Thematic semester on

Mathematics Computer science and biology

CIMI – Centre International de Mathématiques et d'Informatique TOULOUSE

Deterministic and Stochastic Models in Neurosciences

WINTER SCHOOL

Research courses

- François Delarue
- Eva Löcherbach
- Delphine Salort
- Romain Veltz

Invited speakers

- Daniele Avitabile
- Bruno Cessac
- Zachary Kilpatrick
- Carlo Laing
- Cristobal Quininao
- Wilhelm Stannat

Organizers
Patrick Cattiaux & Grégory Faye.

http://www.cimi.univ-toulouse.fr/mib/

From the 11th to the 15th of December 2017

Institut de Mathématiques de Toulouse,
Amphithéâtre Laurent Schwartz,
Bâtiment 1R3

Conception graphique : Benoît Colas, Université Toulouse - Jean Jaurès / CPRS - DAR. Illustration : © Golnaz Behrouznia.

Winter School on Deterministic and Stochastic Models in Neuroscience 11-15/12/17

Invited speakers :

- Daniele Avitabile (`daniele.avitabile@nottingham.ac.uk`)
- Bruno Cessac (`bruno.cessac@inria.fr`)
- François Delarue (`delarue@unice.fr`)
- Zachary Kilpatrick (`zpkilpat@colorado.edu`)
- Carlo Laing (`c.r.laing@massey.ac.nz`)
- Eva Löcherbach (`eva.loecherbach@u-cergy.fr`)
- Cristobal Quiñinao (`cristobal.quininao@uoh.cl`)
- Delphine Salort (`delphine.salort@upmc.fr`)
- Wilhelm Stannat (`stannat@math.tu-berlin.de`)
- Romain Veltz (`romain.veltz@inria.fr`)

Location : Amphitheater Laurent Schwartz, Institut de Mathématiques de Toulouse, Building 1R3

Schedule of the winter school :

	Monday	Tuesday	Wednesday	Thursday	Friday
9h15-10h45	E. Löcherbach	R. Veltz	D. Salort	F. Delarue	D. Salort
11h15-12h45	R. Veltz	E. Löcherbach	B. Cessac	D. Salort	F. Delarue
	Lunch	Lunch	Lunch	Lunch	
14h30-15h30	C. Laing	W. Stannat	Z. Kilpatrick	D. Avitabile	
16h00- 17h30	E. Löcherbach	R. Veltz		F. Delarue	
17h30 - 18h30		C. Quiñinao			
19h		Buffet			

Table 1

⇝ **Monday :** coffee will be served from 8h30 to 9h

⇝ **Monday :** welcome and opening of the winter school from 9h to 9h15

⇝ **Wednesday :** morning sessions will be shifted : 9h30/11h (D. Salort) - 11h30/12h30 (B. Cessac)

Titles & Abstracts

Research courses

François Delarue (Laboratoire J.A. Dieudonné, University of Nice-Sophia Antipolis)

Title : Neuronal networks driven by a singular mean field self-excitation.
Abstract : The goal of these three lectures is to introduce a mean field model for a neural network with excitatory interactions. The key feature of this model is that it exhibits different behaviors depending on the value of the excitation parameter. We shall first introduce the finite model. It includes a large (but finite) population of interacting neurons. Inspired by usual results on McKean-Vlasov equations, we shall derive formally the limiting equation as the number of neurons tends to infinity. We shall address the solvability of the limiting equation in a second step. As a noticeable fact, we shall prove that a singularity may emerge when the excitation parameter driving the interactions between the neurons grows up. Conversely, we shall prove there is no singularity when the excitation parameter is small enough. Finally, we shall justify the passage from the finite to the limiting model. Possibly, we shall also discuss some extensions, including cases when the interaction graph between the neurons is not complete. The lecture will be mostly based on the following two papers :

Delarue F., Inglis J., Rubenthaler R., Tanré, E. (2015). Global solvability of a networked integrate-and-fire model of McKean-Vlasov type. Annals of Applied Probability, 2015, 2096–2133.

