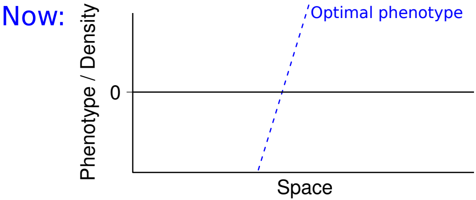


Response to climate shift in **animals**: **spatial** range shift

Pease et al. (1989), Kirkpatrick & Barton (1997), Barton (2001), Polechová et al. (2009), Bridle et al. (2010), Duputié et al. (2012)



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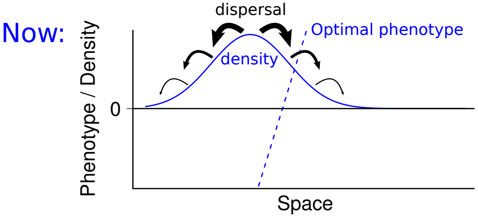

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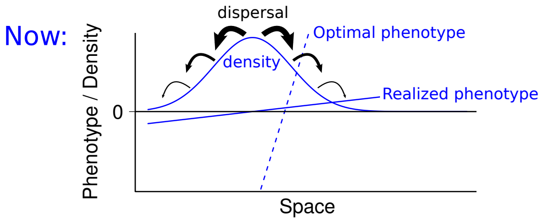

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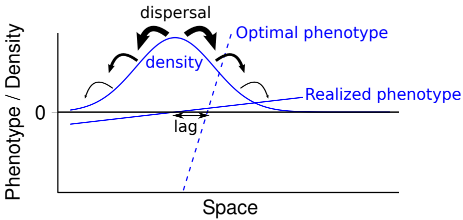

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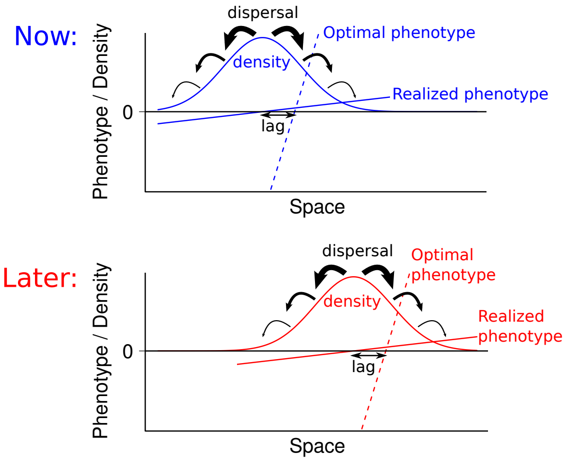

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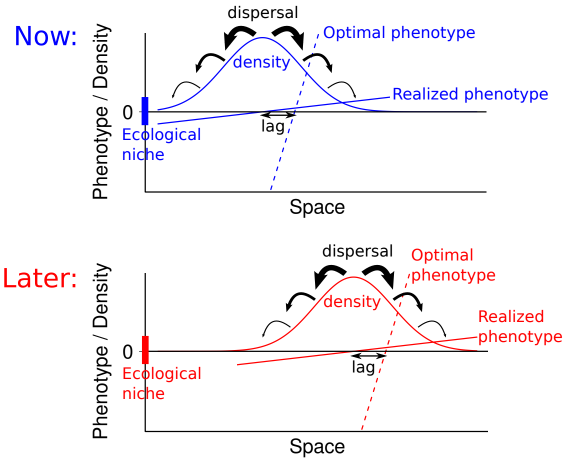
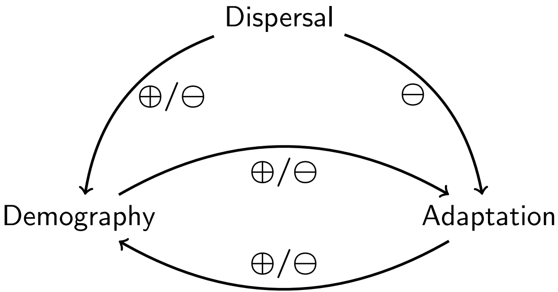


Spatial range shift only (same speed as climate)

Response to climate change with **pollen** dispersal?

Hu & He (2006), Lopez et al. (2008), Aguilée et al. (2013)



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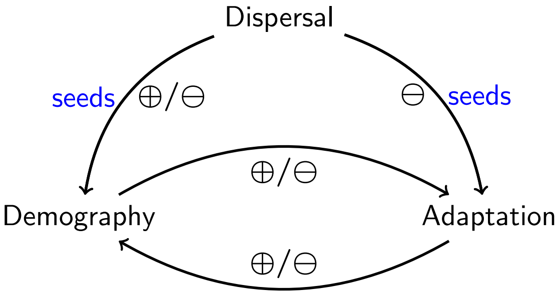

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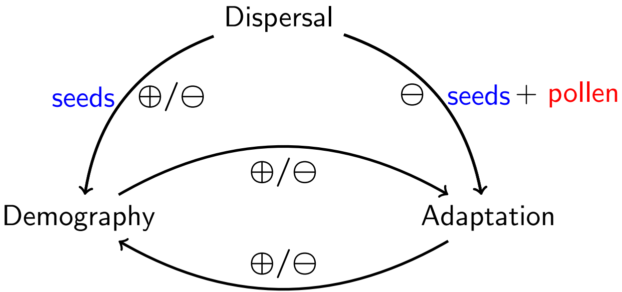

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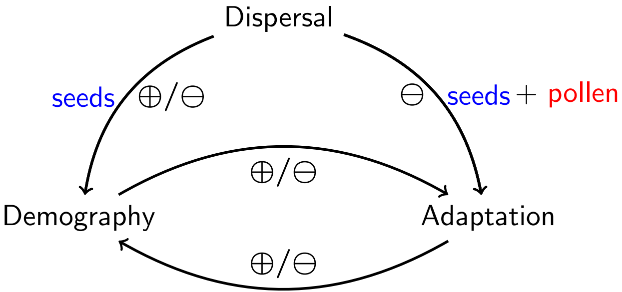

Questions:

- Will a plant species shift its **spatial** range or its **climatic** niche?
- How pollen does affect the **maximal sustainable rate** of climate change?

