Overview of (some) current challenges for fusion plasma modelling

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IMT 25/05/2022 **Matinées scientifiques Maia**

Fluid or kinetic: how to model magnetised plasmas?

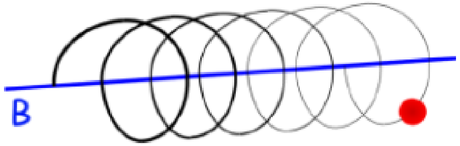
Fluid description appealing. . . yet often incorrect: **3d+time**

$$\rho \mathrm{d}_t \mathbf{v} = -\nabla p + \mathbf{j} \times \mathbf{B} \quad ; \quad \partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad ; \quad + \text{ eqs. } \rho \;\&\; p$$

- Fluid description: $\left| \begin{array}{l} \omega \gg \mathbf{k}\cdot\mathbf{v} \\ \delta_{\text{orbit}} \ll \delta_{\text{mode}} \end{array} \right.$ fails when $\left| \begin{array}{l} \text{wave–particle resonances} \\ \text{finite extension particle orbits} \end{array} \right.$
- Especially critical for low freq. phenomena & small scales
  ↳ **turbulence:** dense resonances $\omega \sim \mathbf{k}\cdot\mathbf{v}$ & $\delta_{\text{orbit}} \sim \delta_{\text{mode}} \sim 1\,\text{cm}$

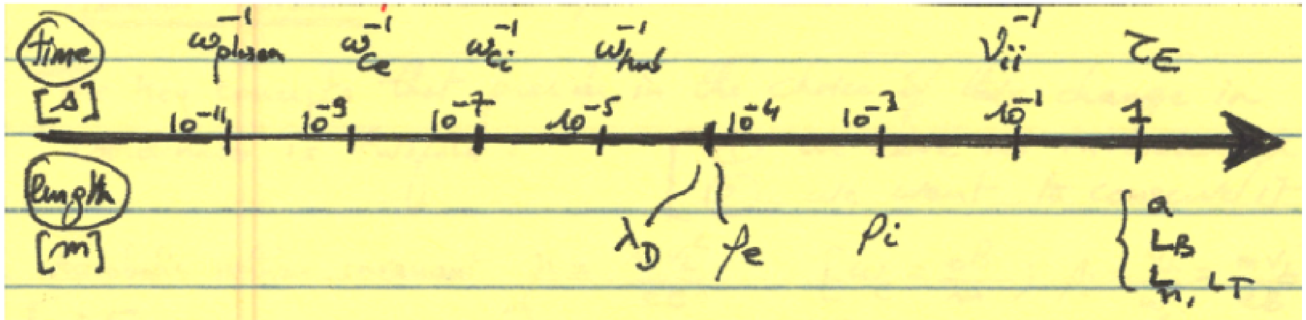
Kinetic description $\partial_t F + \nabla\cdot(\mathbf{v}F) = \mathcal{C}(F) + \mathcal{S}(F)$ + Maxwell **6d+time**

∃ small parameters ≡ focus on **low-freq.** electromagnetic fluctuations



reduction possible

- strong **B** field [~~cyclotron motion~~]
- effective separation of scales



Fluid or kinetic: how to model magnetised plasmas?

- $\partial_t F + \nabla \cdot (\mathbf{v} F) = \mathcal{C}(F) + \mathcal{S}(F) \quad + \text{ Maxwell}$

key idea: eliminate high-freq. processes $\omega > \omega_c$

4 gyrocentre coord. + magn. moment

$$6\text{d} \begin{cases} \bullet\ \omega_{p,s}\,\Delta t < 1 \\ \bullet\ \Delta x < \lambda_{Ds} \end{cases} \Rightarrow \quad 5\text{d} \begin{cases} \bullet\ \omega^\star_{c,s}\,\Delta t < 1 \\ \bullet\ \Delta x < \rho_s \end{cases}$$

- $\partial_t \mathbb{F} + \dot{\mathbf{x}}\,\partial_{\mathbf{x}}\mathbb{F} + \dot{v}_\parallel\,\partial_{v_\parallel}\mathbb{F} + \underbrace{\dot{\mu}}_{=0}\,\partial_\mu \mathbb{F} = \mathcal{C}(\mathbb{F}) + \mathcal{S}(\mathbb{F}) \quad + \text{ Maxwell [pol.; magn.]}$

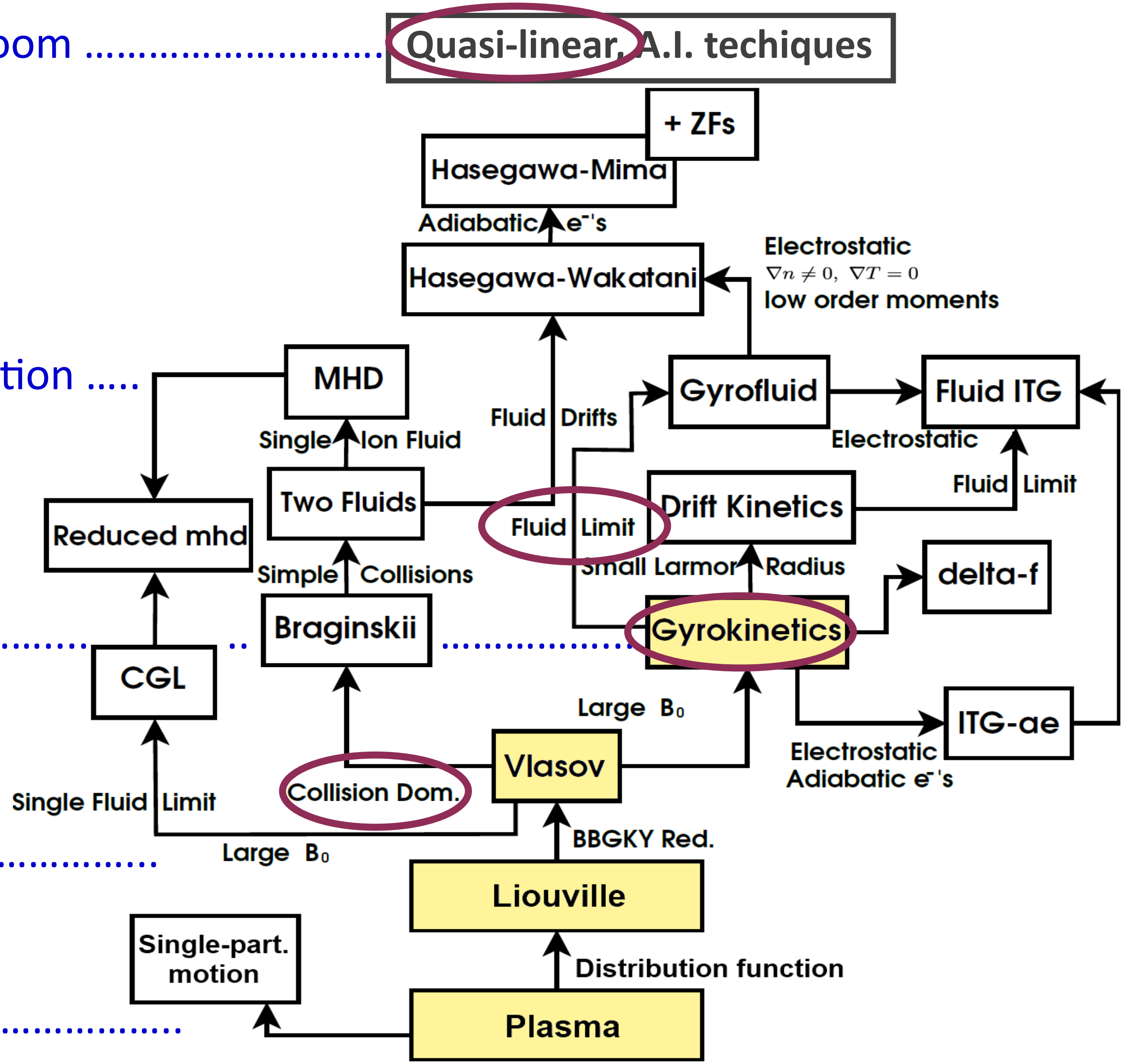
Rq: order 1 expansion → higher order ongoing

Magnetised plasma:

- large reservoirs of free energy | $\nabla T,\ \nabla n \sim 4$ decades, $\nabla I \sim 3$ decades → | externally driven & 'weakly' open
- multiple scales | $\{\text{ions, electrons}\} \sim \{3, 5\}$ decades; time ~ 5 decades
- ~2d turb. with important(?) kinetic features | Landau resonance ↔ dissipation; damping of zonal flows; mom.exchange: collisions, heating

How does the plasma organise? transport?

Confinement: its complexity requires a hierarchy of models



Real time, in the control room ……………………… Quasi-linear, A.I. techiques

Validation, param. exploration …..

Understanding ……………………… .. ………………

Untractable ……………………………….

Nature ……………………………………….

+ ZFs

Hasegawa-Mima

Adiabatic e^-'s

Hasegawa-Wakatani

Electrostatic
$\nabla n \neq 0,\ \nabla T = 0$
low order moments

MHD

Single Ion Fluid

Fluid Drifts

Gyrofluid

Fluid ITG

Electrostatic

Two Fluids

Reduced mhd

Fluid Limit

Drift Kinetics

Fluid Limit

Simple Collisions

Small Larmor Radius

delta-f

Braginskii

Gyrokinetics

CGL

Large B_0

Vlasov

ITG-ae

Electrostatic
Adiabatic e^-'s

Collision Dom.

Single Fluid Limit

Large B_0

BBGKY Red.

Liouville

Single-part. motion

Distribution function

Plasma

Outline

- Fluid approaches: the closure problem
- The quasi-linear reduction: some current problems
- Weakly collisional (gyro)-kinetics
- Boundary conditions: kinetics of plasma-mat. surface interaction

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Reduced dimensionality of fluid approaches appealing…
… yet magn. Plasmas are weakly collisional ➔ closure?

- Kinetics: infinite-dimensional with a continuum of resonances;
- Aim: retain (part of) kinetic properties ↔ resonances
- Seminal method proposed by Hammett-Perkins

[Hammett-Perkins PRL 90]
[Sarazin PPCF 09]
[Gillot PoP 21]

↓

Ex: derivation from simple 1D-1V Vlasov-Poisson system

- Basics on Landau damping (wave-particle resonance)
- Method to account for resonances in fluid closure

- Electron dynamics in 1D periodic system: f(x,v,t)

  - Electrostatic → E = $-\partial_x \phi$
  - Ions: neutralizing & at rest → n_0

$$\begin{cases} \partial_t f + v\,\partial_x f + \partial_x\phi\,\partial_v f = 0 \\ \partial_x^2 \phi = \int_{-\infty}^{+\infty} f \mathrm{d}v - 1 \end{cases}$$

$$\begin{cases} \lambda_D = (\varepsilon_0 T / e^2 n_0)^{1/2} \\ v_T = (T/m_e)^{1/2} \\ \omega_p = v_T / \lambda_D \end{cases}$$

❶ The kinetic problem: 2D Vlasov-Poisson system
Landau damping: resonances & analytic continuation

- Equilibrium + small amplitude perturbations: $f = f_M + \delta f$
  - Maxwellian equilibrium: $f_M = \exp(-v^2/2) \;/\; \sqrt{2\pi}$
  - Fourier modes: $(\delta f, \delta\phi) = \sum_{k,\omega} \left(\hat{f}_{k,\omega}, \hat{\phi}_{k,\omega}\right) \mathrm{e}^{i(kx-\omega t)}$

- Linearized dynamics: $\hat{f}_{k,\omega} = -\dfrac{kv}{\omega - kv}\, f_M\, \hat{\phi}_{k,\omega}$

Dispersion relation:

$$\mathcal{D}(k,\omega) = k^2 - \int_{-\infty}^{+\infty} \frac{\mathrm{d}v}{\sqrt{2\pi}} \frac{kv}{\omega - kv} \mathrm{e}^{-v^2/2} = 0$$

Pole if $\omega \in \mathbb{R}$

- **Vlasov** treatment: ignore resonances $\int_{-\infty}^{+\infty} \ldots \rightarrow \mathrm{P}\int_{-\infty}^{+\infty} \ldots$ [Vlasov 1945]
- **Landau** treatment: analytic continuation $\omega \rightarrow \omega + i0^+$ [Landau 1946]
- Perturbative non linear ➔ same damping [Mouhot-Villani 2011]

② Fluid description: the closure problem

- Fluid transport equations derive from moments of Vlasov eq.

$$M^{(L)} \approx \int \vec{v}^L F \, \mathrm{d}^3\vec{v} \longrightarrow n(\vec{r},t),\ \vec{u}(\vec{r},t),\ \bar{\bar{P}}(\vec{r},t),\ etc...$$

- Each moment *L* is coupled to next order moment *(L+1)*

$$\begin{cases} \partial_t n + \partial_x(un) = 0 \\ \partial_t u + u\,\partial_x u + \dfrac{\partial_x p}{n} - \partial_x \phi = 0 \\ \partial_t p + \partial_x(up) + \partial_x q + 2p\,\partial_x u = 0 \end{cases}$$

$$n \equiv \langle 1 \rangle_f,\ nu \equiv \langle v \rangle_f,\ p \equiv \langle (v-u)^2 \rangle_f$$

$$q \equiv \langle (v-u)^3 \rangle_f$$

$$\langle \ldots \rangle_f \equiv \int_{-\infty}^{\infty} \ldots f \mathrm{d}v$$

Infinite hierarchy of eqs. ⇒ Closure required: $M^{(L)} = G\left(M^{(\ell)}, ..., M^{(L-1)}\right)$

- Distinct regimes, depending on collisionality

Highly collisional ($\nu_*\sim 1$) ⇒ F~$F_{\text{Maxwell.}}$ → [Braginskii, 1965]

Weakly collisional ($\nu_* << 1$) ⇒ ?

Asymptotic preserving ⇒ ∀ collisionality?

