

Structure preserving schemes for the Vlasov-Poisson system and related models

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Thursday 6th August, 2026

[ICERM, Providence \(2026\)](#)

Outline

Physical context and models

Numerical experiments

Numerical schemes for the linearized models

Analysis for nonlinear models

Conclusion & Perspectives

Physical context and models

The question at hand: plasma physics

- **Example : magnetic confinement in a tokamak^a**

^aPhD K. H. Trinh (advisors FF and L.M. Rodrigues) on Particle-In-Cells simulations.

A plasma is a **multiscale problem** and we aim to study its **long time behavior**

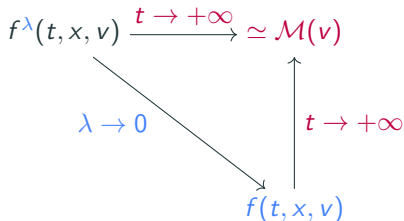
The question at hand: mathematical framework

- Consider the **Vlasov-Poisson** system in $\mathbb{T}^d \times \mathbb{R}^d$:

$$\begin{cases} \partial_t f^\lambda + v \cdot \nabla_x f^\lambda - \underbrace{\nabla_x \phi^\lambda \cdot \nabla_v f^\lambda}_{\text{field interactions}} = \mathcal{L}(f^\lambda), \\ -\lambda^2 \Delta_x \phi^\lambda = \rho^\lambda - \rho_i, \quad \rho^\lambda = \int_{\mathbb{R}^d} f^\lambda dv. \end{cases}$$

- $f^\lambda(t, x, v)$: density of particles at time t , position $x \in \mathbb{T}^d$, velocity $v \in \mathbb{R}^d$.

Possible dynamics : $\rho_i = 1$



- Gaussian** velocity distribution:

$$\mathcal{M}(v) = \frac{1}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2}|v|^2\right),$$

- $f(t, x, v)$ solves

$$\begin{cases} \partial_t f + v \cdot \nabla_x f - \underbrace{\nabla_x \phi \cdot \nabla_v f}_{\text{field interactions}} = \mathcal{L}(f), \\ 0 = \rho - 1, \quad \rho = \int_{\mathbb{R}} f dv. \end{cases}$$

$\lambda \rightarrow 0$: two main difficulties

Difficulty # 1. The quasi-neutral model is not always globally well posed¹.

Difficulty # 2. For $\lambda \ll 1$, fast and large oscillations hold². Set the current j^λ and the kinetic energy $\frac{1}{2}K^\lambda$ as

$$j^\lambda(t, x) = \int_{\mathbb{R}^d} v f^\lambda(t, x, v) dv, \quad K^\lambda(t, x) = \int_{\mathbb{R}^d} |v|^2 f^\lambda(t, x, v) dv.$$

We get the following formal oscillatory behavior

$$\begin{cases} j^\lambda(t, x) \underset{\lambda \ll 1}{\simeq} 0 + \lambda \sin\left(\frac{t}{\lambda}\right) (E^\lambda - \nabla_x K^\lambda)(0, x) + \cos\left(\frac{t}{\lambda}\right) j^\lambda(0, x), \\ E^\lambda(t, x) \underset{\lambda \ll 1}{\simeq} \nabla_x K^\lambda(t, x) + \cos\left(\frac{t}{\lambda}\right) (E^\lambda - \nabla_x K^\lambda)(0, x) - \frac{1}{\lambda} \sin\left(\frac{t}{\lambda}\right) j^\lambda(0, x) \end{cases}$$

¹Bardos (2012), Han Kwan & Hauray (2015)

²Grenier (1996)

$t \rightarrow \infty$: one main difficulty

- Consider the **Vlasov-Poisson-Fokker-Planck** model

$$\begin{cases} \partial_t f^\lambda + v \cdot \nabla_x f^\lambda - \underbrace{\nabla_x \phi^\lambda \cdot \nabla_v f^\lambda}_{\text{field interactions}} = \frac{1}{\tau} \underbrace{\nabla_v \cdot [v f^\lambda + \nabla_v f^\lambda]}_{\text{collisions}}, \\ -\lambda^2 \Delta_x \phi^\lambda = \rho^\lambda - \rho_i, \quad \rho^\lambda = \int_{\mathbb{R}^d} f^\lambda dv. \end{cases}$$