Outline of the model

A quantitative genetic model for population size $n(x, t)$ and mean phenotype $\bar{z}(x, t)$ with:

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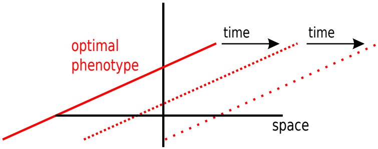
A quantitative genetic model for population size $n(x,t)$ and mean phenotype $\bar{z}(x,t)$ with:

- Linear spatial gradient $\theta(x,t)$ with slope b shifting in time at speed v



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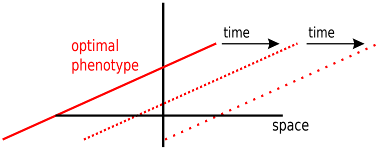

- Seed dispersal σ_s^2 and pollen dispersal σ_p^2 (Gaussian kernel)

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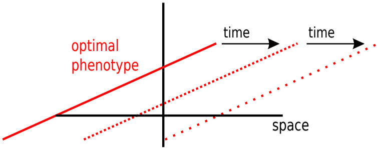

- Seed dispersal σ_s^2 and pollen dispersal σ_p^2 (Gaussian kernel)
- Pollen is not limiting

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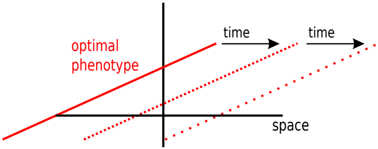

- Seed dispersal σ_s^2 and pollen dispersal σ_p^2 (Gaussian kernel)
- Pollen is not limiting
- Feedbacks between demography and adaptation

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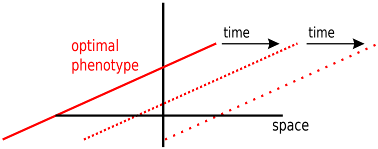


- Seed dispersal σ_{s}^2 and pollen dispersal σ_{p}^2 (Gaussian kernel)
- Pollen is not limiting
- Feedbacks between demography and adaptation

Two strong assumptions:

- Global density dependence
- Constant genetic variance V_{g}

Change in **population size** $n(x, t)$ and mean **phenotype** $\bar{z}(x, t)$

Following Pease et al. (1989), Kirkpatrick & Barton (1997), Polechová et al. (2009), Duputié et al. (2012):

$$\partial_t n = \text{effect of dispersal} + \text{effect of adaptation}$$
$$\partial_t \bar{z} = \text{effect of dispersal} + \text{effect of demography}$$

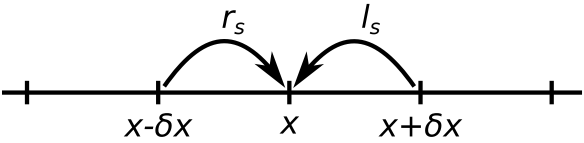
Effect of **dispersal** on **population size** $n(x, t)$

Random walk for seed only (pollen not limiting)

Effect of **dispersal** on **population size** $n(x,t)$

Random walk for seed only (pollen not limiting)

$f(x,t) = \mathbb{P}$(a given seed is at location x at time t)
$l_\mathrm{s} = \mathbb{P}$(seed moves left)
$r_\mathrm{s} = \mathbb{P}$(seed moves right)
δt = small time interval



Effect of **dispersal** on **population size** $n(x,t)$

Random walk for seed only (pollen not limiting)

$f(x,t) = \mathbb{P}$(a given seed is at location x at time t)
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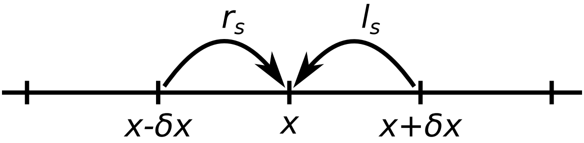

$$f(x,t+\delta t) = r_s f(x-\delta x,t) + l_s f(x+\delta x,t) + (1-r_s-l_s) f(x,t)$$

Effect of **dispersal** on **population size** $n(x,t)$

Random walk for seed only (pollen not limiting)

Using Taylor approximation:

$$
\begin{aligned}
&f(x,t+\delta t) = \\
&\quad r_{\mathrm{s}}\left(f(x,t) - \delta x\,\partial_x f(x,t) + \frac{(\delta x)^2}{2}\partial_{x,x} f(x,t) + o(\delta x)^3\right) \\
&+ l_{\mathrm{s}}\left(f(x,t) + \delta x\,\partial_x f(x,t) + \frac{(\delta x)^2}{2}\partial_{x,x} f(x,t) + o(\delta x)^3\right) \\
&+ (1 - r_{\mathrm{s}} - l_{\mathrm{s}}) f(x,t)
\end{aligned}
$$

Effect of **dispersal** on **population size** $n(x,t)$

Random walk for seed only (pollen not limiting)

Using Taylor approximation:

$$\begin{aligned} & f(x,t+\delta t) = \\ & \quad r_{\mathrm{s}}\left(f(x,t) - \delta x\ \partial_x f(x,t) + \frac{(\delta x)^2}{2}\partial_{x,x} f(x,t) + o(\delta x)^3\right) \\ & +l_{\mathrm{s}}\left(f(x,t) + \delta x\ \partial_x f(x,t) + \frac{(\delta x)^2}{2}\partial_{x,x} f(x,t) + o(\delta x)^3\right) \\ & +(1 - r_{\mathrm{s}} - l_{\mathrm{s}}) f(x,t) \end{aligned}$$

Assuming unbiased dispersal ($l_{\mathrm{s}} = r_{\mathrm{s}} = m_{\mathrm{s}}$):

$$\frac{f(x,t+\delta t) - f(x,t)}{\delta t} = m_{\mathrm{s}} \frac{(\delta x)^2}{\delta t} \partial_{x,x} f(x,t) + \frac{o(\delta x)^3}{\delta t}$$

Effect of **dispersal** on **population size** $n(x, t)$

Random walk for seed only (pollen not limiting)

Taking the limit when $\delta x \to 0$ and $\delta t \to 0$:

$$\partial_t f(x,t) = \frac{\sigma_s^2}{2} \partial_{x,x} f(x,t)$$

where $\sigma_s^2 = \lim_{\delta x \to 0, \delta t \to 0} 2 m_s \frac{(\delta x)^2}{\delta t}$

Effect of **dispersal** on **population size** $n(x,t)$

Random walk for seed only (pollen not limiting)

Taking the limit when $\delta x \to 0$ and $\delta t \to 0$:

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Same reasoning true for all seeds, thus:

$$\partial_t n(x,t) = \frac{\sigma_s^2}{2} \partial_{x,x} n(x,t)$$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x, t)$

Random walk for seed and pollen

$g(x, t)$ = phenotype of a new born individual at t in x
$l_{\mathrm{p}} = \mathbb{P}$(a pollen grain moves left)
$r_{\mathrm{p}} = \mathbb{P}$(a pollen grain moves right)

Effect of **dispersal** on **mean phenotype** $\bar{z}(x,t)$

Random walk for seed and pollen

$g(x,t)$ = phenotype of a new born individual at t in x
$l_\mathrm{p} = \mathbb{P}$(a pollen grain moves left)
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A new born individual in x can originate from:

- a seed dispersing from $x - \delta x$; phenotype $= g(x - \delta x, t)$
- a seed dispersing from $x + \delta x$; phenotype $= g(x + \delta x, t)$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x,t)$

Random walk for seed and pollen

$g(x,t) =$ phenotype of a new born individual at t in x
$l_{\mathrm{p}} = \mathbb{P}$(a pollen grain moves left)
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A new born individual in x can originate from:

- a seed dispersing from $x - \delta x$; phenotype $= g(x - \delta x, t)$
- a seed dispersing from $x + \delta x$; phenotype $= g(x + \delta x, t)$
- an ovule in x fertilized by pollen dispersing from $x - \delta x$; phenotype $= \dfrac{g(x-\delta x, t) + g(x,t)}{2}$
- an ovule in x fertilized by pollen dispersing from $x + \delta x$; phenotype $= \dfrac{g(x+\delta x, t) + g(x,t)}{2}$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x, t)$

Random walk for seed and pollen

Weighting each event by local population density:

$$g(x, t + \delta t) = \frac{\begin{array}{l} r_s g(x - \delta x, t) n(x - \delta x, t) + l_s g(x + \delta x, t) n(x + \delta x, t) \\ + r_p \frac{g(x - \delta x, t) + g(x, t)}{2} n(x - \delta x, t) + l_p \frac{g(x + \delta x, t) + g(x, t)}{2} n(x + \delta x, t) \\ + (1 - r_s - l_s - r_p - l_p) g(x, t) n(x, t) \end{array}}{\begin{array}{l} r_s n(x - \delta x, t) + l_s n(x + \delta x, t) + r_p n(x - \delta x, t) + l_p n(x + \delta x, t) \\ + (1 - r_s - l_s - r_p - l_p) n(x, t) \end{array}}$$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x,t)$

Random walk for seed and pollen

Assuming unbiased dispersal ($l_s = r_s = m_s$ and $l_p = r_p = m_p$) and denoting $m_t = m_s + \frac{m_p}{2}$:

$$g(x, t+\delta t) - g(x,t) =$$

$$\frac{m_t((g(x-\delta x, t) - g(x,t))n(x-\delta x) + (g(x+\delta x, t) - g(x,t))n(x+\delta x))}{(m_s + m_p)(n(x-\delta x, t) + n(x+\delta x, t)) + (1 - 2m_s - 2m_p)n(x,t)}$$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x,t)$

Random walk for seed and pollen

Using Taylor approximation of g and n:

$$\frac{g(x,t+\delta t)-g(x,t)}{\delta t} =$$

$$\frac{m_{\mathrm{t}}\frac{(\delta x)^2}{\delta t}\partial_{x,x}g(x,t)\; n(x,t) + 2m_{\mathrm{t}}\frac{(\delta x)^2}{\delta t}\partial_x g(x,t)\;\partial_x n(x,t) + o(\delta x)^3}{(m_{\mathrm{s}}+m_{\mathrm{p}})(n(x-\delta x,t)+n(x+\delta x,t)) + (1-2m_{\mathrm{s}}-2m_{\mathrm{p}})n(x,t)}$$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x, t)$

Random walk for seed and pollen

Taking the limit when $\delta x \to 0$ and $\delta t \to 0$:

$$\partial_t g(x,t) = \frac{\sigma_{\mathrm{t}}^2}{2} \partial_{x,x} g(x,t) + \sigma_{\mathrm{t}}^2 \partial_x g(x,t)\ \partial_x \log(n(x,t))$$

where $\sigma_{\mathrm{t}}^2 = \sigma_{\mathrm{s}}^2 + \frac{1}{2}\sigma_{\mathrm{p}}^2$ and $\sigma_{\mathrm{p}}^2 = \lim_{\delta x \to 0, \delta t \to 0} 2 m_{\mathrm{p}} \frac{(\delta x)^2}{\delta t}$

Effect of **dispersal** on **mean phenotype** $\bar{z}(x,t)$

Random walk for seed and pollen

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Same reasoning true for all births, thus:

$$\partial_t \bar{z}(x,t) = \frac{\sigma_{\mathrm{t}}^2}{2}\partial_{x,x} \bar{z}(x,t) + \sigma_{\mathrm{t}}^2 \partial_x \bar{z}(x,t)\ \partial_x \log(n(x,t))$$

Effect of **adaptation** on **population size** $n(x,t)$

$$\partial_t n(x,t) = n(x,t)\bar{r}(x,t,\bar{z})$$

where $\bar{r}(x,t,\bar{z})$ is the mean growth rate

Effect of **adaptation** on **population size** $n(x,t)$

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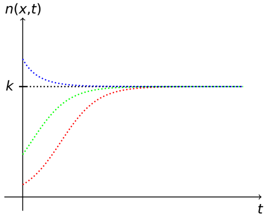
where $\bar{r}(x,t,\bar{z})$ is the mean growth rate

$$\bar{r}(x,t,\bar{z}) \; = r_0\left(1 - \frac{n(x,t)}{k}\right)$$

Density-dependent growth rate



Effect of adaptation on population size $n(x,t)$

$$\partial_t n(x,t) = n(x,t)\bar{r}(x,t,\bar{z})$$

where $\bar{r}(x,t,\bar{z})$ is the mean growth rate

$$\bar{r}(x,t,\bar{z}) = r_0\left(1 - \frac{\lambda}{k}\right)$$

Density-dependent growth rate

Global density-dependance, $\lambda = \int n(x',t)\,dx'$

Effect of **adaptation** on **population size** $n(x,t)$

$$\partial_t n(x,t) = n(x,t)\bar{r}(x,t,\bar{z})$$

where $\bar{r}(x,t,\bar{z})$ is the mean growth rate

$$\bar{r}(x,t,\bar{z}) = r_0\left(1-\frac{\lambda}{k}\right) - \frac{(\bar{z}(x,t)-\theta(x,t))^2}{2V_s}$$

Density-dependent growth rate
Global density-dependance,
$\lambda = \int n(x',t)\,dx'$

Evolutionary load, i.e. maladaptation
$\theta(x,t) = b(x-vt)$

Effect of **adaptation** on **population size** $n(x,t)$

$$\partial_t n(x,t) = n(x,t)\bar{r}(x,t,\bar{z})$$

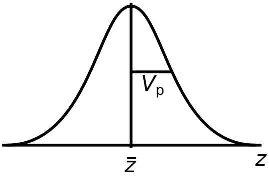
where $\bar{r}(x,t,\bar{z})$ is the mean growth rate

$$\bar{r}(x,t,\bar{z}) = r_0\left(1-\frac{\lambda}{k}\right) - \frac{(\bar{z}(x,t)-\theta(x,t))^2}{2V_s} - \frac{V_p}{2V_s}$$

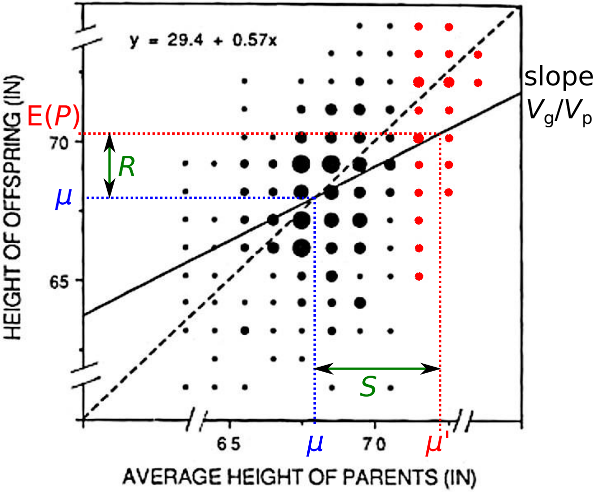
Density-dependent growth rate
Global density-dependance,
$\lambda = \int n(x',t)\,dx'$

