An introduction to nonlinears PDEs

Particles Dynamics and Transport equations

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Preface

Kinetic theory sits at a remarkable crossroads: it is at once one of the most active areas of contemporary mathematics, one of the essential languages of theoretical physics, and a natural meeting ground for the analytic, probabilistic, and numerical branches of mathematics. This course is an invitation to explore that area.

On the one hand, kinetic equations were born from the need to describe systems made of a very large number of interacting particles without tracking every individual trajectory. More than a century later, they remain the natural framework for some of the most fundamental problems in physics: the dynamics of self-gravitating systems and of galaxies, through the Vlasov–Poisson system with attractive interaction; the behavior of ionized gases and magnetically confined plasmas, through the Vlasov–Poisson and Vlasov–Maxwell systems, at the heart of the quest for controlled thermonuclear fusion; and, more recently, a fast-growing range of problems in the life sciences – the collective motion of cells, bacteria, and animal groups, chemotaxis, and models of opinion dynamics – where the same mean-field and kinetic ideas developed for physics capture, with striking fidelity, the emergence of collective behavior in living systems.

On the other hand, few areas of analysis have been so consistently recognized by the highest distinctions in mathematics. P.-L. Lions received the Fields Medal in 1994 for, among other contributions, his work with R. DiPerna on renormalized solutions of transport equations and on the Boltzmann equation – results that underlie much of the rigorous theory developed in this course. C. Villani was awarded the Fields Medal in 2010 for his proof of nonlinear Landau damping and for his work on convergence to equilibrium for the Boltzmann equation, resolving long-standing questions at the very heart of kinetic theory: how irreversible, dissipative behavior can emerge from time-reversible microscopic dynamics. Most recently, Y. Deng was awarded the Fields Medal in 2026 for his work on the rigorous derivation of the Boltzmann kinetic equation from hard-sphere dynamics for rarefied gases – a striking confirmation that the questions first posed by Boltzmann and Maxwell in the nineteenth century remain, a century and a half later, a wellspring of some of the deepest mathematics being produced today.

Finally, what makes kinetic theory particularly rich to teach and to learn is the sheer breadth of mathematics it calls upon. Understanding kinetic equations requires the full toolbox of PDE analysis – existence, uniqueness, regularity, and the passage to hydrodynamic or mean-field limits; it requires probabilistic techniques – propagation of chaos, stochastic particle systems, and the rigorous derivation of the equations themselves from the underlying N-body dynamics; and it requires, as much as any area of applied mathematics, the design and analysis of numerical algorithms – particle, particle-in-cell, and semi-Lagrangian methods chief among them – capable of simulating phenomena far too complex to be understood by analysis alone.

This course is organized to reflect exactly that breadth. Chapter 1 introduces the fundamental kinetic model of plasma physics, the Vlasov–Poisson system, together with some of its most striking phenomena, such as Landau damping and the two-stream instability; Chapter 2 develops the rigorous analytic theory of transport equations and the method of characteristics on which everything that follows relies; Chapter 3 builds the probabilistic bridge between microscopic N-particle systems and their mean-field PDE limits; Chapter 4 turns to the numerical methods used to simulate these equations in practice; and Chapter 5 illustrates, through the example of collective behavior and flocking, how the very same ideas extend from plasma physics and astrophysics to biology.

We hope that, by the end of this course, you will have gained not only the technical tools to analyze and simulate kinetic equations, but also a sense of why this subject continues to attract, generation after generation, some of the most creative minds in mathematics.

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