- $f^\lambda(t, x, v) \longrightarrow f_\infty^\lambda(x, v) = \rho_\infty^\lambda(x) \mathcal{M}(v)$ when $t \rightarrow \infty$.
- Relative entropy equality

$$\frac{d}{dt} \mathcal{H}(f, f_\infty) = -\mathcal{I}(f, f_\infty)$$

- Difficulty : lack of coercivity³

$$\mathcal{H}(f, f_\infty) \not\leq \mathcal{I}(f, f_\infty)$$

—→ **modify the relative entropy functional.**

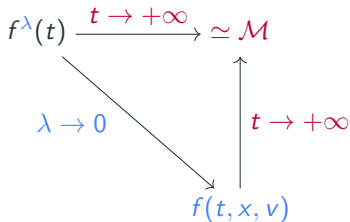
³Villani (2009)

Numerical analysis of the model : motivations

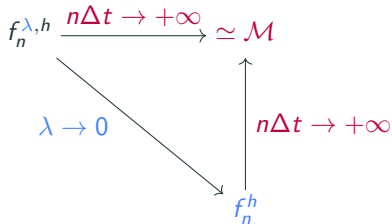
Goal: Design asymptotic and structure preserving in the limit

$\lambda \rightarrow 0$ and $t \rightarrow \infty$ approximations⁴

Continuous model:



Approximations:



where $f_n^{\lambda,h}$ approximation of $f^\lambda(n\Delta t)$, where

- $n \geq 0$ discrete time index, $\Delta t > 0$ time step ;
- h phase space variables (x, v) discretization parameter

⁴Crouseilles, Crispel, Degond, Deluzet , Dimarco, Sonnendrücker, Vignal for Vlasov-Poisson and Badsì, Crouseilles (2025) for Euler-Poisson

Numerical experiments

Quasi-neutral limit problem ($\lambda \rightarrow 0$).

- Consider the **Vlasov-Poisson** system

$$\begin{cases} \partial_t f^\lambda + v \cdot \nabla_x f^\lambda - \nabla_x \phi^\lambda \cdot \nabla_v f^\lambda = 0, \\ -\lambda^2 \Delta_x \phi^\lambda = \rho^\lambda - 1, \quad \rho^\lambda = \int_{\mathbb{R}^d} f^\lambda dv. \end{cases}$$

- We expect that

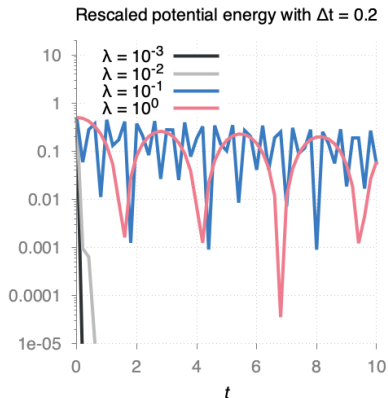
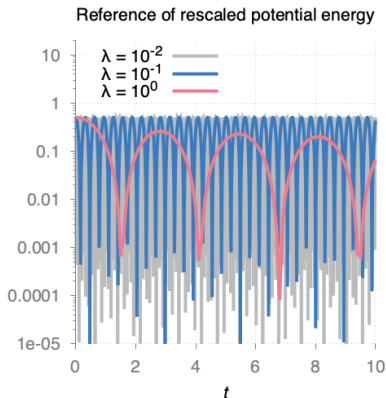
$$E^\lambda(t, x) \underset{\lambda \ll 1}{\simeq} \underbrace{\nabla_x K^\lambda(t, x)}_{E_{\text{slow}}^\lambda \xrightarrow{\lambda \rightarrow 0} \nabla_x K} + \underbrace{\cos\left(\frac{t}{\lambda}\right) (E^\lambda - \nabla_x K^\lambda)(0, x) - \frac{1}{\lambda} \sin\left(\frac{t}{\lambda}\right) j^\lambda(0, x)}_{E_{\text{osc}}^\lambda \xrightarrow{\lambda \rightarrow 0} 0},$$

- Formally speaking, in the limit $\lambda \rightarrow 0$, we expect that the density ρ^λ converges to 1.