Evolutionary load, i.e. maladaptation
$\theta(x,t) = b(x - vt)$

Phenotypic load



Effect of **demography** on **mean phenotype** $\bar{z}(x, t)$



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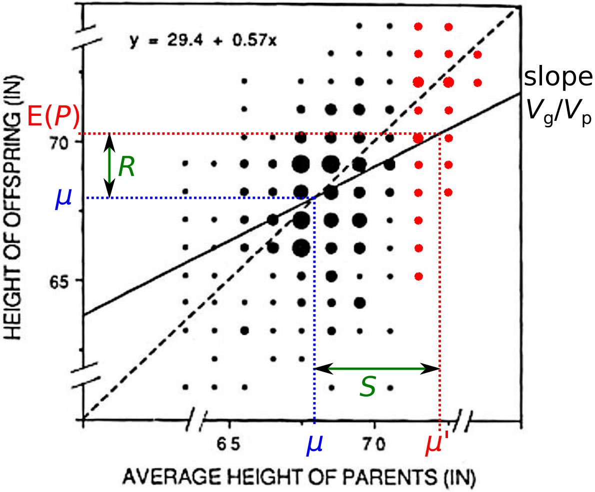
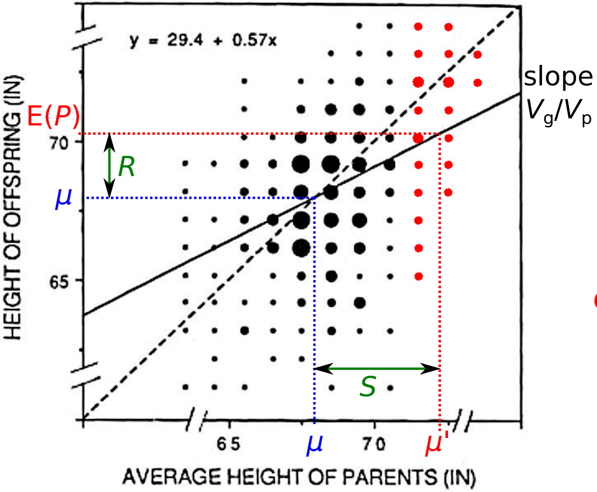


$$R = \frac{V_g}{V_p} S$$

Effect of **demography** on **mean phenotype** $\bar{z}(x,t)$



$$R = \frac{V_\mathrm{g}}{V_\mathrm{p}} S$$

$$\Rightarrow$$

$$\partial_t \bar{z}(x,t) = V_\mathrm{g} \; \partial_{\bar{z}} \bar{r}(x,t,\bar{z})$$

Constant genetic variance

Change in **population size** $n(x,t)$

Following Pease et al. (1989), Kirkpatrick & Barton (1997), Polechová et al. (2009), Duputié et al. (2012):

$$\partial_t n \; =$$

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- Seed dispersal (diffusion)

Change in **population size** $n(x,t)$

Following Pease et al. (1989), Kirkpatrick & Barton (1997), Polechová et al. (2009), Duputié et al. (2012):

$$\partial_t n = \frac{\sigma_s^2}{2}\partial_{x,x} n + n\bar{r}$$

- Seed dispersal (diffusion) → $\frac{\sigma_s^2}{2}\partial_{x,x} n$
- Population growth → $n\bar{r}$

with

$$\bar{r} = r_0\left(1 - \frac{\lambda}{k}\right) - \frac{(\bar{z}-\theta)^2}{2V_s} - \frac{V_p}{2V_s}$$

Density dependence → $r_0\left(1 - \frac{\lambda}{k}\right)$

Evolutionary load → $\frac{(\bar{z}-\theta)^2}{2V_s}$

Phenotypic load → $\frac{V_p}{2V_s}$

Change in mean **phenotype** $\bar{z}(x,t)$

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$$\partial_t \bar{z} \ =$$

Change in mean **phenotype** $\bar{z}(x, t)$

Following Pease et al. (1989), Kirkpatrick & Barton (1997), Polechová et al. (2009), Duputié et al. (2012):

$$\partial_t \bar{z} = \frac{\sigma_{\mathrm{t}}^2}{2} \partial_{x,x} \bar{z} + \sigma_{\mathrm{t}}^2 \, \partial_x \bar{z} \, \partial_x \log(n)$$

- Seed and pollen diffusion
- Seed and pollen asymmetrical dispersal

Total dispersal: $\sigma_{\mathrm{t}}^2 = \sigma_{\mathrm{s}}^2 + \frac{1}{2}\sigma_{\mathrm{p}}^2$

Change in mean **phenotype** $\bar{z}(x,t)$

Following Pease et al. (1989), Kirkpatrick & Barton (1997), Polechová et al. (2009), Duputié et al. (2012):

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- Seed and pollen diffusion
- Seed and pollen asymmetrical dispersal
- Response to selection

Total dispersal: $\sigma_\mathrm{t}^2 = \sigma_\mathrm{s}^2 + \frac{1}{2}\sigma_\mathrm{p}^2$

Rescaled equations

Following Kirkpatrick & Barton (1997):

$$R_0 = r_0 - \frac{V_\mathrm{p}}{2V_\mathrm{s}}$$
$$X = \frac{\sqrt{2R_0}}{\sigma_\mathrm{s}} x$$
$$T = R_0 t$$
$$K = k\frac{R_0}{r_0}$$
$$\Lambda = \frac{\lambda}{K}$$
$$Z = \frac{\bar{z}}{\sqrt{R_0 V_\mathrm{s}}}$$
$$N = \frac{n}{K}$$

Rescaled equations

After rescaling, only 4 parameters:

- $\gamma = \frac{\frac{1}{2}\sigma_p^2}{\sigma_t^2}$ = contribution of pollen to dispersal
- $V = v\frac{\sqrt{2}}{\sigma_s\sqrt{R_0}}$ speed of climate change
- $A = \frac{V_g}{R_0 V_s}$ adaptive potential
- $B = b\frac{\sigma_s}{R_0\sqrt{2V_s}}$ slope of the optimal gradient

$$\partial_T N = \partial_{X,X} N + NR$$
$$\partial_T Z = \frac{1}{1-\gamma}\partial_{X,X} Z + \frac{2}{1-\gamma}\partial_X Z \; \partial_X \log(N) - A\;\partial_Z R$$

with $R = 1 - \Lambda - \frac{1}{2}(Z-\Theta)^2$ and $\Theta = B(X - VT)$

The 3 solutions of the model

- Extinction of the population

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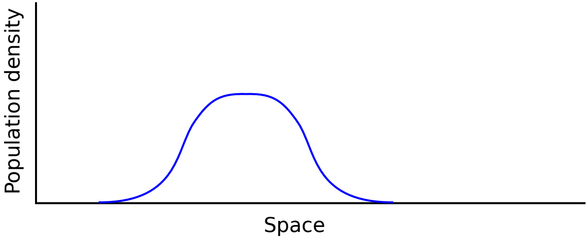
- Extinction of the population
- Invasion of the whole space