General procedure: matched response function in kin.limit

- Linearisation: $\hat{q}_{k,\omega} = \alpha^{(n)}_{k,\omega}\hat{n}_{k,\omega} + \alpha^{(u)}_{k,\omega}\hat{u}_{k,\omega} + \alpha^{(p)}_{k,\omega}\hat{p}_{k,\omega}$ → *L* unknowns $\alpha^{(\ell)}_{k,\omega}$

- kinetic & fluid response functions

[Fried & Conte, Academic Press NY (1961)]

$$\hat{n}_{k,\omega} = -R_{k,\omega}\,\hat{\phi}_{k,\omega} \quad \rightarrow \quad R^{kin}_{k,\omega}\ ;\ R^{fluid}_{k,\omega}$$

- Order by order matching in the kinetic limit $\zeta = \omega/(\sqrt{2}|k|) \ll 1 \;\Leftrightarrow\; \omega \ll k\,v_{th}$

$$R_{k,\omega} = R^{(0)}_{k,\omega} + \zeta\,R^{(1)}_{k,\omega} + \zeta^2\,R^{(2)}_{k,\omega} + \ldots$$

Captures some physics of resonance

$$Z(\zeta) \approx i\pi^{1/2}\mathrm{e}^{-\zeta^2} - 2\zeta\left[1 - \frac{2}{3}\zeta^2 + \frac{4}{15}\zeta^4 + \ldots\right]$$

closure: $\alpha^{(n)}_{k,\omega} = -\alpha^{(p)}_{k,\omega} = i\sqrt{\frac{8}{\pi}}\,\mathrm{sign}(k) \;\rightarrow\; \boxed{\hat{q}_{k,\omega} = -i\sqrt{\frac{8}{\pi}}\,\mathrm{sign}(k)\,\hat{T}_{k,\omega}}$

[Hammett & Perkins, PRL **64** (1990) 3019]

Rk: Similar closure when trying to match the kinetic & fluid entropy production rates [Sarazin et al., PPCF **51** (2009) 115003]

Collisionless closures: a (too?) large variety of choices

linear, non-linear, local, non-local, ...

- Collisional closure: local operator in space [Braginskii 1965]

$$\hat{q}^{coll}_{k,\omega} = -i\chi_{coll}\ k\ \hat{T}_{k,\omega} \quad \rightarrow \quad q_{coll} = -\chi_{coll} \lim_{h\to 0+} \frac{T(x+h)-T(x-h)}{2h}$$

- Matching response functions in the kinetic limit [Hammett & Perkins PRL 1990; Sarazin PPCF 2018]

  sign(k) ⇒ non-local operator in space

$$\hat{q}_{k,\omega} = -i\sqrt{\frac{8}{\pi}}\ \mathrm{sign}(k)\ \hat{T}_{k,\omega} \quad \rightarrow \quad \boxed{q = -\left(\frac{2}{\pi}\right)^{3/2} \lim_{\varepsilon\to 0+} \int_{\varepsilon}^{+\infty} \frac{T(x+h)-T(x-h)}{h}\, \mathrm{d}h}$$

- Generalisation: interpolation method (« moment matching ») [Antoulas 2010, Beattie 2017, Vuillemin 2019]

  interpolate response functions @ «well-chosen » frequencies [Gillot PoP 2021]

- Balanced truncation: define « reachable » & « observable » states [Mullis 1976, Moore 1981, Gugercin 2004]

  ➔ only keep most reachable & observable

- …

<u>YET:</u> discrepancies between gyrokinetic & fluid results (e.g. Dimits upshift)

Problems for fluid approaches @low freq. or small scales

- Weaknesses of (Gyro)Fluid models
  - Hardly account for wave-particle resonances (important at n*<<1)
  - Cannot distinguish *a priori* passing, trapped & fast particles

    & Limited to large modes $\delta_{orbit} << \delta_{mode}$
  - Overdamp Zonal Flows

- Not a benign problem….

A few words of history…

- 1990- : Collisionless closures [Hammett & Perkins, PRL (1990); Beer & Hammett, PoP (1996)]
- 1996: Simulations based on such closures

  → "Turbulence May Sink Titanic Reactor" !

  [Kotschenreuther, Dorland, Science (1996)]
- 1998: USA quit ITER

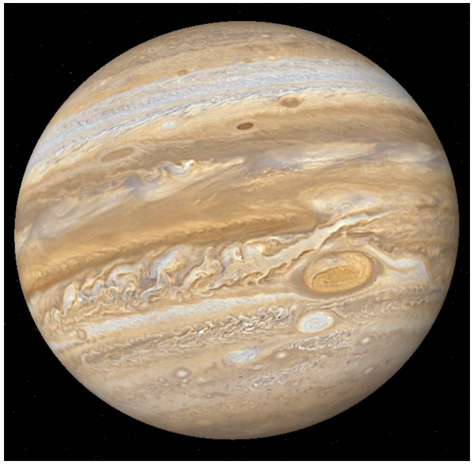
Theoretical proof that Zonal Flows are linearly undamped in the collisionless regime → fluid codes miss this physics

[Rosenbluth & Hinton, PRL (1998)]

[Lin et al., Science (1998)]

ZF efficiently contribute to turbulence saturation by tearing apart convective cells

[Diamond, Itoh, Itoh, Hahm, PoP (2005)]



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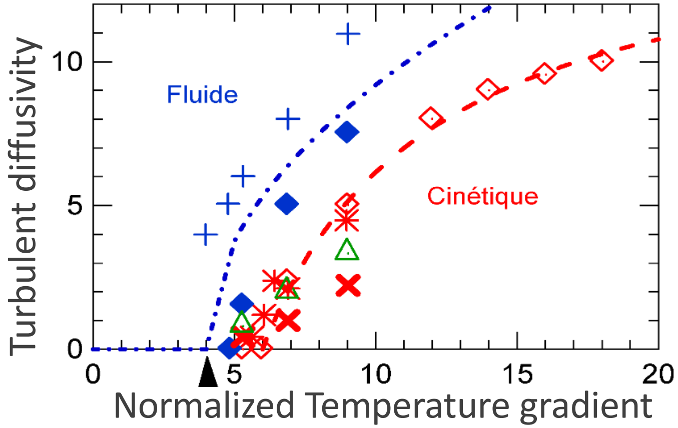
- 2000: Gyrokinetic simulations

  → **Fluid codes overestimate turbulent diffusivity**

  "Dimits' upshift" = non-linear upshift of threshold due to ZF

[Dimits et al., PoP (2000)]



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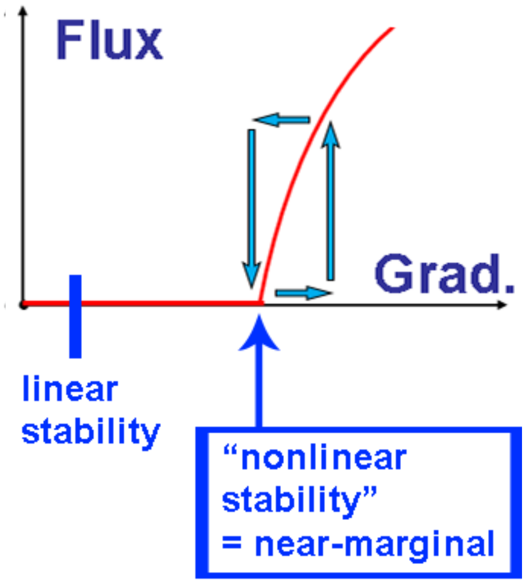
Reduced models: a practical requirement… « fidelity »?

- **Open system**
  - Slow forcing out-of-eq. → heat/momentum/matter/vorticity sources
  - Fast relax. Processes → (many) instabilities with thresh. behaviour



Near-marginality = an important regime of operation
→ Wonderland for self-organisation: slow modes key
→ Zonal (mean) flows
→ similarities SOC (fronts, spreading/nibbling, …)
→ Secondary structures (staircase),
→ Bifurcations

- **Hierarchy of models**
  - ① Flux-driven primitive equations (Vlasov—Maxwell)

  → impractical cost for parameter exploration & interpretation
  - 2 popular reductions: **Quasi-Lin.** ③ & **scale-separation** ② ("gradient-driven")

Reduced modelling: required for magn. plasmas

①

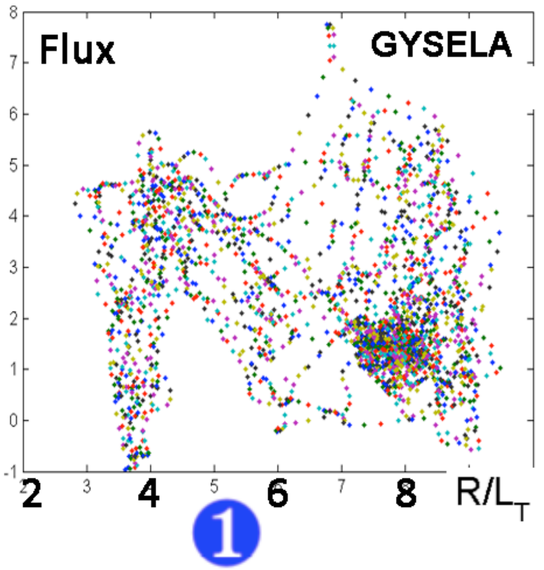
$$\partial_t(\mathbb{F}+\mathfrak{f}) - [\mathbb{H}+h, \mathbb{F}+\mathfrak{f}] = \mathcal{S}(\mathbb{F}+\mathfrak{f}) + \mathcal{C}(\mathbb{F}+\mathfrak{f})$$

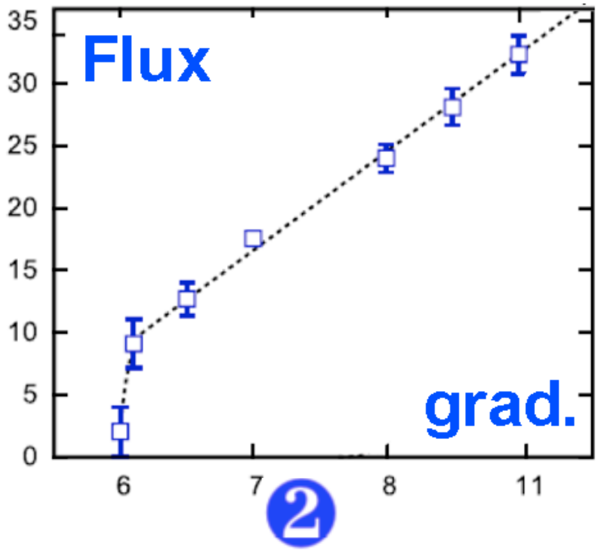
② **Scale-separation ("gradient-driven")**

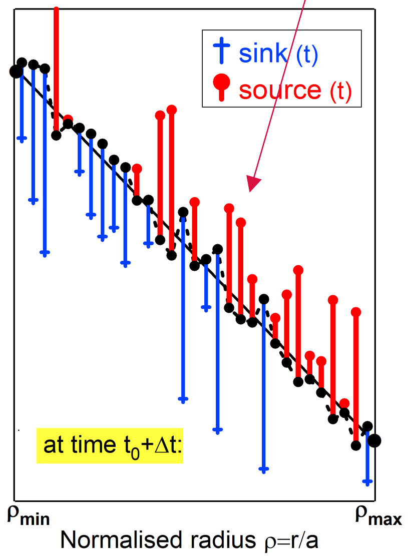
$$\cancel{\partial_t \mathbb{F} - \langle[h, \mathfrak{f}]\rangle = \mathcal{S}(\mathbb{F}+\mathfrak{f})}$$

$$\partial_t \mathfrak{f} - [\mathbb{H}, \mathfrak{f}] - \{[h, \mathfrak{f}] - \cancel{\langle[h, \mathfrak{f}]\rangle}\} = [h, \mathbb{F}] + \mathcal{C}(\mathbb{F}+\mathfrak{f}) \quad \mathbf{+S}$$

- Background fixed ←→ scale sep. = no back-reaction of fluct. on mean







Reduced modelling: required for magn. plasmas

① $$\partial_t(\mathbb{F}+\mathfrak{f}) - [\mathbb{H}+h, \mathbb{F}+\mathfrak{f}] = \mathcal{S}(\mathbb{F}+\mathfrak{f}) + \mathcal{C}(\mathbb{F}+\mathfrak{f})$$

③ **Quasi-linear**

$$\partial_t\mathbb{F} - \langle [h,\mathfrak{f}] \rangle = \mathcal{S}(\mathbb{F}+\mathfrak{f})$$

$$\partial_t\mathfrak{f} - [h,\mathbb{F}] - [\mathbb{H},\mathfrak{f}] - \mathcal{C}(\mathbb{F}+\mathfrak{f}) = \cancel{[h,\mathfrak{f}] - \langle [h,\mathfrak{f}] \rangle}$$

- Each Fourier mode is an eigenmode of fluct. linear equation
- perturb. is lin. func. of pot. spectrum & mean $\mathfrak{f}_{n,\omega} = \mathcal{L}_{n,\omega} \cdot (\phi\mathbb{F})_{n,\omega}$

→ freq. dependant product of fluct. $\phi_{n,\omega}$ and ambient $\mathbb{F}_{n,\omega}$

- Consistency: mean profile evol << fluct. evolution

↳ $\mathcal{L}_{n,\omega} \cdot (\phi\mathbb{F})_{n,\omega} \approx [\mathcal{L}_{n,\omega} \cdot \phi_{n,\omega}]\,\mathbb{F}$

- Heat flux $Q_r = \int_{\text{fast}} d\omega \sum_{n\neq 0} \int \mathcal{E}\, v_{n,\omega}^{E,r,\star}\, \mathfrak{f}_{n,\omega} \approx g(\phi_{n,\omega}^2)$
- Closure: spectrum
  - i. Double power law k_y & unstable modes only → damped modes?
  - ii. Fit on ref. nonlin. runs → inherit shortcomings & not self-consistent
  - iii. Static spectrum → should respond to free energy & flow evol.