Quasi-neutral limit : near equilibrium $E^\lambda = O(1)$

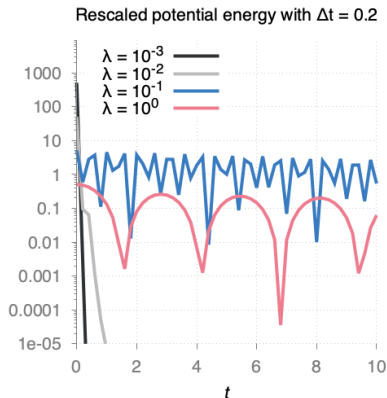
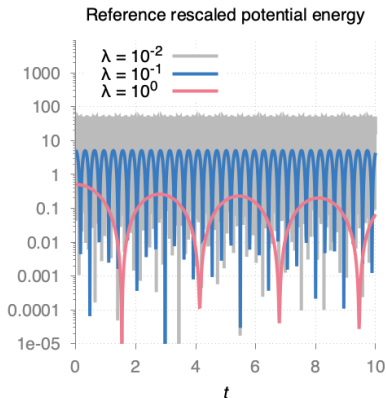
We choose $E^\lambda(0)$ bounded and f^λ closed to $\mathcal{M}$.

Then, we have that (f^λ, E^λ) weakly converges to $(\mathcal{M}, 0)$ as $\lambda \rightarrow 0$.



Quasi-neutral limit : near equilibrium $E^\lambda = O(\lambda^{-1/2})$

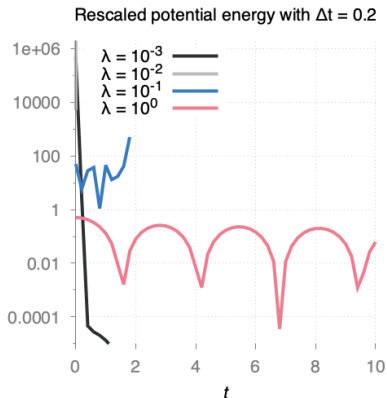
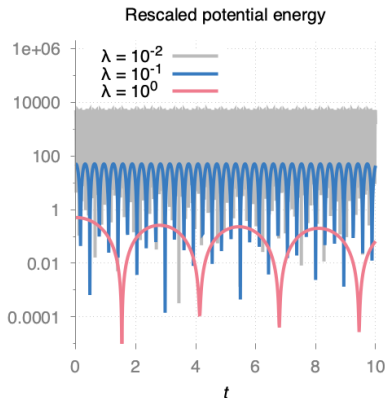
We choose $E^\lambda(0)$ is $O(\lambda^{-1/2})$ and f^λ closed to $\mathcal{M}$.



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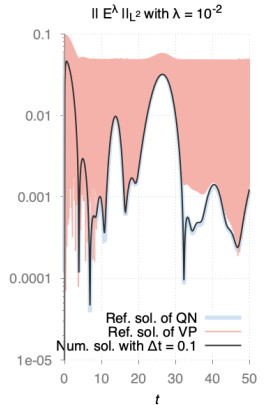
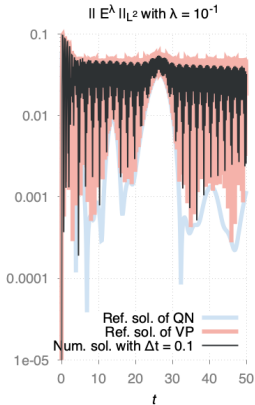
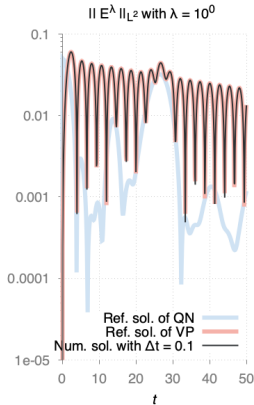
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Oscillatory perturbation

