Selected topics in statistics
Spatial Statistics
Homework 3

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Exercice 1

Let $y_1, ..., y_n$ be n *iid* standard Gaussian variables.

a) Prove that the random vector $y = (y_1, ..., y_n)$ is a Gaussian vector, and give its mean vector and its covariance function.

b) Let A be a matrix so that $AA^t = I_n$. Prove that $z = Ay$ is a Gaussian vector, and give its mean vector and covariance matrix.

Exercise 2

Let Y be a Gaussian process on $\mathbb{R}$, with mean function 0 and with covariance function $K(x, y) = e^{-(x-y)^2}$. Let Z be the stochastic process on $\mathbb{R}$ defined by $Z(x) = 1 + x + Y(2x)$.

a) Prove than Z is a Gaussian process.

b) Calculate the mean function and the covariance function of Z.

c) Prove that Z is mean square differentiable on $\mathbb{R}$.