NB: 2nd order cumulant eq. (3rd order closure) not considered

The goal of precise cross-model comparison

Comparing…

- Flux-driven ①
- Grad-driven (scale separation) ②
- Quasilinear ③

…we wish to test assumptions of:

1. Linearity of turb. fluctuations
2. Choice for saturation rule
3. Assumptions of locality/scale separation

Procedure:

1. Run flux-driven model
2. extract profiles, run them as initial (& equil.) state of the 2 others

QL marginally valid based on Kubo number estimation → near-marginal regime ~more problematic

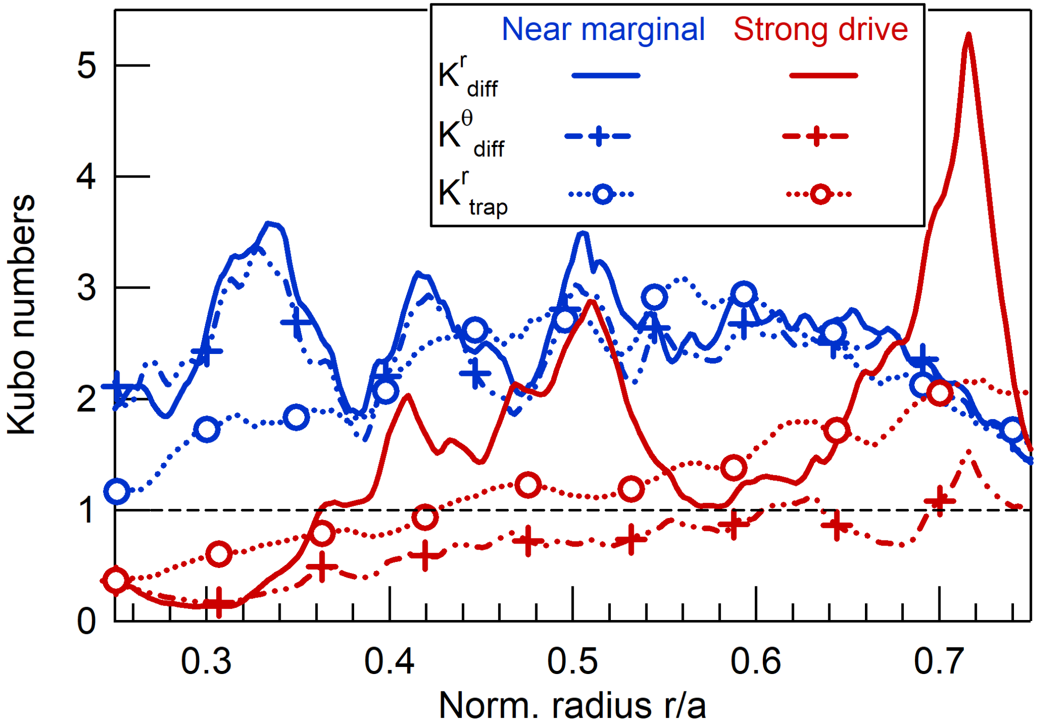
- QL = fluct. obey linear dyn → nonlinarities weak
  → modest upset structure of turb. cells

$$Q_r = \boxed{\int_{\text{fast}}} \mathrm{d}\omega \sum_{n\neq 0} \int \mathcal{E}\, v^{E,r,\star}_{n,\omega}\, \mathfrak{f}_{n,\omega}$$

- $\mathrm{Ku} = \dfrac{\text{charact. timescale of turb. field}}{\text{trapping/wave−part interact. time}}$

- Several estimates (Lagrangian time computed from 3D data)

$$\left\{ \begin{aligned} K^r_{trap} &= \frac{\tau^{eddy}_{turb}}{\tau^*_{wp}} \sim \frac{B/\langle|\nabla^2_\perp \tilde{\phi}|^2\rangle^{1/2}}{|L_\theta e B/\nabla T|} \\ K^x_{diff} &= \frac{\tau^{corr.}_{wp}}{\tau^{diff,x}_{turb}} \sim \frac{\tau^{corr.}_{wp}}{L_x/\langle|v_{E,x}|^2\rangle^{1/2}} \end{aligned} \right.$$



- Marginal QL valid. **Ku ~ O(1)**

  non discriminating

- Larger near-marginality
  → nonlinearity wins over stochasticity
  → spreading / avalanche / fronts <u>not irrelevant</u>

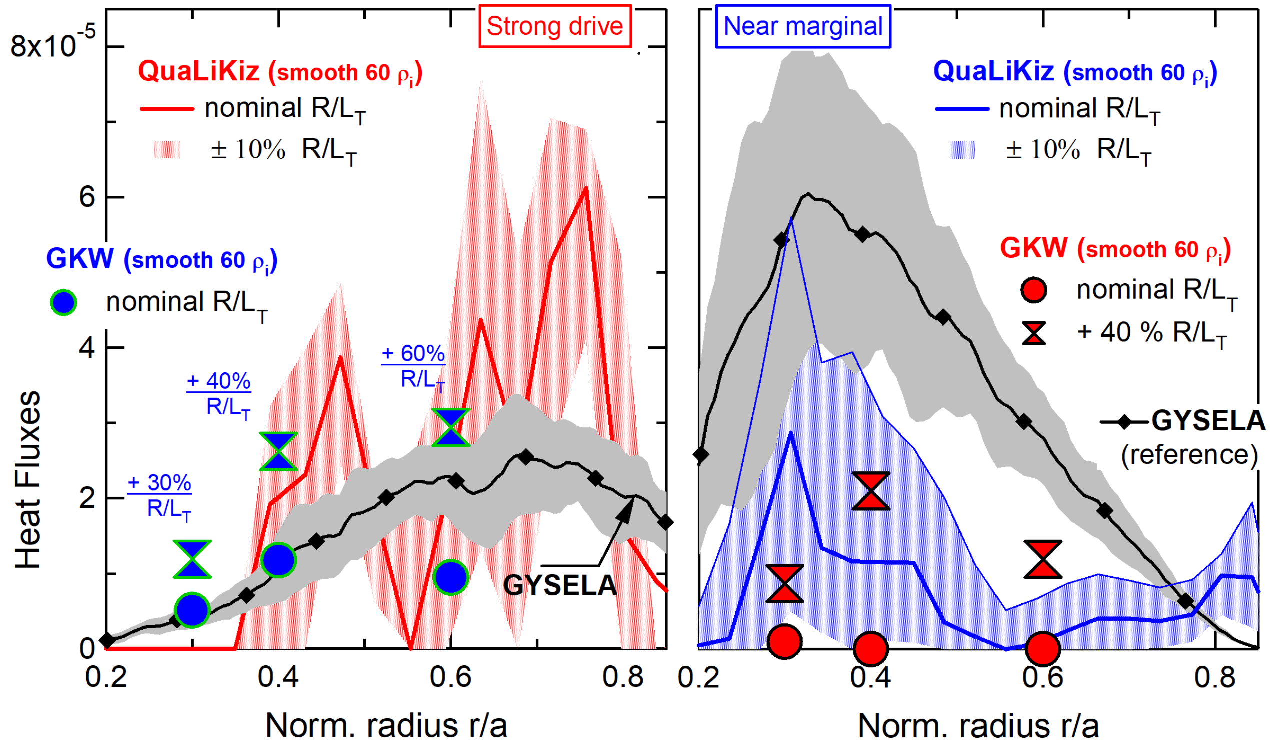
⇒ Input from mathematics?

Contrasted success for model reduction <u>near marginality</u>
→ *"again": ZFs & secondary patterns to blame?*

- ▶ reasonable agreement when strongly driven
- ▶ Important <u>mismatch near marginality</u> (outside sensitivity scan)
  - ↳ Confirmed in flux-driven evolution of QLK

<u>NB1:</u> **inherent shortcoming of QL**? Inheritance **scale separation/locality** assump.?

<u>NB2:</u> QL especially = large stiffness & sensitivity to shear → caution w "prediction"



NB:

- ▶ Local simulations (QL & GKW) fed with time averaged GYSELA profiles
- ▶ Shaded area = RMS fluctuations (GYSELA) or sensitivity scan (QLK)

Closure (sat. rule) seems to fail … again
➔ *here: <u>near marginality</u> ; structure formation to blame*

- Assumption of linearity of fluct. ~valid, despite K~1

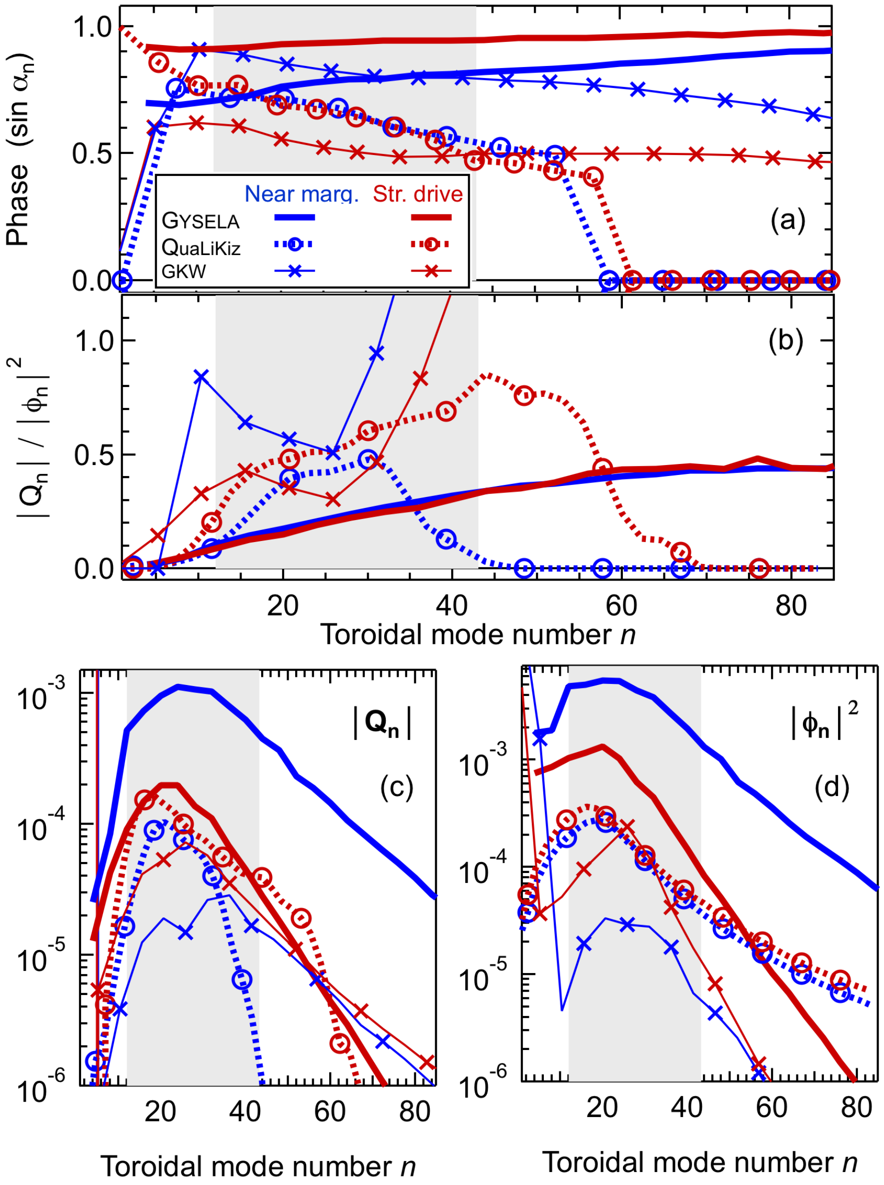
$$Q_n = \left\langle \frac{3}{2} p_n^* \, v_{E,n}^r \right\rangle_\theta = -i |Q_n| \, e^{i\alpha_n}$$

- Failure of **closure** (again!)
  ➔ **quasilinear spectrum**

ZFs & secondary patterns to blame

- Complementing QL models with e.g. dynamical equation for **spectrum,$\mathbf{I_{turb}}$**
  ➔alternative to current closures?

  e.g. k-Ɛ ; I.A (neural networks)



Take-home messages & discussion

- forcing influences accessible states [long history, e.g. St-Michel PRL 2013]

- different dynamics & different stat. equilibria, depending on distance to marginal stability
  → Significant flux at/below NL instability threshold → spreading & flow patterning key

- Quasilinear approx.
  - Marginally valid Ku = 0(1) ➔ more discriminating math. criteria? …
  - … yet robust fundamentals of QL theory in nonlinear regimes
  - Problem: closure (again!) ≡ saturation rule ➔ structure formation

Way forward → need improve closures/sat. rules for QL models:

- Account for **turb. spreading**/avalanches & **flow patterning**
- Train on GK flux-driven simulations and/or add dynamics of turb. intensity (e.g. bi-stability, reaction-diffusion, k-ε, etc.) and/or seriously consider cumulant expansion

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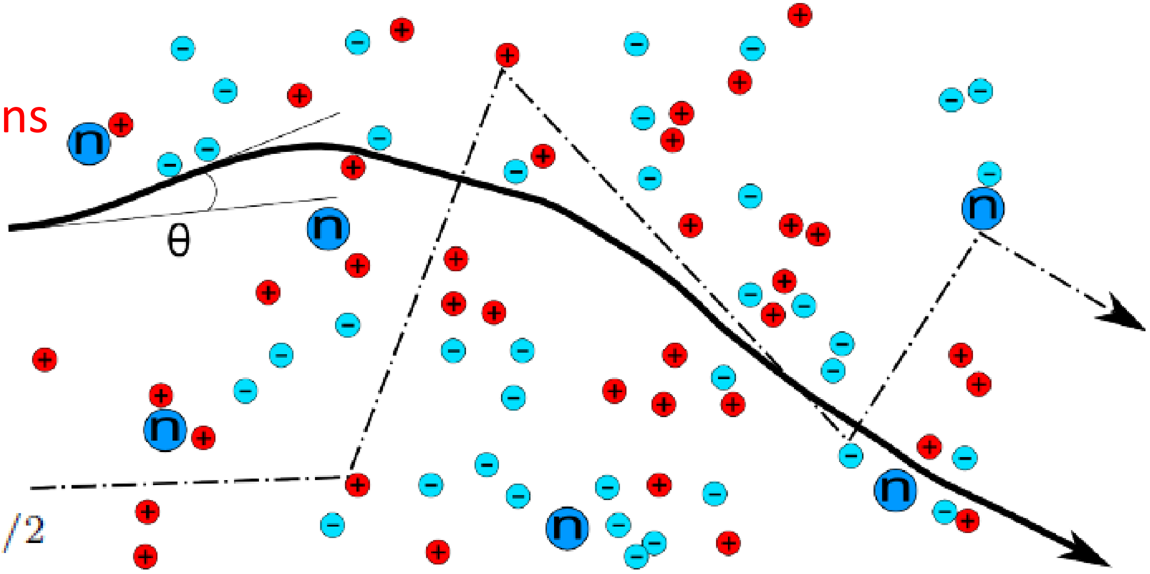
Weak collisions… yet essential $[\nu_* \rightarrow 0] \neq [\nu_* = 0]$