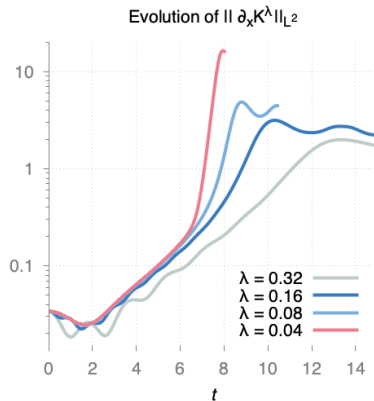
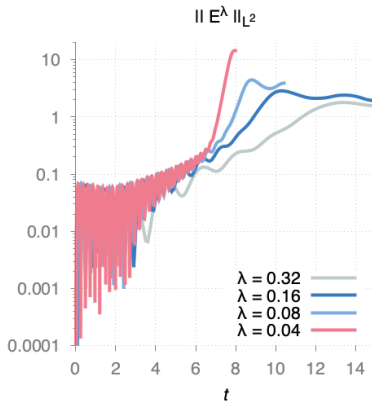
Choose (f^λ, E^λ) which weakly converges to (f, E) .



Two-stream instability with $\lambda = 0.1$

Two-stream instability with $\lambda \ll 1$ ($\lambda = 0.1$)

We choose an initial data for (f^λ, E^λ) such that the quasi-neutral limit (f, E) is not globally well posed.



About asymptotic preserving schemes

Find a **subtle balance** between

- **stability criteria** with respect to the physical parameters λ and t and numerical parameters (time step, mesh size).
- **consistency** with respect to the original and asymptotic models

Numerical schemes for the linearized models

The Vlasov-Poisson system

Consider $f_{\infty}^{\lambda}(x, v) = \rho_{\infty}^{\lambda}(x) \mathcal{M}(v)$ and

$$\rho_{\infty}^{\lambda} = e^{-\phi_{\infty}^{\lambda}} \quad \text{and} \quad -\lambda^2 \partial_x^2 \phi_{\infty}^{\lambda} = \rho_{\infty}^{\lambda} - \rho_i.$$

Let us introduce $\psi^{\lambda} = \phi^{\lambda} - \phi_{\infty}^{\lambda}$ and

$$\begin{cases} \partial_t f^{\lambda} + v \partial_x f^{\lambda} - \partial_x \phi_{\infty}^{\lambda} \partial_v f^{\lambda} - \partial_x \psi^{\lambda} \partial_v f^{\lambda} = 0, \\ -\lambda^2 \partial_x^2 \psi^{\lambda} = \rho^{\lambda} - \rho_{\infty}^{\lambda}, \quad \rho^{\lambda}(t, x) = \int_{\mathbb{R}} f^{\lambda}(t, x, v) dv, \end{cases}$$

Equilibrium and energy estimate

The equilibrium is $(f_{\infty}^{\lambda}, \psi_{\infty}^{\lambda})$ where $\psi_{\infty}^{\lambda} \equiv 0$ and

$$\frac{d}{dt} \mathcal{H}(f^{\lambda}, f_{\infty}^{\lambda}) = 0,$$

$$\mathcal{H}(f^{\lambda}, f_{\infty}^{\lambda}) := \int_{\mathbb{T} \times \mathbb{R}} \ln \left(\frac{f^{\lambda}}{f_{\infty}^{\lambda}} \right) f^{\lambda} dx dv + \frac{\lambda^2}{2} \|\partial_x \psi^{\lambda}\|_{L^2(\mathbb{T})}^2.$$

The linearized Vlasov-Poisson system

Plugging the following formal expansion into the free energy

$$f^\lambda \ln \left(\frac{f^\lambda}{f_\infty^\lambda} \right) \underset{f^\lambda \rightarrow f_\infty^\lambda}{\sim} f^\lambda - f_\infty^\lambda + \frac{|f^\lambda - f_\infty^\lambda|^2}{2 f_\infty^\lambda}$$

we define a new functional,

$$\mathcal{E}^\lambda(t) = \|f^\lambda(t) - f_\infty^\lambda\|_{L^2((f_\infty^\lambda)^{-1})}^2 + \lambda^2 \|\partial_x \psi^\lambda(t)\|_{L^2(\mathbb{T})}^2.$$

Unfortunately, this functional is not conserved for the solution to the nonlinear system.