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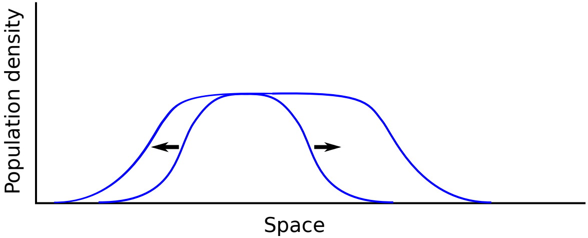

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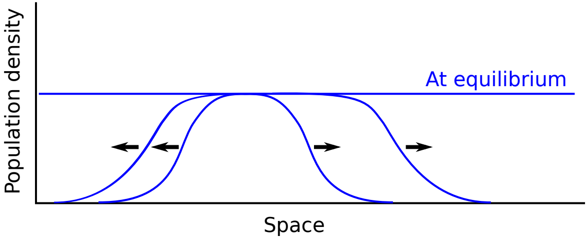
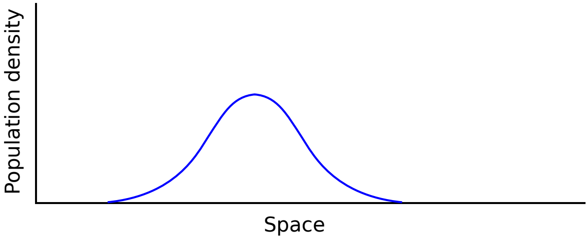


The 3 solutions of the model

- Extinction of the population
- Invasion of the whole space
- A travelling wave



The 3 solutions of the model

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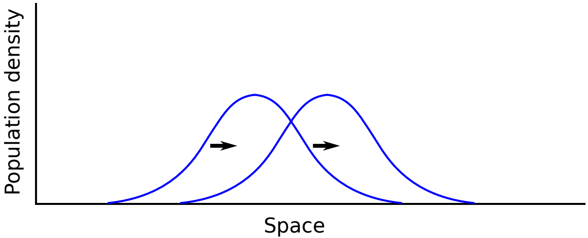
- A travelling wave



With pollen: spatial range shift **and** climatic niche shift

Let's assume there is a solution with spatial range shift and climatic niche shift:

$$N(X,T) = N_0 \exp\left(-\frac{(X - CT - L_{\mathrm{n}})^2}{2V_{\mathrm{n}}}\right)$$

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$$Z(X,T) = S(X - CT - L_{\mathrm{n}})$$

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$$N(X,T) = N_0 \exp\left(-\frac{(X - CT - L_n)^2}{2V_n}\right)$$

Spatial range shift at speed C

$$Z(X,T) = S(X - CT - L_n) + DT$$

Ecological niche shift at speed D

With pollen: spatial range shift **and** climatic niche shift

Such solution indeed exists with:

$$S = \mathrm{sign}(B)\frac{A}{\sqrt{2}}(1-\gamma)$$

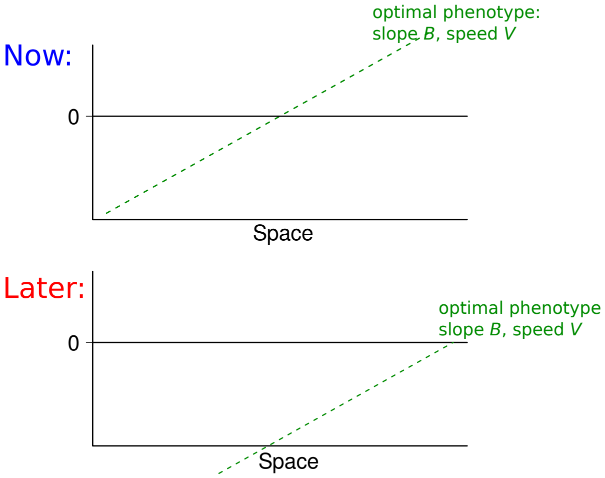
$$V_\mathrm{n} = \frac{2}{|B|\sqrt{2} - A(1-\gamma)}$$

$$C = \frac{V}{1 + \frac{A}{|B|\sqrt{2}}\gamma}$$

$$L_\mathrm{n} = -\frac{V}{|B|\sqrt{2} + A\gamma}$$

$$D = -\frac{ABV\gamma}{|B|\sqrt{2} + A\gamma}$$

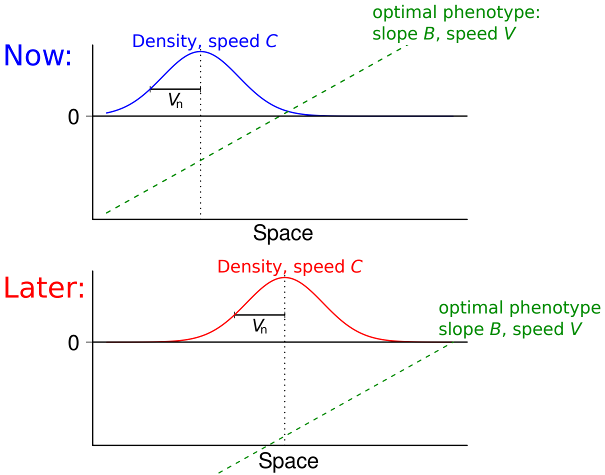
With pollen: spatial range shift **and** climatic niche shift



With pollen: spatial range shift **and** climatic niche shift

V_{n} = size of the range

C = speed of spatial range shift: **different** from climate



$\Rightarrow$ spatial range shift

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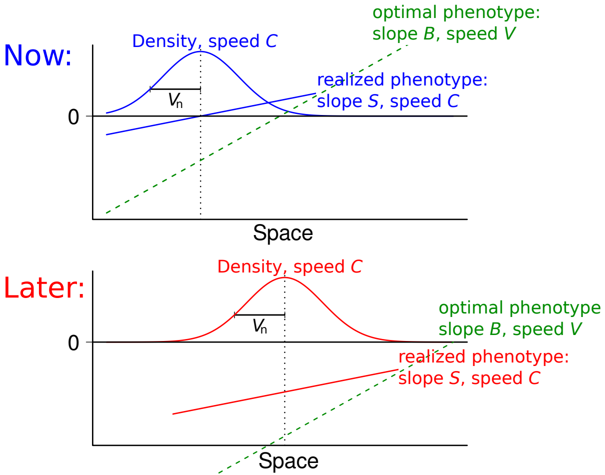