- **Dilute & hot plasma:**
  - **No** head-on **fusion** collisions
  - Most Coulomb collisions are grazing
  - Weak core collisionality:

$$\nu_* = \nu_{coll}\tau_{\|} \ll 1$$

$$\nu_{coll} \sim nT^{-3/2} \qquad \tau_{\|} \sim L_{\|}T^{-1/2}$$



- **Critical issues governed by collisions** (NS vs. Euler):
  - Relaxation towards Maxwellian; regul. of filamentation
  - Flow damping: **friction** on trapped particles (→ transition between turbulent regimes, …), **Zonal Flow** damping (→ turbulence saturation)
  - Impurity transport, especially high Z + possible synergy turb./coll.
  - Momentum & Energy exchanges between species

General form of collision operator

- **Landau formulation** (1936) of the Fokker-Planck operator
  (limit of small angle scattering events & cut-off at Debye shielding distances)

$$\frac{\partial f_a}{\partial t} = \sum_b \mathcal{C}_{ab}(f_a, f_b)$$

Coulomb logarithm: $\ln\Lambda = \ln\left(\frac{\lambda_D}{b_0}\right) \gg 1$

$$\mathcal{C}_{ab}(f_a, f_b) = \frac{\partial}{\partial \mathbf{v}} \cdot \left\{ \frac{2\pi e_a^2 e_b^2 \ln\Lambda}{m_a} \int \mathrm{d}^3\mathbf{v}' f_a(\mathbf{v}) f_b(\mathbf{v}') \frac{\mathbb{I}u^2 - \mathbf{u}\otimes\mathbf{u}}{u^3} \cdot \left(\frac{1}{m_a}\frac{\partial \ln f_a(\mathbf{v})}{\partial \mathbf{v}} - \frac{1}{m_b}\frac{\partial \ln f_b(\mathbf{v}')}{\partial \mathbf{v}'} \right) \right\}$$

$$\mathbf{u} = \mathbf{v} - \mathbf{v}'$$

- Particle, momentum & energy **conservation** → exchanges between species
- Satisfies Boltzmann **H-theorem** (1872): Entropy production & C($f_{Maxwell}$)=0

[Helander-Sigmar'05, Hirschman-Sigmar '76 '77 '81, Hinton-Hazeltine '76, Hinton '83,
Rosenbluth-Hazeltine-Hinton '72, Abel '08, Barnes '09, Donnel '19]

Towards a nonlinear collision operator

- Usually **linearized**: $f_s = f_{M,s} + \delta f_s$ with $\delta f_s \ll f_{M,s}$

$$\mathcal{C}_{ab}(f_a, f_b) \approx \mathcal{C}_{ab}(f_{Ma}, f_{Mb}) + \mathcal{C}_{ab}(\delta f_a, f_{Mb}) + \mathcal{C}_{ab}(f_{Ma}, \delta f_b)$$

Test particle op. Field op.

NB: also { norm **|v|** + pitch of **v** + mom. exchange a ↔ b}

- **Full operator (nonlin.)** required at the edge (strong fluctuations, departure F_{Maxwell})

$$\mathcal{C}_{ab}(f_a, f_b) = -\frac{4\pi e_a^2 e_b^2 \ln\Lambda}{m_a m_b} \frac{\partial}{\partial \mathbf{v}} \cdot \left(\frac{m_a}{m_b} \frac{\partial h_b}{\partial \mathbf{v}} f_a - \frac{1}{2} \frac{\partial^2 g_b}{\partial \mathbf{v} \partial \mathbf{v}} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right)$$

Rosenbluth potentials (1957)

$$\nabla_v^2 h_b = -4\pi f_b \qquad \nabla_v^2 g_b = 2 h_b$$

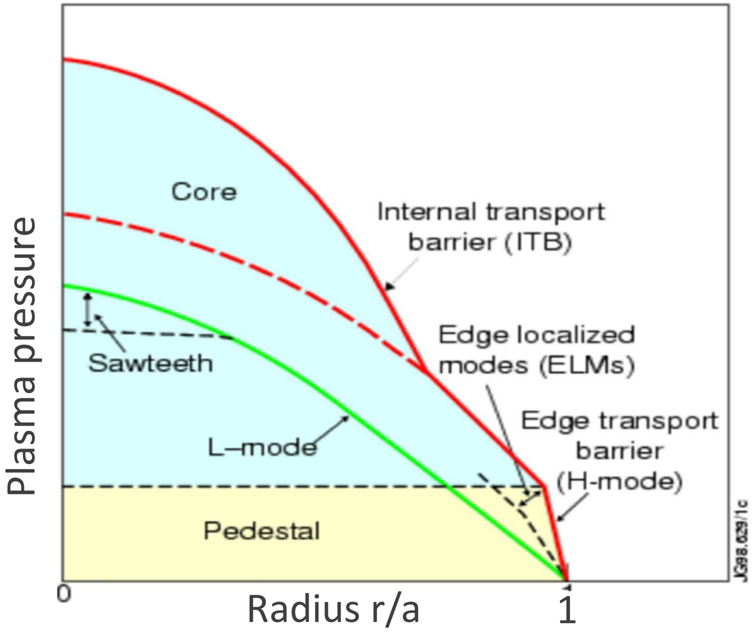
- **Physical** issues:
  - Conservation properties
  - compromise between **canonical** & **Maxwellian** distrib.
  - **Neoclassical** results **D**f_a/**D**t = $\mathbf{C_{ab}}$ → χ_{neo}, $V_{\theta,\mathrm{neo}}$, $\Gamma_{Z,\mathrm{neo}}$, …
- **Numerical** issues:
  - Whatever mass ratio m_a/m_b and charge states e_s
  - To be adapted to actual choice of variables, e.g. $(v, v_\parallel/v)$, $(v_\parallel, \mu)$
  - Moderate CPU time → parallelization

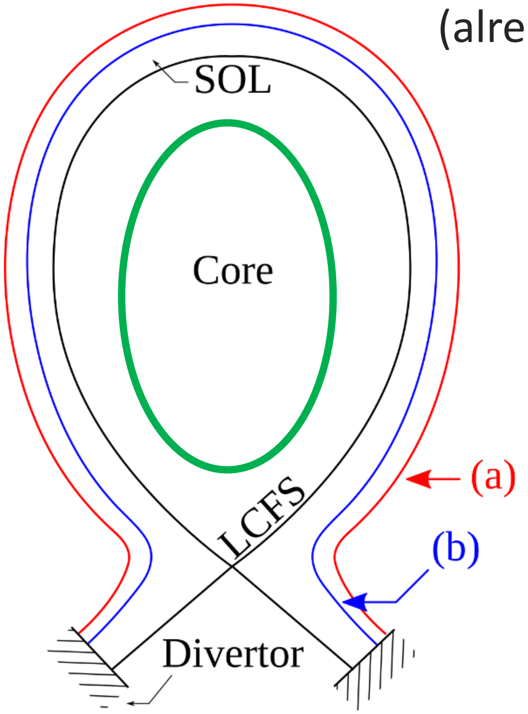
Outline

- Fluid approaches: the closure problem
- The quasi-linear reduction: some current problems
- Weakly collisional (gyro)-kinetics
- Boundary conditions: kinetics of plasma-mat. surface interaction

SOL-Core-edge coupling is key to assessing confinement

- In tokamaks, global confinement much dependent on edge plasma:
  - Edge transport barrier = spontaneous reduced turbulent transport
  - Critical role of sheared electric field $v_{E\theta} = -E_r/B$ → vortex stretching (already in L-mode)





$$\boxed{E_r \sim \nabla p/(ne) < 0}$$

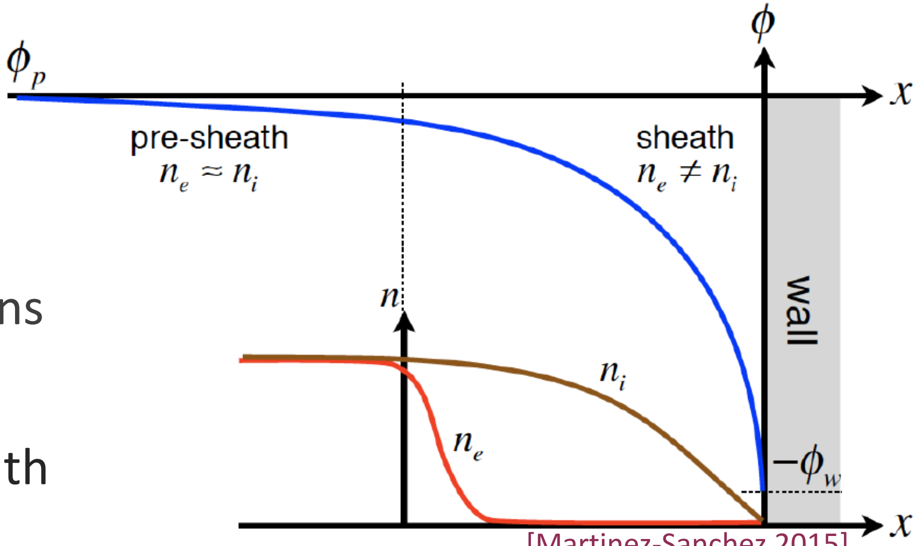
VS

$$\boxed{\boxed{E_r \sim -\nabla T_e > 0}}$$

Physics of plasma-wall interaction = // BC

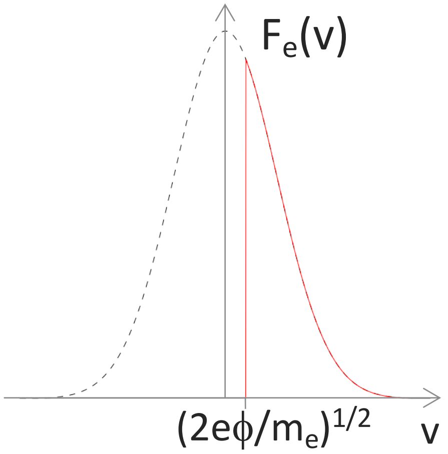
- **Mainly governed by // motion:**
  - Different electron & ion inertia
    - ⇒ Space electric charge @ wall
  - Large $E_{//}$ in the sheath
    - ⇒ Accelerates ions / confines electrons
    - ⇒ Vanishing current in the wall
  - Bohm (fluid) criterion: $v_i = \pm c_s$ at sheath entrance

[Bohm 1949; …; Riemann 1991; Stangeby 2000; Ghendrih 2011]



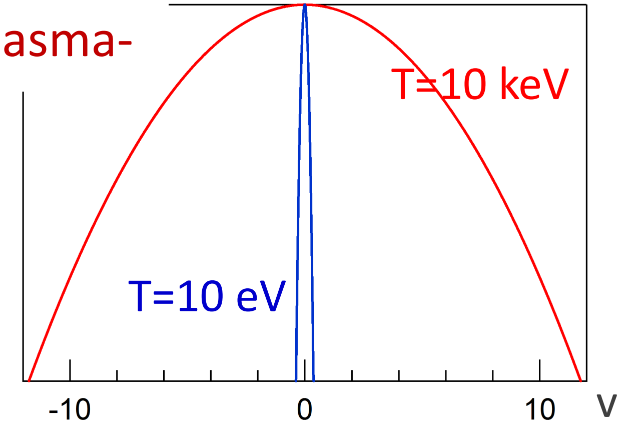
[Martinez-Sanchez 2015]

- **Physical & numerical issues:**
  - Strongly distorted distribution functions (non-Maxwellian)
  - Broad range of scales: $L_{//} \approx 2\pi qR >> \lambda_{sheath} \approx \lambda_D$
    - ≈ 100m $\approx 10^{-4}$m
  - Kinetic Bohm criterion?
  - Important role of reflected ions

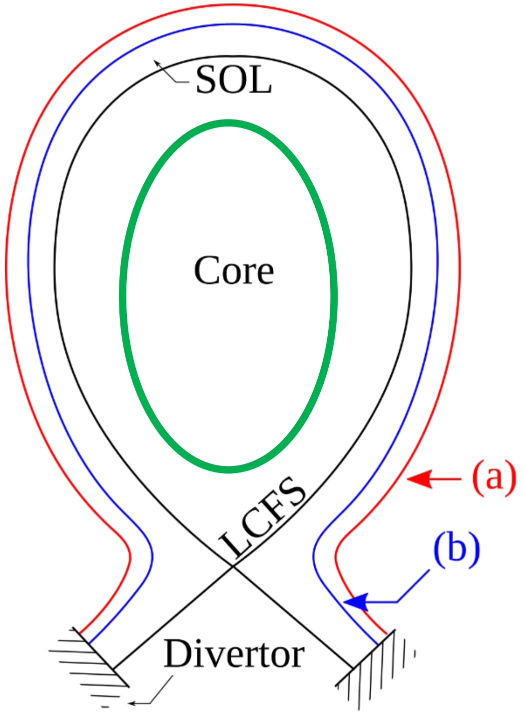


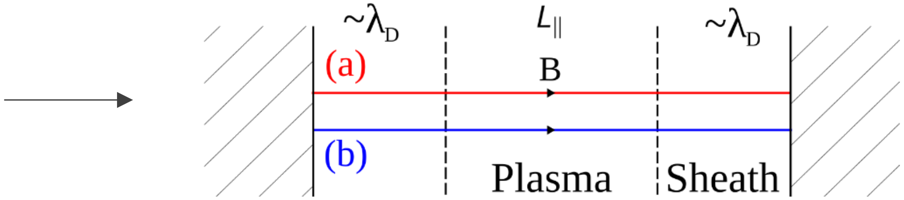
Boundary conditions: influence felt deep within the plasma

- Key challenges when addressing core & edge plasma transport:
  - Closed vs. open field lines ⇒ loss of periodicity, plasma-wall interaction
  - Complex 3D geometry
  - Huge variations of temperature (10 keV → a few eV)



1.