But only for its linearized version

$$\begin{cases} \partial_t f^\lambda + v \partial_x f^\lambda - \partial_x \phi_\infty^\lambda \partial_v f^\lambda - \partial_x \psi^\lambda \partial_v f_\infty^\lambda = 0, \\ -\lambda^2 \partial_x^2 \psi^\lambda = \rho^\lambda - \rho_\infty^\lambda, \quad \rho^\lambda(t, x) = \int_{\mathbb{R}} f^\lambda(t, x, v) dv, \end{cases}$$

From the key estimate to the functional space

Conservation of the relative L^2 -norm

$$\frac{d}{dt} \left(\int \left| \frac{f^\lambda}{\sqrt{\rho_\infty^\lambda} \mathcal{M}} - \sqrt{\rho_\infty^\lambda} \right|^2 dx \mathcal{M} dv + \lambda^2 \left\| \partial_x \psi^\lambda(t) \right\|_{L^2(\mathbb{T})}^2 \right) = 0.$$

→ Functional space :

$$\frac{f^\lambda}{\sqrt{\rho_\infty^\lambda} \mathcal{M}} \in L^2(dx \mathcal{M}(v) dv) .$$

- Spectral decomp. in Hermite basis $(H_k)_{k \in \mathbb{N}}$ of $L^2(\mathcal{M} dv)$

$$\frac{f^\lambda}{\sqrt{\rho_\infty^\lambda} \mathcal{M}}(t, x, v) = \sum_{k \in \mathbb{N}} D_k^\lambda(t, x) H_k(v) .$$

- **No weight** with respect to dx so

$$D_k^\lambda \in L^2(dx) .$$

Discrete linearized setting for $v \in \mathbb{R}$

In this framework, we set $D_\infty = (\sqrt{\rho}_\infty^\lambda, 0, \dots)$ and the discrete linearized system rewrites for $0 \leq k \leq N$,

$$\begin{cases} \partial_t D_k^\lambda + \sqrt{k} \mathcal{A} D_{k-1}^\lambda - \sqrt{k+1} \mathcal{A}^* D_{k+1}^\lambda + \sqrt{k} \mathcal{A} \omega^\lambda D_{\infty, k-1} = 0, \\ -\lambda^2 \partial_{xx} \psi^\lambda = \sqrt{\rho_\infty^\lambda} D_0^\lambda - \rho_\infty^\lambda. \end{cases}$$

where

$$\mathcal{A}u = \partial_x u + \frac{\partial_x \phi_\infty^\lambda}{2} u$$

and the modified potential ω^λ is given by

$$\omega^\lambda = \sqrt{\rho_\infty^\lambda} \psi^\lambda.$$

This formulation is well suited to preserve the energy estimate...

Discrete framework

We design discrete approximations $(f_n^{\lambda,h}, \rho_\infty^{\lambda,h})$ of $(f^\lambda, \rho_\infty^\lambda)$ such that

$$\begin{cases} \frac{D_k^{n+1} - D_k^n}{\Delta t} + \sqrt{k} \mathcal{A}_h D_{k-1}^{n+1} - \sqrt{k+1} \mathcal{A}_h^* D_{k+1}^{n+1} + \sqrt{k} \mathcal{A}_h \omega_h^{n+1} D_{\infty,k-1} = 0, \\ \lambda^2 \partial_h \partial_h^* \psi_h^{n+1} = \sqrt{\rho_\infty^\lambda} D_0^{n+1} - \rho_\infty^\lambda, \end{cases}$$

where $\omega_h^\lambda = \sqrt{\rho_\infty^\lambda} \psi_h^\lambda$ and discrete operators $\mathcal{A}_h$ and $\mathcal{A}_h^*$ verify⁵