Delarue F., Inglis J., Rubenthaler R., Tanré E. (2015). Particle systems with a singular mean-field self-excitation. Application to neuronal networks. Stochastic Processes and their Applications, 125, pp.2451-2492

Eva Löcherbach (Département de Mathématiques, University of Cergy-Pontoise)

Title : Modeling interacting networks as processes with variable length.
Abstract : A class of recently introduced models to describe networks of neurons as stochastic processes with memory of variable length will be presented. These are non-Markovian processes in high or infinite dimension in which the past dependence of transition probabilities or intensities has a range that is finite but depends on the particular history. Starting from existence results and results on perfect simulation, we study related mean-field models in continuous time and their large population limits, and discuss the relation with associated Piecewise Deterministic Markov Processes (PDMP's) and state results concerning their longtime behavior. Finally, we will look at two important problems of statistical inference in such models : estimation of the spiking rate function and estimation of the neuronal interaction graph.

Delphine Salort (Laboratory of Computational and Quantitative Biology, University of Pierre and Marie Curie)

Title : Mathematical deterministic models in neurosciences.
Abstract : The aim of this course is to present some mathematical deterministic tools and models to study qualitative dynamics of single neuron and interacting neurons in a network. We will first consider some simple ordinary differential equation models for the modelling of single neurons in order

to show how very simple mathematical models are able to capture some classical dynamic observed in a neuron. We will then explore the case of interacting neurons which communicate between them via they mean activity, using partial differential equations. We will particularly focus on two choices of descriptions : the time elapsed model and the Leaky Integrate and Fire model.

Romain Veltz (MathNeuro Team, Inria Sophia-Antipolis)

Title : Invariants manifolds for dynamical models in neurosciences.
Abstract : In these lectures, we focus on two methods for the analysis of bifurcations in infinite dimensions e.g.the center manifold and the normal form theory. The center manifold in a finite dimensional invariant manifold for the dynamics and the normal form theory allows the study of the flow restricted to it. We then study various applications of these tools to neural hallucinations in the visual cortex and to the effects of propagation delays in spatially extended neural networks, both applications involve spontaneous symmetry breaking.

Invited talks

Daniele Avitabile (School of Mathematical Sciences, University of Nottingham)

Title : Analysing coherent structures via interfacial dynamics : from spatio-temporal canards to coarse-grained computations.
Abstract : I will discuss level-set based approaches to study the existence and bifurcation structure of spatio-temporal patterns in biological neural networks. Using this framework, which extends previous ideas in the study of neural field models, we study the first example of canards in an infinite- dimensional dynamical system, and perform a computational reduction of dimensionality in certain neural network models.

I will initially consider a spatially-extended network with heterogeneous synaptic kernel. Interfacial methods allow for the explicit construction of a bifurcation equation for localised steady states, so that analytical, closed-form expressions for a classical "snakes and ladders" bifurcation scenario can be derived. When the model is subject to slow variations in the control parameters, a new type of coherent structure emerges : the structure displays a spatially-localised pattern, undergoing a slow-fast modulation at the core. Using interfacial dynamics and geometric singular perturbation theory, we show that these patterns follow an invariant repelling slow manifold, hence we name them "spatio-temporal canards". We classify spatio-temporal canards and give conditions for the existence of folded-saddle and folded-node canards. We also find that these structures are robust to changes in the synaptic connectivity and firing rate. The theory correctly predicts the existence of spatio-temporal canards with octahedral symmetries in a neural field model posed on a spherical domain.

I will then discuss how the insight gained with interfacial dynamics may be used to perform coarse-grained bifurcation analysis on neural networks, even in models where the network does not evolve according to an integro-differential equation. As an example I will consider a well-known event-driven network of spiking neurons, proposed by Laing and Chow. In this setting, we construct numerically travelling waves whose profiles possess an arbitrary number of spikes. An open question is the origin of the travelling waves, which have been conjectured to form via a destabilisation of a bump solution. We provide numerical evidence that this mechanism is not in place, by showing that disconnected branches of travelling waves with countably many spikes exist, and terminate at grazing points ; the grazing points correspond to travelling waves with an increasing number of spikes, a well-defined width, and decreasing propagation speed.