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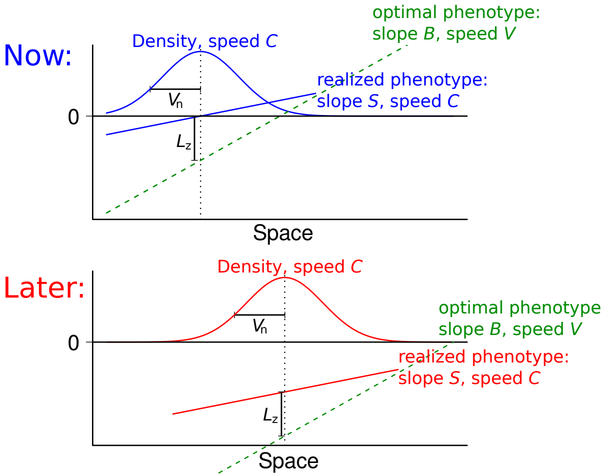

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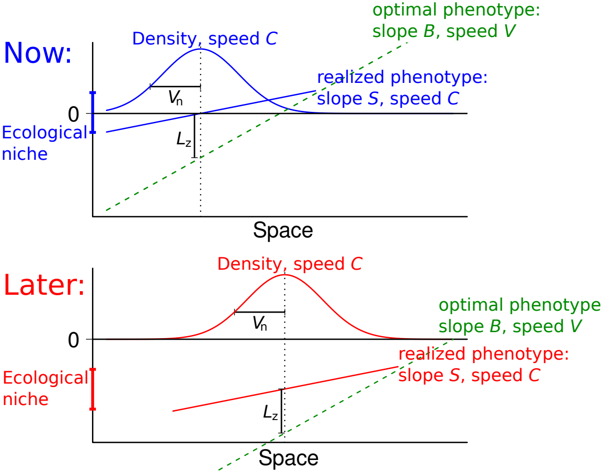


$\Rightarrow$ spatial range shift and ecological niche shift

With pollen: spatial range shift **and** climatic niche shift

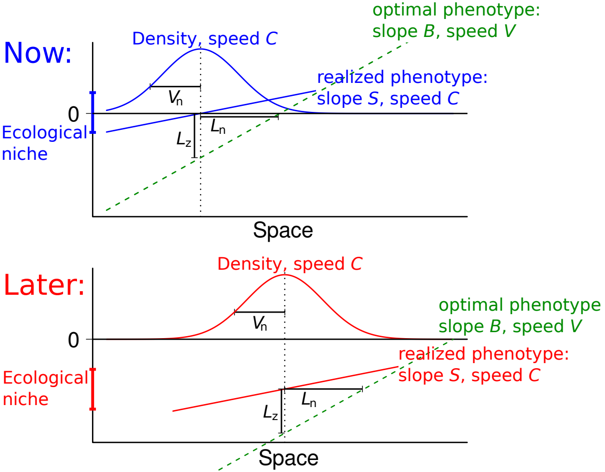
V_n = size of the range

C = speed of spatial range shift: **different** from climate

S = slope of the realized phenotype

L_z = maladaptation at the core of the range

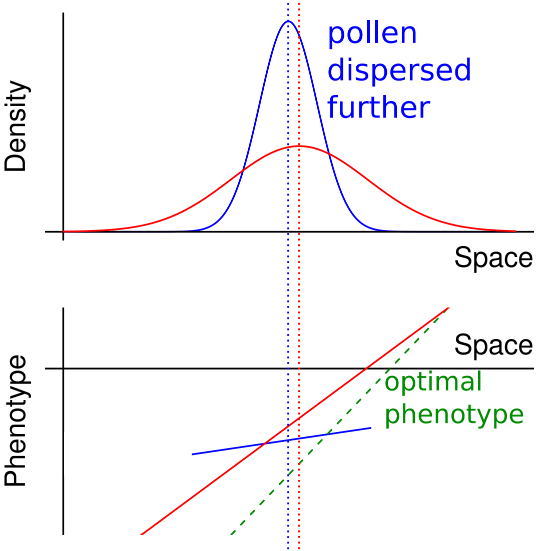
L_n = spatial lag



$\Rightarrow$ spatial range shift and ecological niche shift

Effect of pollen (γ) on the features of the travelling wave

Blue = pollen dispersed further



Effect of pollen (γ) on the features of the travelling wave

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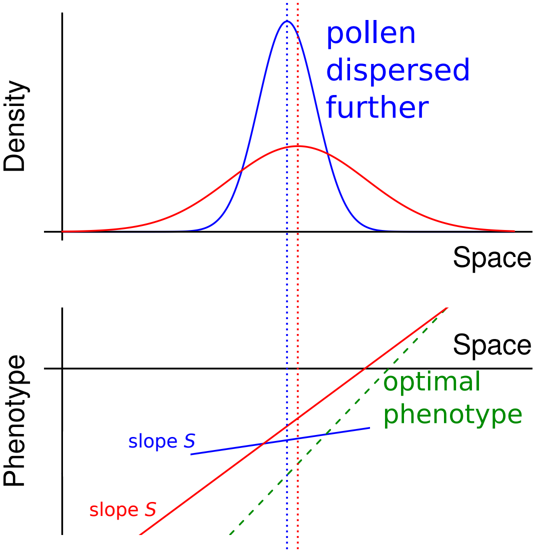
- Flatter phenotypic cline ($|S|$)



Effect of pollen (γ) on the features of the travelling wave

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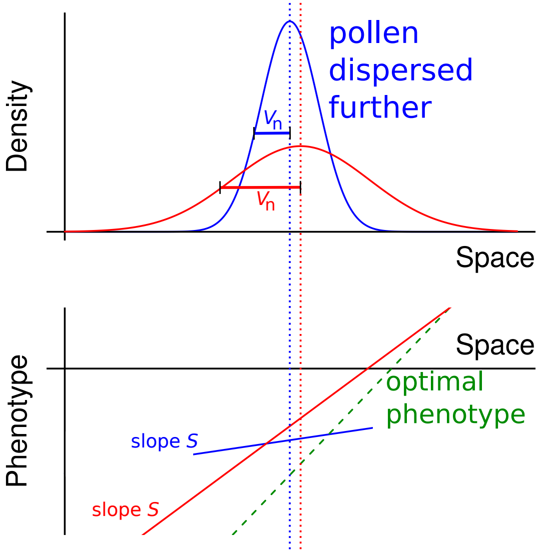
- Flatter phenotypic cline ($|S|$)
- Narrower range (V_n)