Properties	Preservation
$\langle \mathcal{A}_h u, v \rangle_{L^2} = \langle u, \mathcal{A}_h^* v \rangle_{L^2}$	duality structure
$\mathcal{A}_h \sqrt{\rho_\infty^\lambda} = 0$	equilibrium state
$\sum_j \Delta x_j (\mathcal{A}_h^* u)_j \sqrt{\rho_\infty^\lambda}_{,j} = 0$	invariants

⁵Y. Cai, A. Blaustein, T. Xiong and FF for Discontinuous Galerkin and A. Blaustein and FF for Finite Difference.

Asymptotic analysis $\lambda \rightarrow 0$

Discrete energy estimate

Theorem (Long time behavior)

$$\left\| f_n^{\lambda,h} - \rho_\infty^{\lambda,h} \mathcal{M} \right\|^2 + \lambda^2 \left\| \partial_h \psi_n^{\lambda,h} \right\|^2 \lesssim \left\| f_0^{\lambda,h} - \rho_\infty^{\lambda,h} \mathcal{M} \right\|^2 + \lambda^2 \left\| \partial_h \psi_0^{\lambda,h} \right\|^2.$$

- This first result allows to prove the quasineutral limit $\lambda \rightarrow 0$ when the solution is closed to a $\mathcal{M}$.

Formal limit $\lambda \rightarrow 0$

$$\begin{cases} \frac{D_0^{n+1} - D_0^n}{\Delta t} - \mathcal{A}_h^* D_1^{n+1} = 0, \\ \frac{D_1^{n+1} - D_1^n}{\Delta t} + \mathcal{A}_h D_0^{n+1} - \sqrt{2} \mathcal{A}_h^* D_2^{n+1} + \mathcal{A}_h \omega_h^{n+1} D_{\infty,0} = 0, \\ \lambda^2 \partial_h \partial_h^* \psi_h^{n+1} = \sqrt{\rho_\infty}^\lambda D_0^{n+1} - \rho_\infty^\lambda, \quad \omega_h^{n+1} = \sqrt{\rho_\infty}^\lambda \psi_h^{n+1}. \end{cases}$$

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Long time behavior $n\Delta t \rightarrow \infty$

For simplicity, let us take $\psi \equiv 0$ and add a Fokker-Planck operator, for which Hermite functions are eigenfunctions. It leads to

Long time behavior ($n\Delta t \rightarrow +\infty$)

$$\frac{D_k^{n+1} - D_k^n}{\Delta t} + \sqrt{k} \mathcal{A}_h D_{k-1}^{n+1} - \sqrt{k+1} \mathcal{A}_h^* D_{k+1}^{n+1} = -\frac{k}{\tau} D_k^{n+1},$$

We go back to the L^2 estimate

$$\frac{\|D^{n+1} - D_\infty\|_{L^2}^2 - \|D^n - D_\infty\|_{L^2}^2}{\Delta t} \leq -\frac{2}{\tau} \|D_\perp^{n+1}\|_{L^2}^2,$$

with $D_\perp^{n+1} = (0, D_1^{n+1}, D_2^{n+1}, D_3^{n+1}, \dots)$.

Lack of coercivity⁶

$$\|D^{n+1} - D_\infty\|_{L^2}^2 \not\leq \|D_\perp^{n+1}\|_{L^2}^2$$

⁶Shizuta-Kawashima (1984), Villani (2009), Beauchard-Zuazua (2011), Dolbeault, Mouhot and Schmeiser (2010) and Liu-Jin (2018)

Illuminating example

Consider

$$\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = -x - \frac{1}{\tau} v \end{cases}.$$