Bruno Cessac (Biovision Team, Inria Sophia-Antipolis)

Title : Multiscale dynamics in retinal waves.
Abstract : Spontaneous waves of spiking activity are observed in the retina during development. This activity plays a central role in shaping the visual system and retinal circuitry. Waves first occur at early embryonic stages of development and gradually disappear upon maturation. This process involves several time and space scales from molecular level (neurotransmitters), to neuron, to neurons population in the retina. I will present a model describing these different scales, accurate enough to reproduce experiments and to predict experimental results. This model can also be studied by tools from dynamical systems theory and bifurcations analysis. In my talk I will present part of this analysis linking it to biophysics and experiments.

Zachary Kilpatrick (University of Colorado Boulder)

Title : Interacting bumps model of working memory limitations.
Abstract : Working memory (WM) is limited both in its temporal length and capacity. Classic conceptions of WM capacity assume the system possesses a finite number of slots, but recent evidence suggests WM may be a continuous resource. Resource models typically assume there is no hard upper bound on the number of items that can be stored, but WM fidelity does decrease with the number of items. We analyze a neural field model of multi-item WM that associates each item with the location of a bump in a finite spatial domain, considering items that span a one-dimensional continuous feature space. Our analysis relates the neural architecture of the network to accumulated errors and capacity limitations arising during the delay period of a multi-item WM task. In particular, we can develop explicit expressions for how errors relate to the vicinity of items to one another. This model not only provides a neural circuit explanation for WM capacity, but also speaks to how capacity relates to the geometry of stored items in a feature space. This is work is in collaboration with Nikhil Krishnan, also at University of Colorado Boulder.

Carlo Laing (Institute of Natural and Mathematical Sciences, Massey University)

Title : Neural field models which include gap junctions.
Abstract : Neural field models are nonlocal PDEs used to describe macroscopic dynamics of the cortex. They are normally derived under the assumption that connections between neurons are synaptic rather than via gap junctions. I will show how to derive a new type of neural field model from a network of quadratic integrate and fire neurons with both synaptic and gap junction connectivity.

Cristobal Quiñinao (Institute of Engineering Sciences, University of O'Higgins)

Title : Large-scale dynamics for the FitzHugh-Nagumo model : the effects of strong coupling.
Abstract : In this talk, we present a nonlocal PDEs that is inspired as the limit of large dynamics of a neuronal network. This large scale of modelling is useful for understanding the role of mean connectivity and we focus on the study of the long term solutions when the coupling is small/large. In a first part we present the kinetic Fitzhugh-Nagumo equation, which appear in the modeling of the evolution of a neuronal network interacting through electrical synapses. The Fitzhugh-Nagumo (FhN) model, has gained the status of canonical model of excitable cells in neuroscience. In this setting, each

neuron is described by two variables : the membrane potential and a recovery function related to the ionic-gates of the cells. As coupling is strong, numerical simulations show that solutions converge to a Dirac mass, and depending on the parameters of the system this distribution might present oscillatory behaviour. By using a simplification of the kinetic equation we are able to get some insights of why this behavior is expected. In a second part we focus on the solutions when strong coupling is present. We present some preliminary results on this system, in particular, the equivalence with a Hamilton-Jacobi equation as the connectivity goes to infinite. This talk is based on some ongoing works done in collaboration with S. Mirrahimi and G. Faye and a publication done in collaboration with S. Mischler, J. Touboul.

Wilhelm Stannat (Institute of Mathematics, Technische Universität Berlin)

Title : Stochastic mean-field theories of cortical networks .

Abstract : I will discuss a new approach to stochastic mean-field theories for the population activity in the visual cortex within the mathematical framework of recurrent binary neural networks. Our mathematical analysis provides a complete description of nonequilibrium fluctuations in networks with finite size and finite degree of interactions and allows the investigation of systems for which a deterministic mean-field theory breaks down.

To demonstrate this, a novel dynamical state is described in which a recurrent network of binary units with statistically inhomogeneous interactions, along with an asynchronous behavior, also exhibits collective nontrivial stochastic fluctuations in the thermodynamical limit.

This is joint work with Farzad Farkhooi.

References :

F. Farkhooi and W. Stannat : A complete mean-field theory for dynamics of binary recurrent neural networks, Phys. Rev. Lett., to appear (2017).

Liste des participants

1. Eric Agius (Uniersity of Toulouse 3)
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8. Patrick Cattiaux (University of Toulouse 3)
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