Effect of pollen (γ) on the features of the travelling wave

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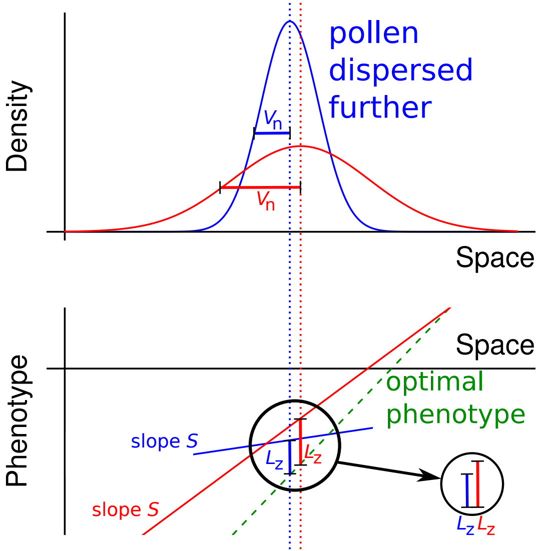

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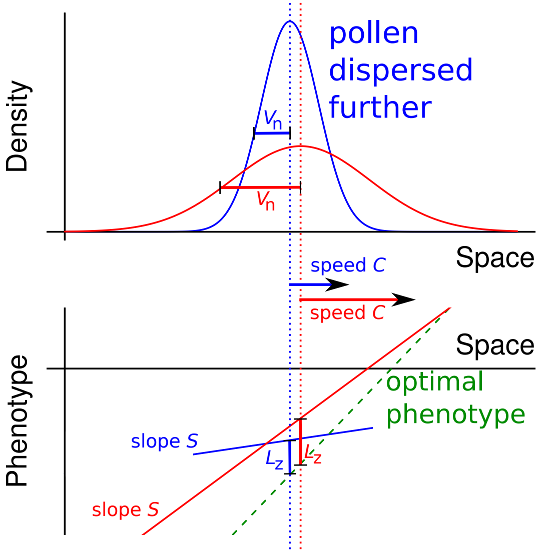
- Slower wave (C)



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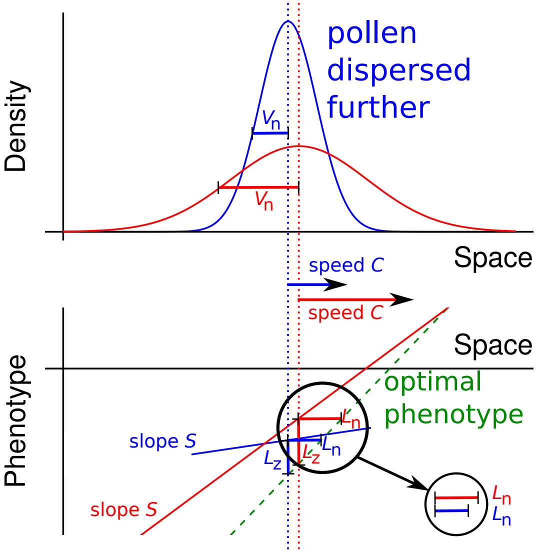
- Slower wave (C)
- Closer track of the optimum ($|L_\mathrm{n}|$)



Maximal **sustainable** rate of climate change

Sustainable climate change if $\Lambda = 1 - \frac{1}{V_{\mathrm{n}}} - \frac{{L_{\mathrm{z}}}^2}{2} > 0$, i.e. if:

$$V < V^{\mathrm{crit}} = 2\left(1 + \frac{A}{|B|\sqrt{2}}\gamma\right)\sqrt{1 - \frac{|B|\sqrt{2}}{2} + \frac{A}{2}(1-\gamma)}$$

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- Positive effect of better adaptation at the core ($|L_z|$)
- Negative effect of smaller range size (V_n)

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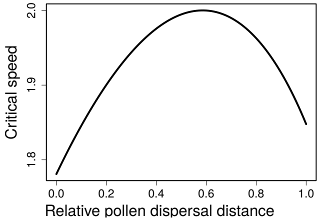

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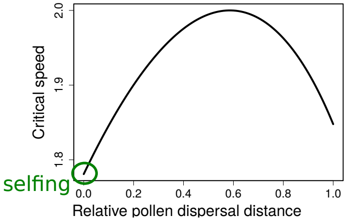

Pollen dispersal may allow to persist under faster climate changes than without pollen dispersal

Robustness of the results

Two strong assumptions to relax:

1. Density dependence: global $\rightarrow$ local
2. Genetic variance: constant $\rightarrow$ evolving

With **local** density dependence: methods

Global density dependence

$$\bar{r}(x,t,\bar{z}) = r_0\left(1-\frac{\lambda}{k}\right) - \frac{(\bar{z}-\theta)^2}{2V_\mathrm{s}} - \frac{V_\mathrm{p}}{2V_\mathrm{s}}$$

With **local** density dependence: methods

Global density dependence

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Local density dependence

$$\bar{r}(x,t,\bar{z}) = r_0\left(1 - \frac{n(x,t)}{k}\right) - \frac{(\bar{z}-\theta)^2}{2V_s} - \frac{V_p}{2V_s}$$

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- Resolution of the equations with numerical integration

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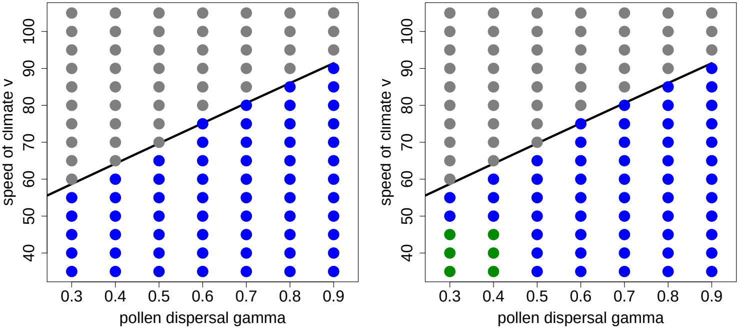
- Resolution of the equations with numerical integration
- Parameters value as estimated for Sitka spruce (*Picea sitchensis*) (Mimura and Aitken, 2007; Aitken et al., 2008)

With local density dependence: results are **robust**

Maximal sustainable rate of climate change:

Global density dependence Local density dependence

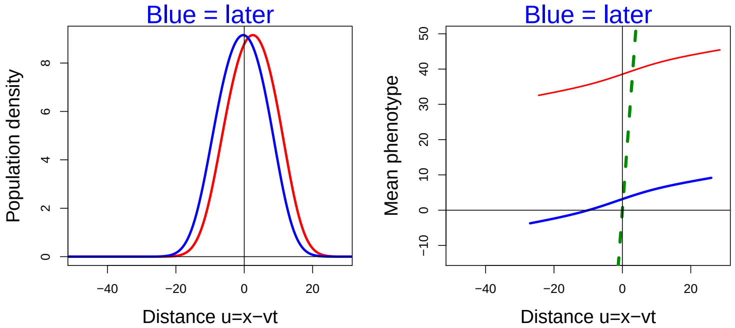


Blue = Travelling wave Gray = Extinction

Green = Invasion of the whole space

With local density dependence: results are **robust**

A travelling wave with spatial range shift and ecological niche shift:



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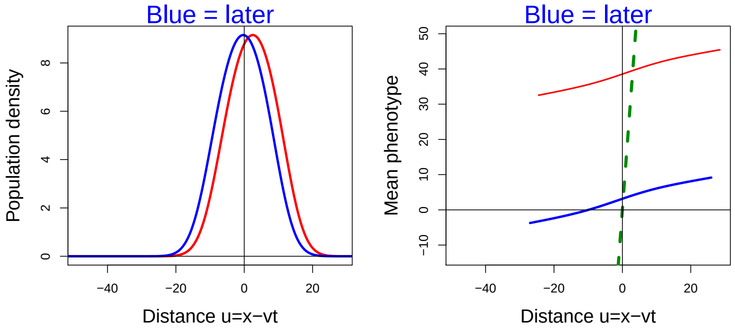
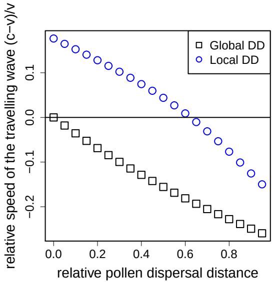


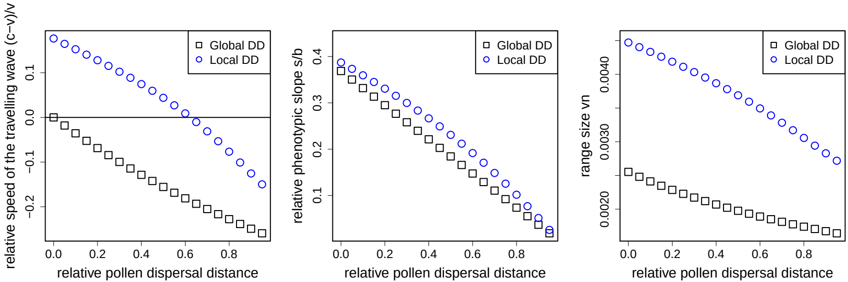
Possibly asymmetrical travelling wave (more individuals at the leading edge)

With local density dependence: results are **robust**



- Faster travelling wave
- Pollen still decreases the speed of the travelling wave (and magnifies the climatic niche shift)

With local density dependence: results are **robust**



- Faster travelling wave
- Pollen still decreases the speed of the travelling wave (and magnifies the climatic niche shift)
- Quite small quantitative effect

With **evolving** genetic variance: methods

- Genotype centered model

$$\partial_t n(x, t, g) = \text{dispersal} + \text{births} + \text{deaths}$$

With **evolving** genetic variance: methods

- Genotype centered model

$$\partial_t n(x, t, g) = \text{dispersal} + \text{births} + \text{deaths}$$

- Explicit model of genes inheritance

With **evolving** genetic variance: methods

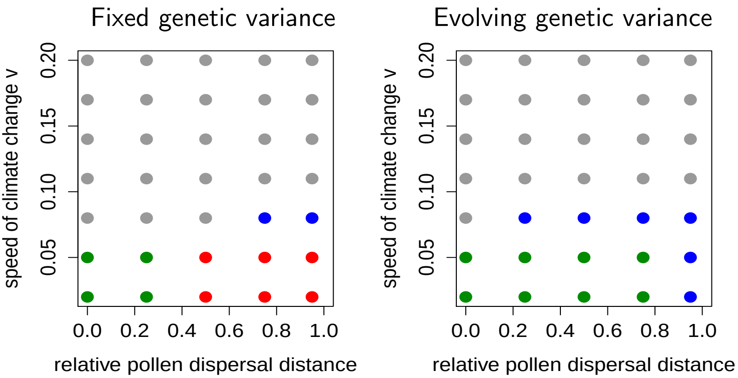
- Genotype centered model

$$\partial_t n(x, t, g) = \text{dispersal} + \text{births} + \text{deaths}$$

- Explicit model of genes inheritance
- Local density dependence
- Numerical resolution
- Parameters value as estimated for Sitka spruce

With evolving genetic variance: results are **robust**

Maximal sustainable rate of climate change:



Gray = Extinction

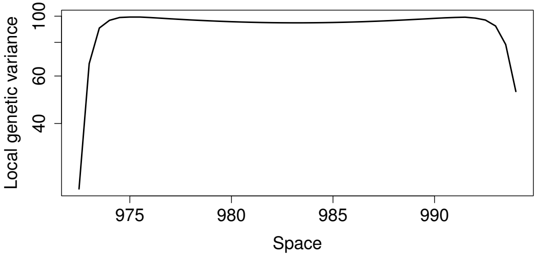
Blue = Travelling wave

Green = Invasion of the whole space

Red = Travelling wave or Invasion of the whole space

With evolving genetic variance: results are **robust**

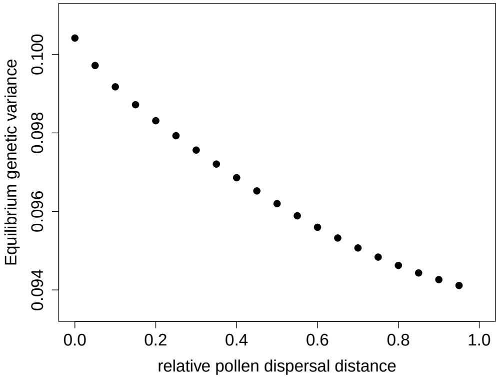
An equilibrium genetic variance is reached:



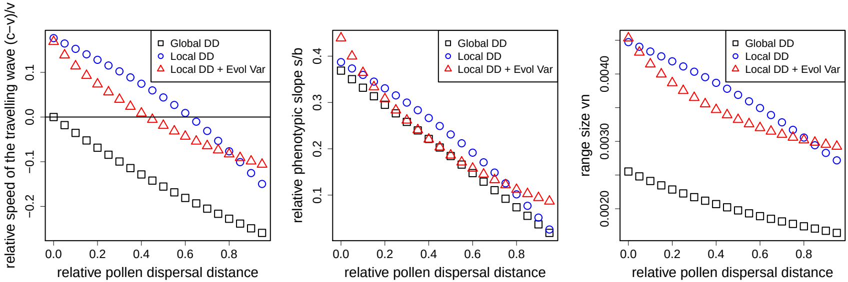
- Lower genetic variance at the very edges
- Slightly lower genetic variance at the core

With evolving genetic variance: results are **robust**

The equilibrium genetic variance slightly decreases with the relative pollen dispersal distance:



With evolving genetic variance: results are **robust**



- Qualitative effect of pollen dispersal unchanged
- Quantitative effect quite small

Take Home Messages (for biologists)

- Genetic effect of pollen dispersal and feedbacks with demography: worsens adaptation at the margins but improves adaptation at the core

Take Home Messages (for biologists)

- Genetic effect of pollen dispersal and feedbacks with demography: worsens adaptation at the margins but improves adaptation at the core
- Response to climate change in plants: spatial range shift and climatic niche shift

Take Home Messages (for biologists)

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- Don't you want to work with me?