2.

Prepare *implementation of kinetic SOL in GYSELA*

- No curvature, no ⊥ transport, no **B** line angle
- Simulated domain : Plasma and Wall (sink)
- 1D space and 1D velocity model (1D-1V)

Kinetic modeling of plasma-wall interaction (VOICE)

Medium term = prepare implementation of kinetic Scrape-off-Layer in GYSELA

Ion & electron // dynamics governed by **(1D,1V) Vlasov-Poisson**

[Valade JPCS 2016, CPP 2017; Bourne in preparation, Munschy in preparation]

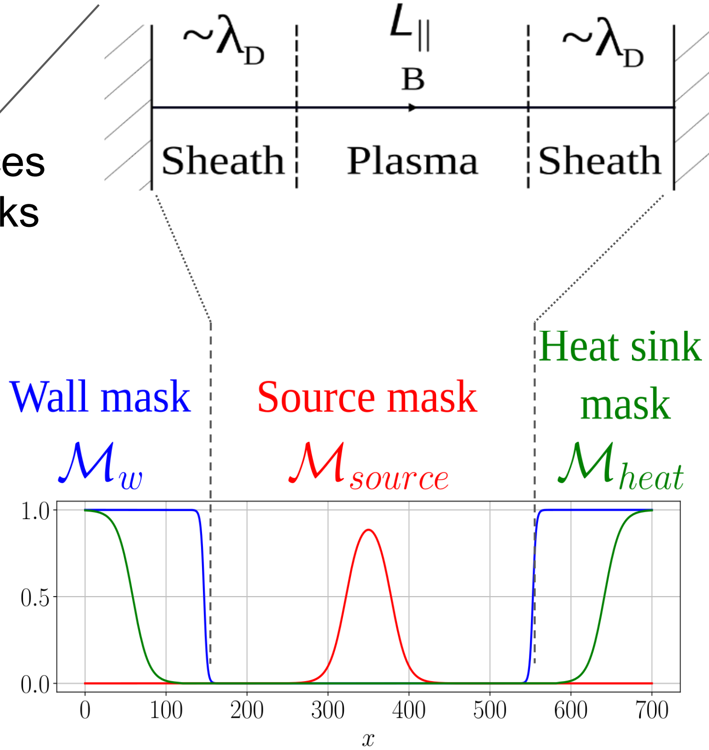
$$\partial_t f_s + \sqrt{\frac{m_e}{m_s}}\left(v_s \partial_x f_s - Z_s \partial_x \phi \, \partial_{v_s} f_s\right) = \sum_{s'} C_{ss'}(f_s) + S(f_s)$$

$$\partial_x^2 \phi = -\sum_s Z_s \int \mathrm{d}v_s f_s$$

Collisions

[Dif-Pradalier PoP (2011)]

Sources & Sinks



- **Wall = perfect particle, momentum & energy sink**

$$S_w(f_s) = -\mathcal{M}_w \, \nu_{ws}(f_s - g_w)$$

Mask ∈{0,1}

Restoring freq. ⇒ vanishing j_{wall}

Centered Maxwellian of ~vanishing density

Kinetic modeling of plasma-wall interaction (VOICE)

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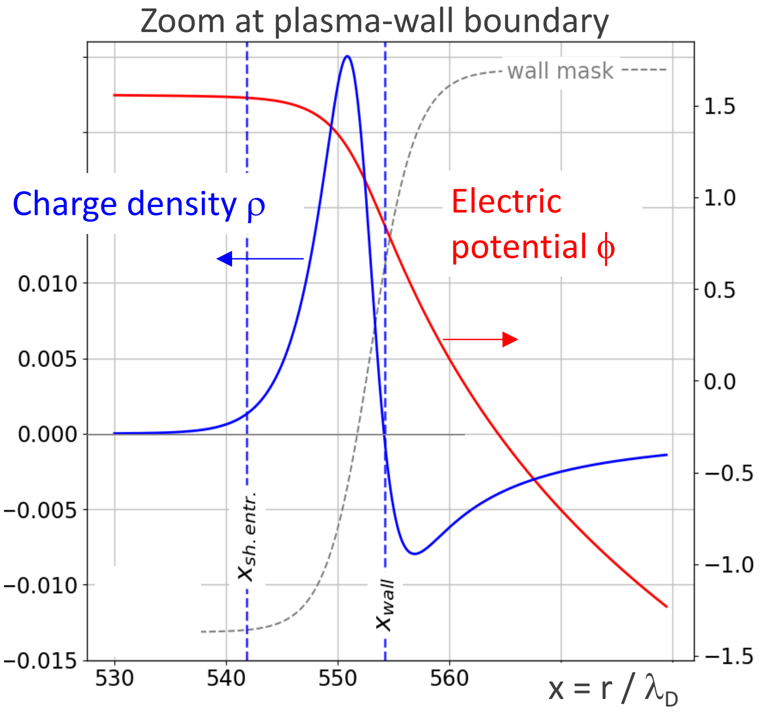
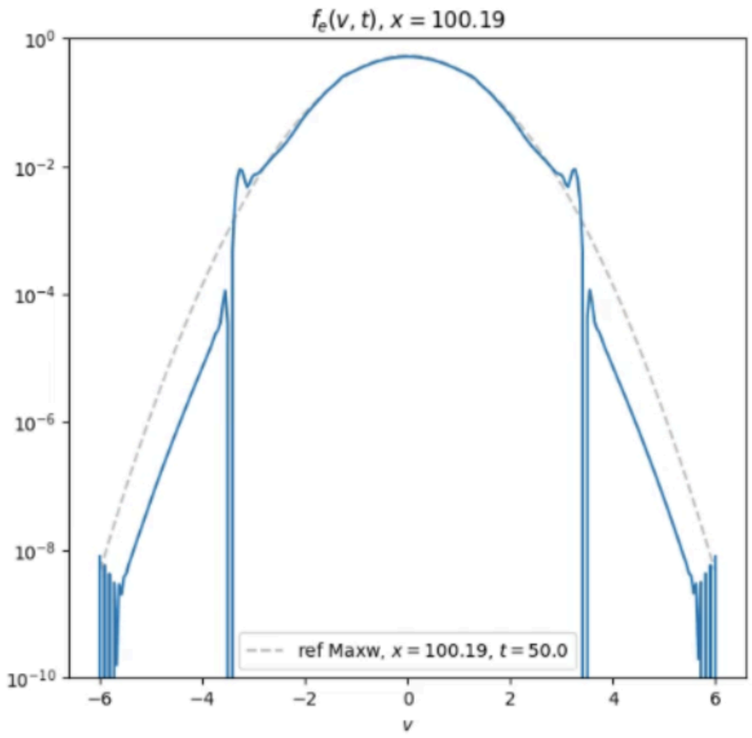
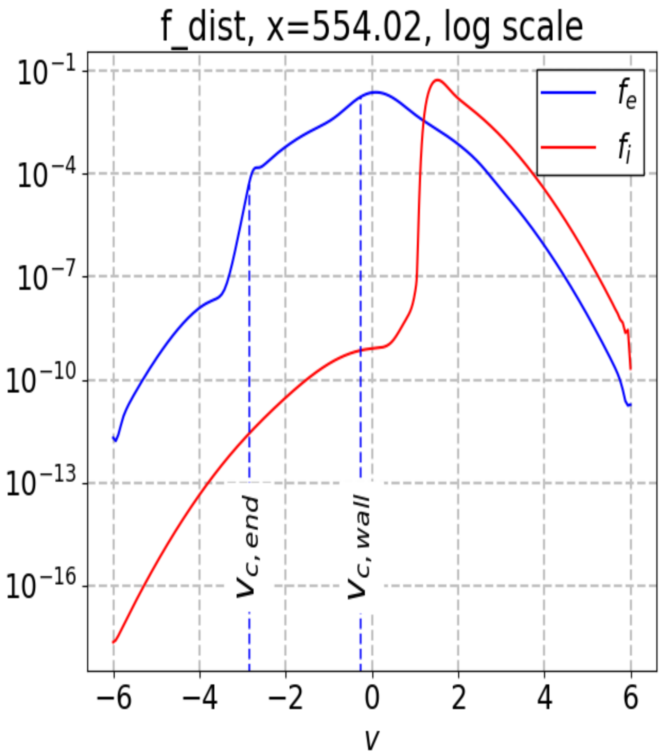


- Charge density ρ>0 in sheath ⇒ **wall position** where ρ=0

The (somewhat puzzling) central role of collisions…





Collisions ensure convergence / positivity → repopulate phase space

$[\nu_*=0]$ ≠ $[\nu_* \to 0]$

- **C(f) important math & numerics**…yet:
  - Low temperature
  - Small (Debye) scales

→ ν_* should ≈ 0, in most experimental cases

Conclusions

- First-principles simulations: crucial for ITER (pulse preparation, post-pulse analysis)
- At the limits of current HPC supercomputers (memory & CPU time)
- A hierarchy of complementary approaches (parameter exploration & real-time)

Reduced 1D models (D,V), QuasiLinear, K-ε, … → **3D Fluid, MHD** → **5D Gyrokinetic** → **6D kinetic**

Eulerian, PIC/MC, SL

- Mathematical bottlenecks: closures (fluid & quasilinear saturation rule)
- Physical bottlenecks: high $\boldsymbol{\nu}_*$ yet grazing coll.; nonlin. C(f); **B** angle w/ wall; …
- Numerical bottlenecks: coupling $\neq$ approaches, core $\rightarrow$ wall (e.g. T varies $10^{3\text{-}4}$)
- Other issues: optimization of 3D magnetic configurations (stellarators),
  free boundary magnetic equil., …

Inputs from Maths essential!

Back-up slides

Conclusions - perspectives

Kinetic (VOICE) & GyroKinetic (GYSELA) modeling within TSVV #1 & #4

(also in TSVV #6)

- Effort to **understand** and **predict plasma flows** at the **edge** of tokamak plasmas
  - Impact of **ripple** in ≠ regimes and w.r.t. turbulence [Varennes, submitted to PCF (2022) submitted to PRL (2022)]
  - **Build up of E_r** in the vicinity of separatrix [Dif-Pradalier, Comm. Phys. (2022)]
- Ongoing effort towards **Core-edge-SOL simulations**
  - Plasma-wall interaction → immersed boundary conditions
  - Parallel kinetic physics unraveled with VOICE [Munschy, in reparation (2022)]

Next step = **implementation in GYSELA**

Issues:

PWI without resolving λ_D? $E_{//}$ vs. $E_{\perp}$ in quasi-neutral.
Model for source of particles

} TSVV #4

Electromagnetic effects (critical in H-mode)
From limiter to X-point

} EoCoE-III ?

3 motion invariants at equilibrium

- **Total energy H** .... (provided $\partial_t \phi = 0 = \partial_t$ **A**)

→ $H = \frac{1}{2} m_s v_\parallel^2 + \mu B + e_s \phi$

Magnetic moment μ (adiabatic invariant): ..... (provided $\partial_t \log(B) << \omega_c$ & $\rho_s \nabla \log(B) << 1$)

→ $\mu = \frac{m_s v_\perp^2}{2B}$

~ magnetic flux embrassed by gyro-motion

- **Toroidal kinetic momentum P_φ** .... (provided axisymmetry)

→ $P_\varphi = e_s \psi + m_s R v_\varphi$

⇒ particles do NOT stick to magnetic flux surfaces

Label of flux surface

$$\psi = \frac{1}{2\pi} \oint \mathbf{B}.\mathrm{d}\mathbf{S}_{tor}$$

Particle trajectories are integrable (confined)

Loss of integrability if $\partial_t \phi \neq 0$ and/or $\partial_\varphi \neq 0$ ⇐ e.g. turbulence!

Basics on gyrokinetics

6D Vlasov equation → 4D+1D gyrokinetic equation

$$f(\mathbf{x}, \mathbf{v}, t) \quad \rightarrow \quad \bar{f}(\mathbf{x}_G, v_{G\parallel}, \mu, t)$$

$$\frac{\partial \bar{f}}{\partial t} + (\mathbf{v}_{G\perp} + \mathbf{v}_{G\parallel}).\boldsymbol{\nabla} \bar{f} + \frac{dv_{G\parallel}}{dt} \frac{\partial \bar{f}}{\partial v_{G\parallel}} = C(\bar{f}) + S$$

$$\simeq \quad \frac{\mathbf{B} \times \boldsymbol{\nabla}\langle\phi\rangle}{B^2} + \frac{m{v_{G\parallel}}^2 + \mu B}{eB} \frac{\mathbf{B} \times \boldsymbol{\nabla} B}{B^2} + o(\beta)$$

+ Jacobian / metric corrections to preserve conservative form

$$= -\frac{e}{m}\nabla_\parallel\langle\phi\rangle - \frac{\mu}{m}\nabla_\parallel B - v_{G\parallel} \frac{\mathbf{B} \times \boldsymbol{\nabla} B}{B^3} \cdot \boldsymbol{\nabla}\langle\phi\rangle + o(\beta)$$

Poisson *vs.* quasi-neutrality equation

- **Poisson** equation:

~ (few ρ_i)$^{-2}$

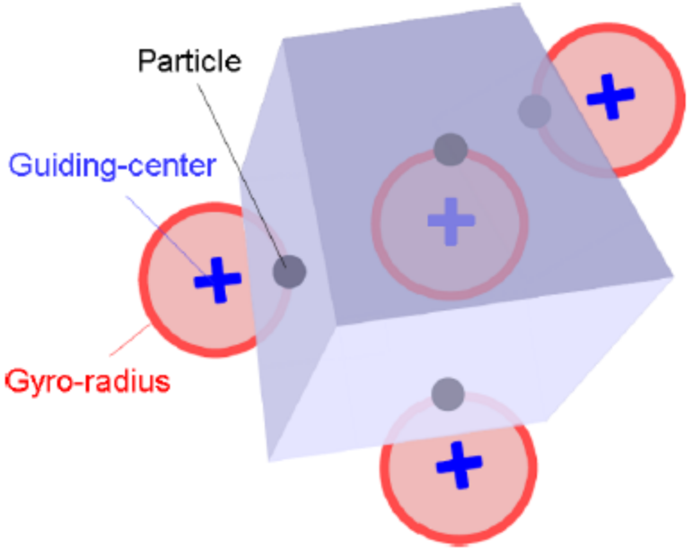
~ (few 4.10^{-3})$^{-2}$

~ few % in the core

$$\lambda_D^2\ \nabla^2\left(\frac{e\phi}{T_0}\right) = \frac{n_e - n_i}{n_0}$$

$$\lambda_D \approx 2.35\,10^{-5}(T_{[keV]}/n_{10^{20}m^{-3}})^{1/2}\ m$$

$\approx 10^{-4}\ m$ for Deuterium ions in ITER



- Safely replaced by **quasi-neutrality** (at least for ion turb.):