- Relative entropy estimate:

$$\frac{d}{dt} \left(|x(t)|^2 + |v(t)|^2 \right) = -\frac{2}{\tau} |v(t)|^2.$$

- Modified entropy: $\mathcal{H} = |x|^2 + |y|^2 + \alpha \tau x v$

$$\frac{d}{dt} \mathcal{H}(t) = -\frac{2}{\tau} |v(t)|^2 + \alpha \tau \left(|v(t)|^2 - |x(t)|^2 - \frac{1}{\tau} x(t) v(t) \right).$$

We deduce $\frac{d}{dt} \mathcal{H}(t) = -\kappa \mathcal{H}(t)$ and $|x(t)|^2 + |v(t)|^2 \lesssim e^{-\kappa t}$.

Long time behavior ($n\Delta t \rightarrow +\infty$)

Using a **discrete Poincaré inequality on the operator $\mathcal{A}_h$** , we prove convergence to equilibrium for the fully discrete system.

Analysis for nonlinear models

Time splitting scheme to solve the nonlinear equation

It leads to the scheme :

- **first step** is the linearized equation

$$\begin{cases} \frac{D_k^{(1)} - D_k^n}{\Delta t} + \sqrt{k} \mathcal{A}_h D_{k-1}^{(1)} - \sqrt{k+1} \mathcal{A}_h^* D_{k+1}^{(1)} + \mathcal{A}_h \omega_h^{(1)} D_{\infty, k-1} = 0, \\ \lambda^2 \mathcal{A}_h^* (\rho_\infty^\lambda)^{-1} \mathcal{A}_h \omega_h^{(1)} = D_0^{(1)} - \sqrt{\rho_\infty^\lambda}, \end{cases}$$

- **the second step**

$$D_0^{n+1} = D_0^{(1)}, \quad \omega_h^{n+1} = \omega_h^{(1)}$$

and for $k \geq 1$,

$$\frac{D_k^{n+1} - D_k^{(1)}}{\Delta t} + \sqrt{k} \mathcal{A}_h \omega_h^{n+1} (D_{k-1}^{n+1} - D_{\infty, k-1}) = 0.$$

Goal: Apply a Strang splitting scheme with L-stable second order SDIRK method for each step for numerical experiments.

Nonlinear setting $\rho_\infty^\lambda = 1$

We define the following quantities $E_{\text{slow}}^\lambda := \sqrt{2} \partial_x D_2^\lambda$.

$$\begin{cases} \mathcal{E}_0^\lambda(t) := \|E^\lambda(t) - E_{\text{slow}}^\lambda(t) - E_{\text{osc}}^\lambda(t)\|_{L^2(\mathbb{T})}, \\ \mathcal{E}_1^\lambda(t) := \|D_1^\lambda(t) - D_{1,\text{osc}}^\lambda(t)\|_{L^2(\mathbb{T})}, \end{cases}$$

where E_{osc}^λ and $D_{1,\text{osc}}^\lambda$ are previously given.

Theorem ($\lambda \rightarrow 0$).

$$\mathcal{E}_0^\lambda(t) \leq C \lambda^{1-\alpha} \quad \text{and} \quad \mathcal{E}_1^\lambda(t) \leq C \lambda^{2-\alpha},$$

Proposition ($\Delta t > 0$ is fixed and $\lambda \rightarrow 0$).

We have for all $\lambda > 0$

$$\sup_{1 \leq n \leq T/\Delta t} \|E^n - \sqrt{2} \partial_h D_2^n\|_{L^2(\mathcal{J})} \leq C \lambda,$$

and

$$\sup_{2 \leq n \leq T/\Delta t} \|D_1^n\|_{h^1(\mathcal{J})} \leq C \lambda,$$

Conclusion & Perspectives

Conclusion & Perspectives

- Build a class of structure preserving schemes for the Vlasov-Poisson system with collisions ;
- Quantitative estimates for the linearized model ;
- Quantitative estimates and numerical results for the nonlinear model in a perturbative setting;

Perspectives

- Show that the present method can be applied for high dimensional problems : smart splitting scheme based on divide and conquer;
- Include nonlinear collision operators closer to physics (ex: Landau)