# Selected topics in statistics
# Spatial Statistics
# Homework 1

Lecturer: François Bachoc, PhD

## Exercise 1

Let $(X_i)_{i \in \mathbb{N}}$ be a sequence of *iid* random variables with standard Gaussian distributions. Let $Y(i) = X_i$ for $i \in \mathbb{N}$. Then $Y$ is a stochastic process on $\mathcal{D} = \mathbb{N}$. Let $Z(k)$ be defined by $Z(k) = \max_{i=0\dots k} Y(i)$.

- Show briefly that $Z$ is a stochastic process.
- Draw a typical trajectory of $Y$ on $\{1, ..., 10\}$ for an element $w$ of the probability space. Draw the corresponding trajectory for $Z$.
- Is $Y$ stationary? Is $Z$ stationary? Explain your answer.

## Exercise 2

Let $T \in \mathbb{R}$ and $U$ be a random variable with uniform distribution on $[0, T]$. Let $Y$ be the stochastic process on $\mathcal{D} = \mathbb{R}$ defined by $Y(x) = cos(U + x)$. Show that $Y$ is stationary if and only if $T$ can be written $T = 2k\pi$ with $k \in \mathbb{Z}$.

## Exercise 3

Let $Y$ be a stationary stochastic process on $\mathbb{R}$ for which all the trajectories are non-decreasing and for all $x \in \mathbb{R}$, $Y(x)$ has a Bernouilli distribution on $\{0, 1\}$. Show that, for any $x_1 < x_2 \in \mathbb{R}$, $P\left(Y(x_1) = Y(x_2)\right) = 1$.