$n_e(x,t) = Z_i\, n_i(x,t)$ with $n_s(\mathbf{x},t) \doteq \int \mathrm{d}^3\mathbf{v}\,\boxed{f_s}(\mathbf{x},\mathbf{v},t)$

Gyrocenter density/current ≠ Particle density/current

Non trivial link between particle f_s & gyrocenter $\bar{f}_s$ distribution function

Quasi-neutrality equation in gyrokinetic

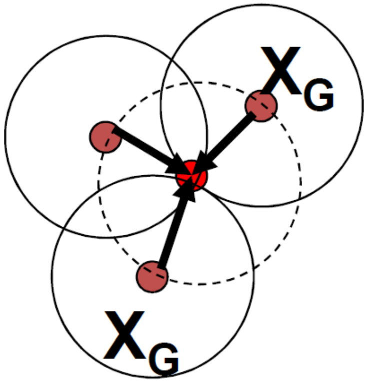
- Gyrokinetics is based on a change of variable

→ non trivial link between particle f & gyrocenter $\bar{f}$ distribution functions

$$\begin{aligned} f(\mathbf{x},\mathbf{v},t) &= \bar{f}(\mathbf{x}_G,\mathbf{v}_G,t) \\ &+ \frac{e}{B}\left\{\phi(\mathbf{x},t) - \langle\phi(\mathbf{x}_G,\mathbf{v}_G)\rangle\right\}\partial_\mu \bar{f}_{eq}(\mathbf{x}_G,\mathbf{v}_G) \end{aligned}$$

- Two contributions to particle density:

$$n_s(\mathbf{x},t) = \underbrace{\int \mathrm{d}^3\mathbf{v}\,\bar{f}_s(\mathbf{x}_G,\mathbf{v}_G,t)}_{} + \underbrace{\int \mathrm{d}^3\mathbf{v}\,\frac{e_s}{B}\bar{f}_{eq,s}(\mathbf{x}_G,\mathbf{v}_G)\,\partial_\mu\bar{\phi}(\mathbf{x}_G,\mathbf{v}_G,t)}_{}$$



Gyro-center density

$n_{Gs}(x,t)$

Gyro-radius gets polarized due to electric field

Polarization density

$n_{pol,s}(x,t)$

↓

Limit of large wave length ($k_\perp \rho_i << 1$)

$$= \nabla_\perp \cdot \left(\frac{m_s n_{eq,s}}{e_s B^2}\nabla_\perp \phi(\mathbf{x},t)\right) \simeq n_{eq,s}\,\rho_s^2\nabla_\perp^2\left(\frac{e_s\phi}{T_s}\right)$$

Limitations of gyrokinetic approach

There are important issues at the frontier of gyrokinetic framework

- High frequency processes $\omega \sim \omega_c$ (GK: $\omega << \omega_c$)
  - Radio frequency heating
  - High frequency instabilities (Mirror, Fire-Hose)

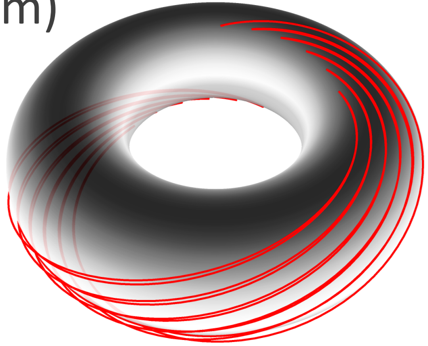
- Large amplitude fluctuations $e\delta\phi/T \sim \delta B/B \sim 1$ (GK: $e\delta\phi/T \sim \rho_*$)
  - Turbulence in the vicinity of the last closed field surface
  - MHD modes (MHD ordering: $v_E \sim v_{th}$)

- Large equilibrium gradients (gradient lengths ~ ρ_c) (GK: $\rho_c/L_\perp \sim \rho_*$)
  - Edge (H-mode) & Internal transport barriers

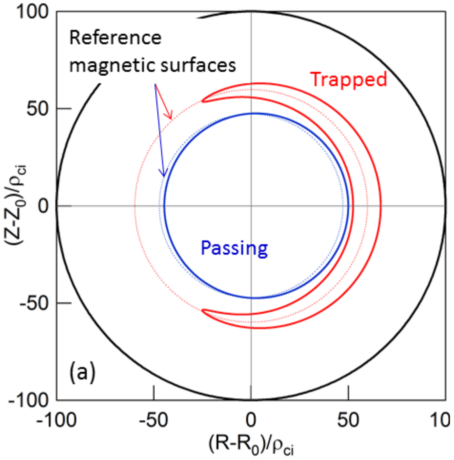
Open issue: Relevance of gyrokinetic description beyond its domain of validity ?

Challenging electron-ion turbulence simu.

- Electrostatic turbulence mainly at ion scale (few $\rho_i \sim$ cm)



- Yet, kinetic electrons are mandatory:
  - Modify Ion Temperature Gradient (ITG) turbulence
  - **Trapped electrons** → Trapped Electron Modes
  - **Passing electrons** → ETG turb., Electromagnetic effects



- **Numerical issues**: $m_D/m_e \approx 3\,672$

$\Rightarrow \rho_e \sim \rho_i/60$, $v_{th,e} \sim v_{th,i} \times 60 \Rightarrow$ **memory, CPU time**

- **Staggered approach:** [Idomura 2016; Lanti 2020]
  - Trapped electrons → Kinetic
  - Passing electrons → Boltzmann response (or fluid)

Particle conservation

$$\underbrace{n_{e,trapped}}_{\int_{trapped} \mathrm{d}^3\mathrm{v}\, F_e} + \underbrace{n_{e,passing}}_{\propto f_p \exp(e\,\delta\phi/T_e)} = \underbrace{n_i}_{\int \mathrm{d}^3\mathrm{v}\, J[F_i] + n_{pol}}$$

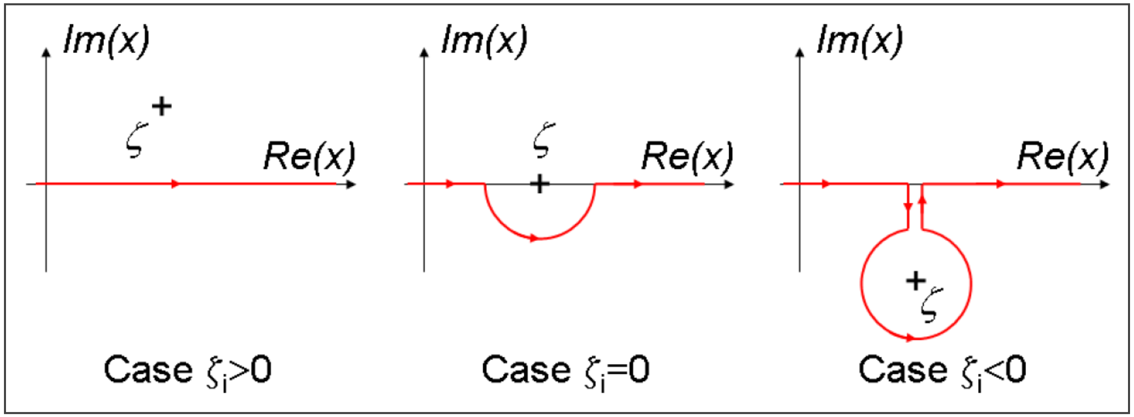
Landau damping: mathematics (II)

- Landau's rationale: causality principle ($\delta\phi \to 0 \text{ when } t \to -\infty$)
  And assumes the distribution function remains analytic…

- Causality principle governs the contour choice

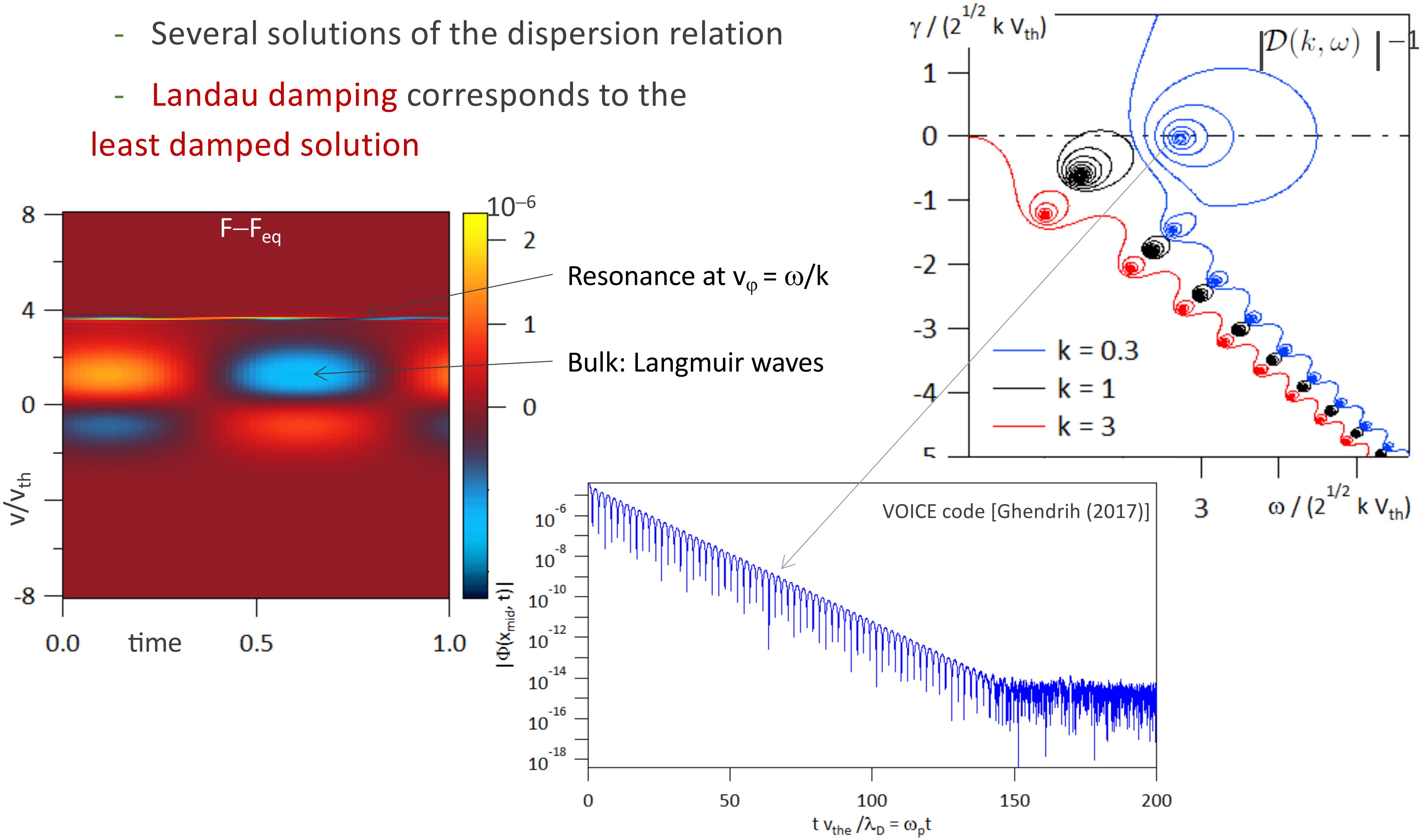
$$\mathcal{D}(k,\omega) = 1 + k^2 + \zeta\, Z(\zeta)$$

$$\left[\begin{array}{l} \zeta = \omega/(\sqrt{2}|k|) \\ Z(\zeta) = \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\infty}^{+\infty} \frac{\mathrm{e}^{-x^2}}{x-\zeta}\,\mathrm{d}x \end{array}\right.$$



Landau damping: numerical analysis

- Several solutions of the dispersion relation
- Landau damping corresponds to the least damped solution



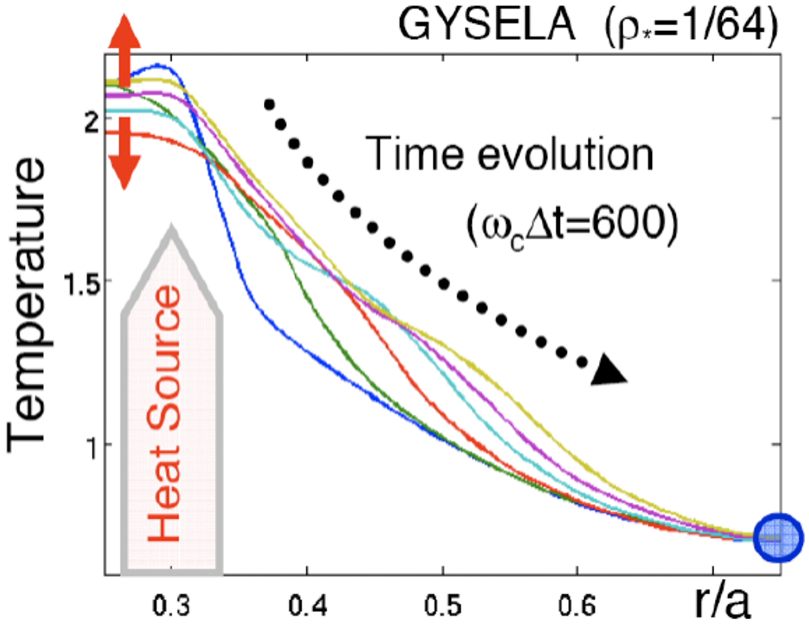
Profile relaxation & modeling

Critical ingredients for self-consistent modeling of turbulence:

1. Self-consistency ⇒ NO scale separation

F_{eq} and δF treated on equal footing → full-F

$$\partial_t F - [H,F] = \mathcal{C}(F) + S \Leftrightarrow \begin{cases} \partial_t \delta F - [H,\delta F] = [\delta H, F_{eq}] + \mathcal{C}(\delta F) \\ \partial_t F_{eq} - \langle[\delta H,\delta F]\rangle = S \end{cases}$$



Source / Sinks

⇓

- ❑ Reservoir of energy for turbulence
- ❑ Opposes profile relaxation on τ_E time scale

[Diamond-Hahm PoP (1995)]

Profile relaxation & modeling

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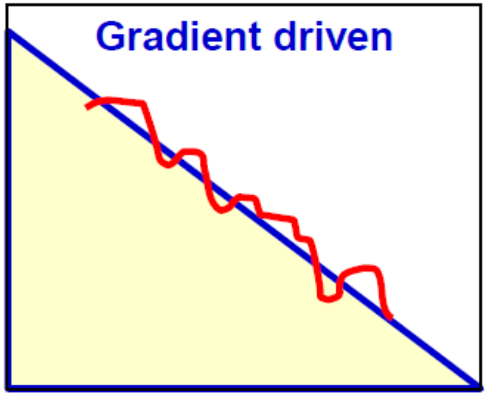
2. Adiabatic Sources & Sinks → must NOT annihilate turbulent transport

Gradient driven ⇒ inconsistent solution

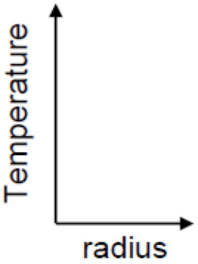
$$\langle [\delta H, \delta F] \rangle + S = 0 \rightarrow \partial_t F_{eq} = 0$$

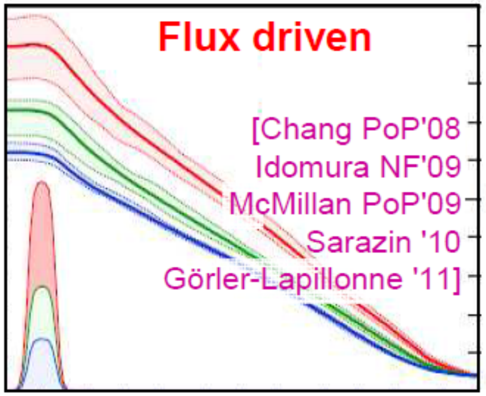
Flux driven OK: prescibed sources

$$\langle \langle [\delta H, \delta F] \rangle + S \rangle_{\tau_E} = 0 \rightarrow \langle \partial_t F_{eq} \rangle_{\tau_E} = 0$$



No profile relaxation





Towards exp. conditions

Profile relaxation & modeling

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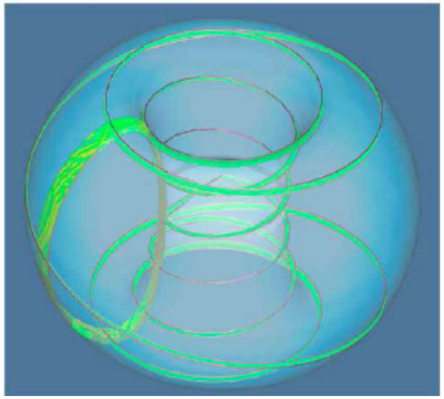
$$\partial_t F - [H, F] = \mathcal{C}(F) + S \Leftrightarrow \begin{cases} \partial_t \delta F - [H, \delta F] = [\delta H, F_{eq}] + \mathcal{C}(\delta F) \\ \partial_t F_{eq} - \langle [\delta H, \delta F] \rangle = S \end{cases}$$

2. Adiabatic Sources & Sinks → must NOT annihilate turbulent transport

3. Global ⇒ boundary conditions (open system)

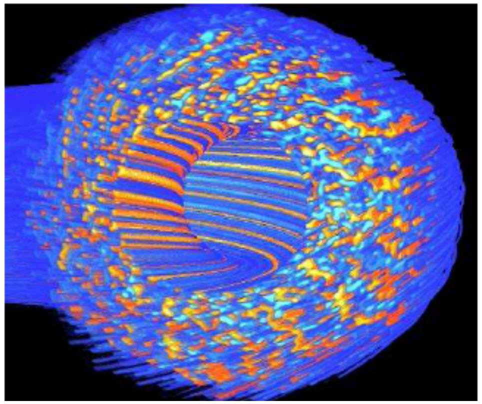
Local (flux tube) → periodic radial B.C.

GS2 [Kotschenreuther CPC (1995)]



Global

GYSELA [Grandgirard JCP (2006)]



Routes towards turbulence saturation

Saturation mechanisms:

- "Quasi-linear" profile relaxation → Reduction of Linear Drive
- Nonlinear mode-mode coupling → - Energy Cascade to damped/stable modes
  - Turbulent heating
- Energy transfer to zonal modes → Energy stored in zero-transport modes
- …

Quasi-Linear transport

- Basics on Quasi-Linear theory: $f = \langle f\rangle + \delta f$ with $\langle\ldots\rangle = \int_t^{t+\tau}\frac{dt'}{\tau}\int_0^{L_x}\frac{dx}{L_x}\ldots$

2D Vlasov-Poisson syst.
$$\begin{cases}\partial_t\langle f\rangle = -\partial_v\langle\delta f\;\partial_x\delta\phi\rangle \\ \partial_t\delta f + v\partial_x\delta f + \partial_x\delta\phi\;\partial_v\langle f\rangle = \cancel{\partial_v\langle\delta f\;\partial_x\delta\phi\rangle - \partial_v(\delta f\;\partial_x\delta\phi)}\end{cases}$$

Nonlinear coupling neglected for fluct.

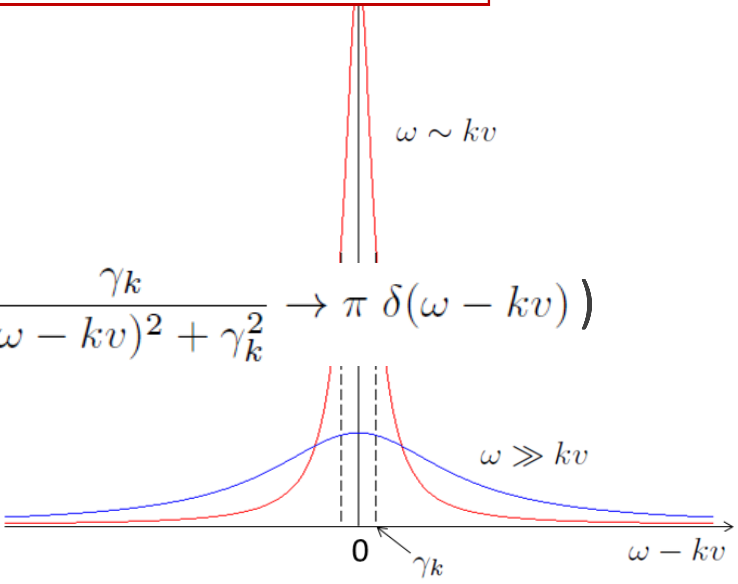
$$\Rightarrow\quad \partial_t\langle f\rangle = \partial_v\left(D_v\partial_v\langle f\rangle\right)$$

$$\boxed{D_v = \sum_k |k\hat{\phi}_{k,\omega}|^2\frac{|\gamma_k|}{(\omega-kv)^2+\gamma_k^2}}$$

$\Rightarrow$ Quasi-Linear transport is diffusive

- 2 distinct contributions:
  - Resonant particles $\omega\sim kv$
  - Non-resonant particles

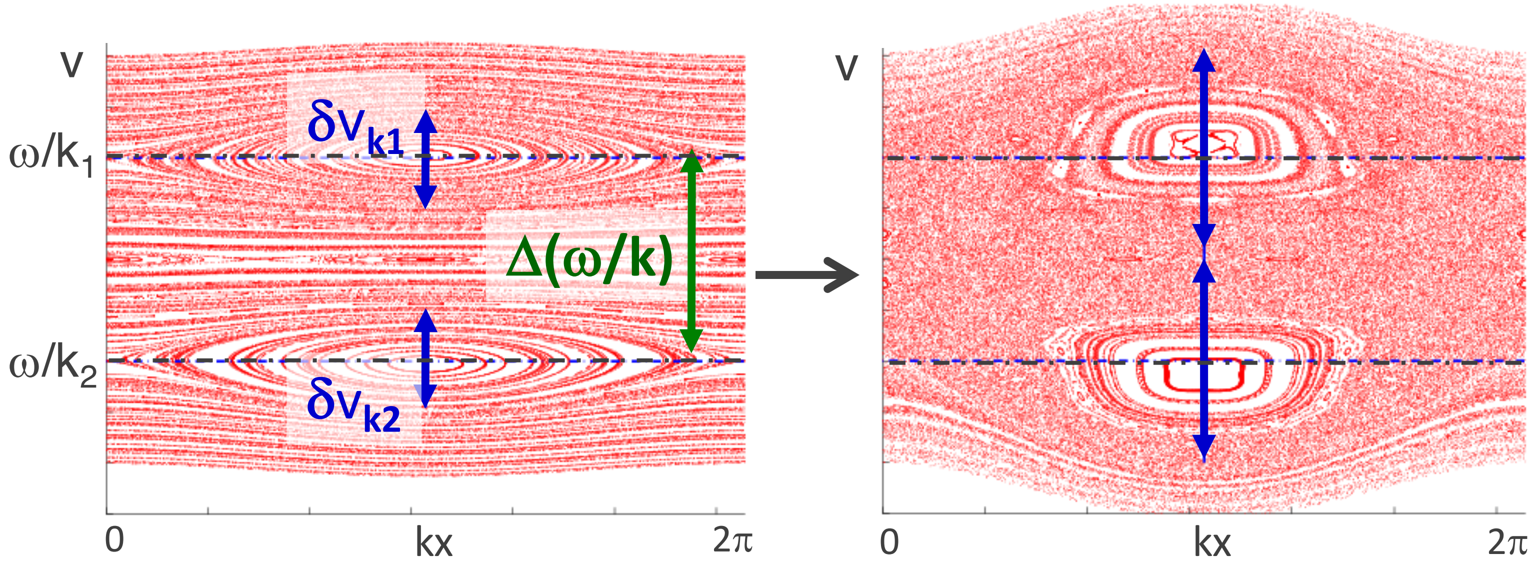
$$\left(\lim_{\gamma_k\to 0}\frac{\gamma_k}{(\omega-kv)^2+\gamma_k^2}\to\pi\,\delta(\omega-kv)\right)$$



Properties & validity of QL theory

- Efficient transport when resonances overlap → stochasticity
- Typical width of resonance (island): $\delta v_k \sim 4\ |\phi_k|^{1/2}$
- Approximate overlap criterion: $\frac{1}{2}(\delta v_{k1}+\delta v_{k2}) > |\omega/k_1 - \omega/k_2|$

[Chirikov (1960)]



- Validity of linearization → field pattern changes faster than trapping (Kubo<<1)

$$\Rightarrow \tau_{ac}^{-1} \sim |k\ \Delta(\omega/k)| = |[v_g(k) - v_\varphi(k)]\Delta k| \gg \tau_b^{-1} \sim k|\hat{\phi}_{k,\omega}|^{1/2}$$

[Diamond, Itoh, Itoh (2010)]

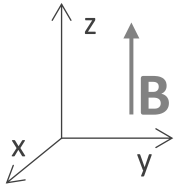
QL transport for Drift-Waves (3D)

- Example of slab ITG (instability governed by resonance, ≠ resistivity in HW)

$$\partial_t f + v\partial_z f + [\phi, f] - \partial_z\phi\,\partial_v f = 0$$

Parallel streaming along field lines

Transverse advection by Drift Waves



- Quasi-linear transport in x & v: $\partial_t\langle f\rangle - \partial_x\langle\tilde f\partial_y\tilde\phi\rangle - \partial_v\langle\tilde f\partial_z\tilde\phi\rangle = 0$
  - Radial diffusion: $\partial_x\langle\tilde f\partial_y\tilde\phi\rangle = \partial_x(D_x\partial_x\langle f\rangle)$

$$D_x = \mathrm{Re}\sum_{k\quad k}|k_y\hat\phi_k|^2\frac{i}{\omega_k - k_z v} \quad\longrightarrow\quad k_\theta = k_{\theta 0} + \Delta k_\theta \qquad k_z = k_{z0} + \Delta k_z$$

$$\simeq \mathrm{Re}\frac{i\,|k_{y0}\hat\phi_0|^2}{\omega_0 - k_{z0}v + \underbrace{(d\omega/dk_z - v)\Delta k_z + (d\omega/dk_\theta)\Delta k_\theta}_{\text{resonance broadening}}}$$

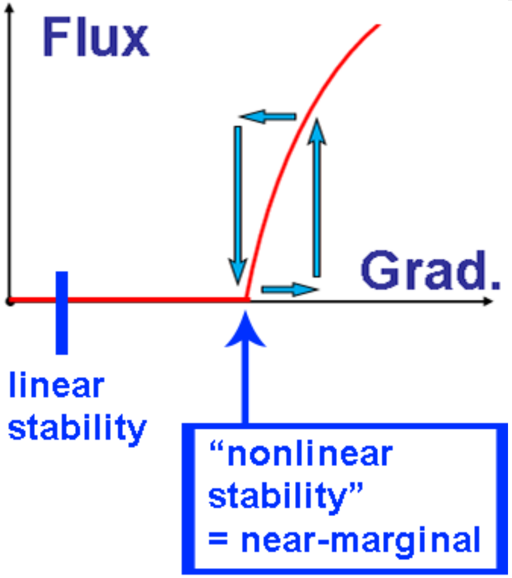
[Diamond, Itoh, Itoh (2010)]

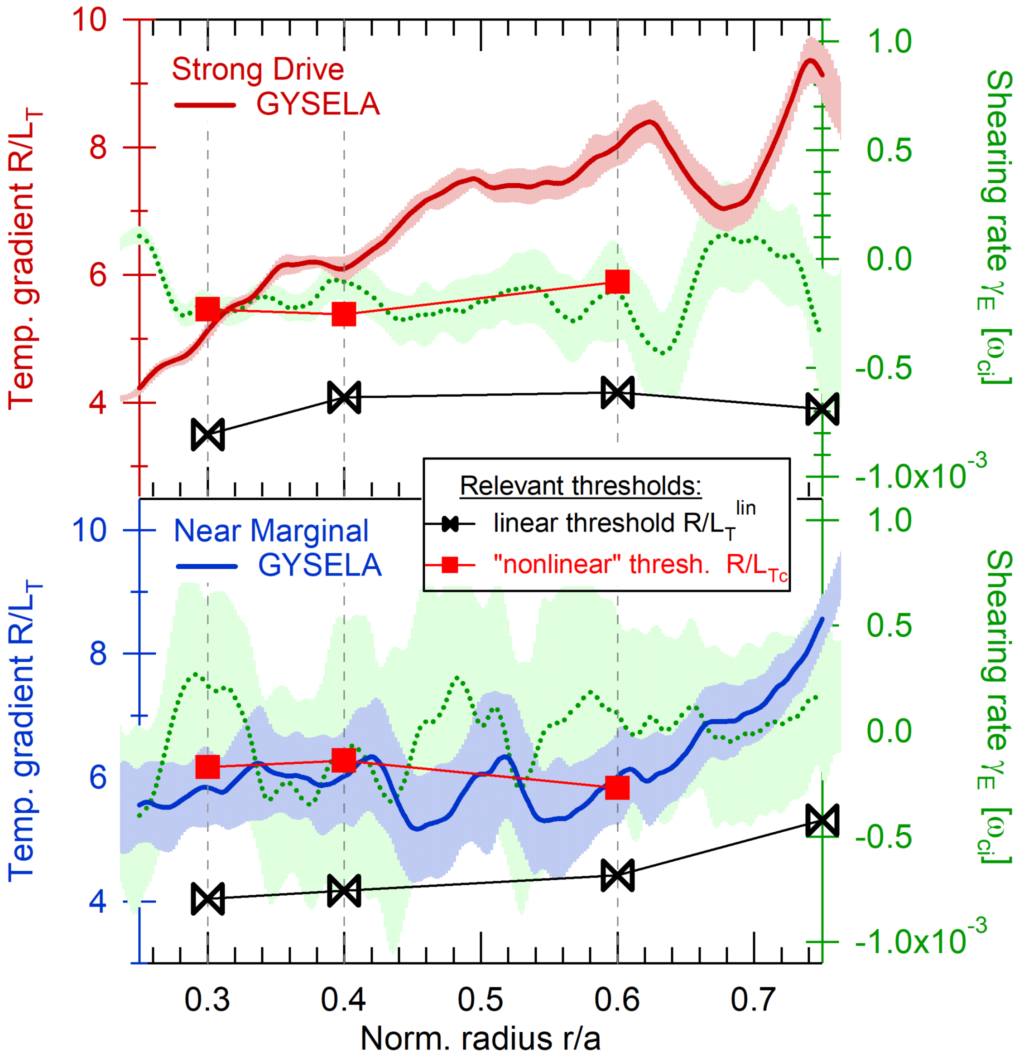
Effective decorrelation rate:

$$\tau_{ac}^{-1} \sim |v_T\Delta k_z| + \left|\frac{d\omega}{dk_\theta}\Delta k_\theta\right|$$

→ combined effect of parallel dispersion & poloidal propagation at v_{de}

Compare 3 ``s-o-t-a” models, in 2 distinct regimes





- "Strong drive"
  - Far from linear & nonlinear thresholds
- "Near marginal"
  - Meandering staircases & flow patterning
  - Close to nonlinear threshold

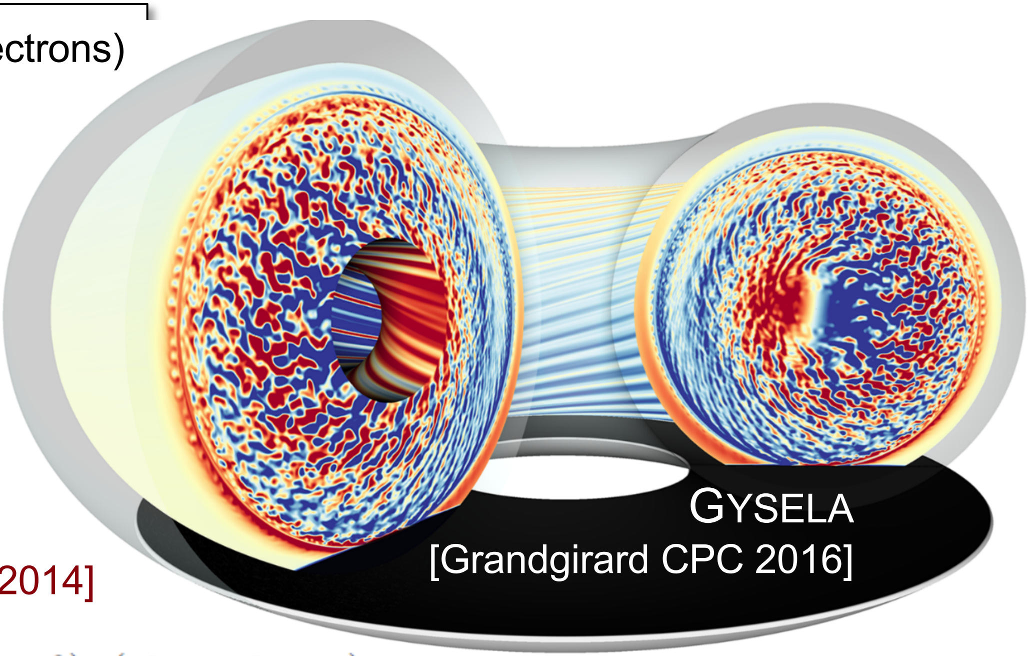
→ operationnally-relevant?

Gyrokinetic simulation with limiter

Flux-driven ITG turbulence (adiab. electrons)

$1/\rho^{*}=250$, $\nu^{*}=0.14$

Global domain: $0 < \rho < 1.3$



GYSELA
[Grandgirard CPC 2016]

- Poloidally localized toroidal limiter in between $1<\rho<1.3$
- Penalization technique = Krook operator in Vlasov RHS

[Isoardi JCP 2010; Paredes JCP 2014]

$$\frac{Df}{Dt} = \mathcal{C}_{coll} + \mathcal{S}_{heat} - \nu \, \mathrm{M}_{Lim}(r,\theta)\,(f - f_{Lim})$$

Strength of restoring force ($\nu \to \infty \;\Rightarrow\; f = f_{Lim}$)

Mask → defines Limiter region

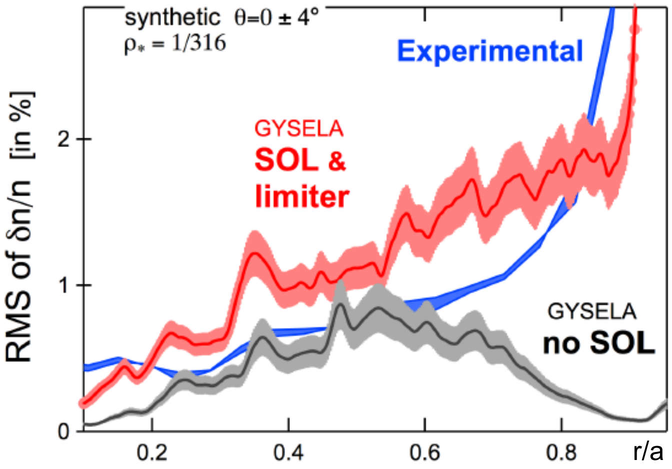
Centered Maxwellian at low Temp. ($T_{Lim} \sim 0.1\, T_{core}$)

- Modified quasi-neutrality in the SOL (adiabatic electron case):

Bohm condition is enforced → $\delta n_e/n_e = e\phi/T_e - \Lambda$

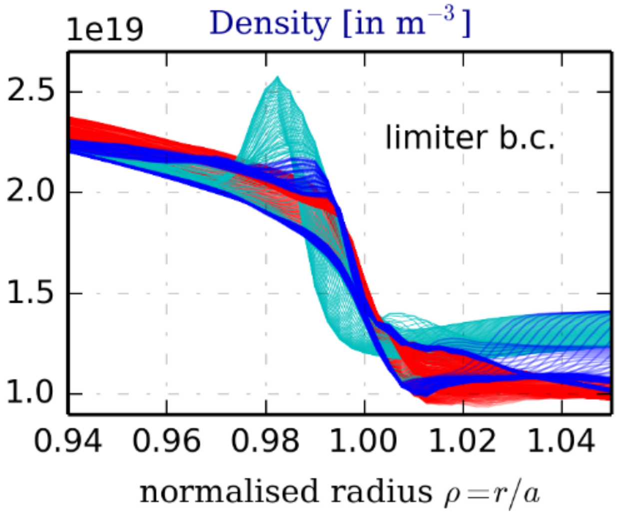
Evidence of core-edge interplay

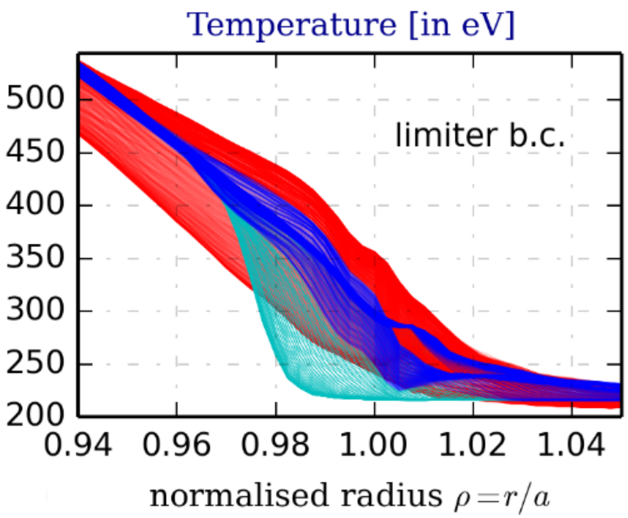
[Dif-Pradalier, Phys. Comm. (2022)]

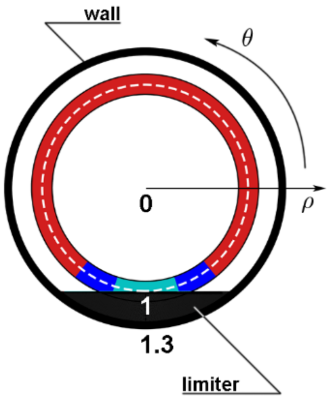


Core→edge (beach effect) AND edge→core turbulence spreading are critical to recover experimental density fluctuation increase

Steep n & T profiles at separatrix (and limiter)







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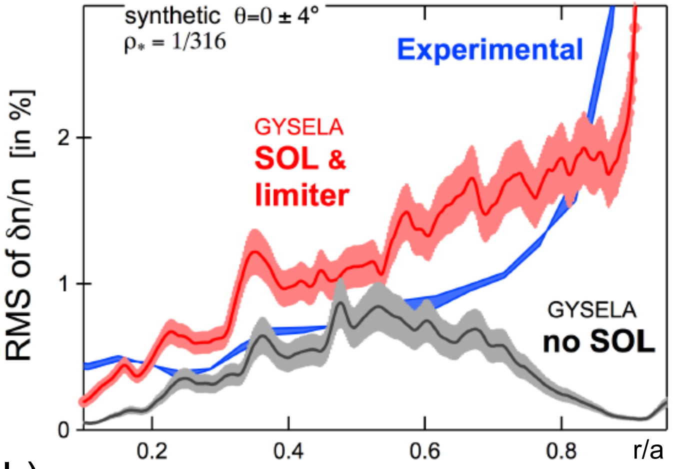


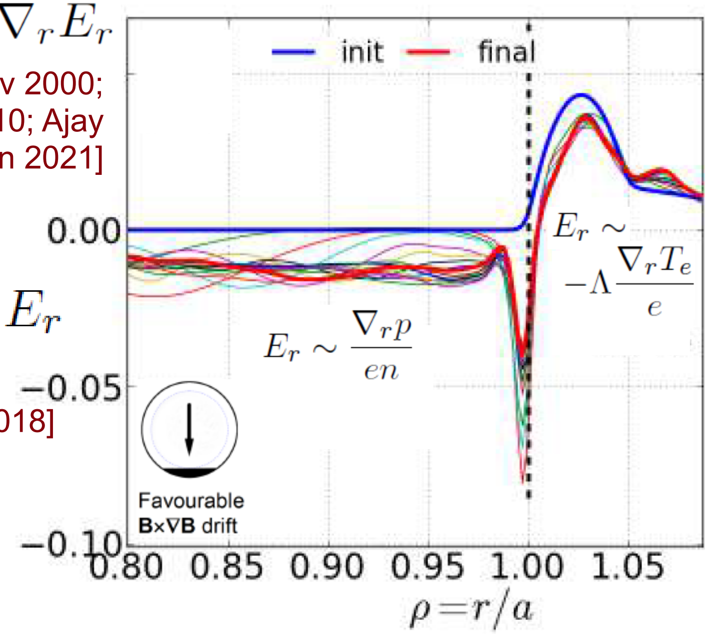
- Core→edge (beach effect) AND edge→core turbulence spreading are critical to recover experimental density fluctuation increase
- Steep n & T profiles at separatrix (and limiter)
- Deep well of E_r close to separatrix (<0 in core, >0 in SOL)

Results from complex dynamics of vorticity $\Omega_r = -\nabla_r E_r$

$$\partial_t \langle \Omega_r \rangle \approx -\nabla_r \big(\underbrace{\langle v_{Er}\, \Omega_r \rangle}_{\text{usual RS}} + \underbrace{\langle v_r^*\, \Omega_r \rangle}_{\text{diamagnetic RS}}$$

[Smolyakov 2000; McDevitt 2010; Ajay 2021; Sarazin 2021]

$$- \langle v_\theta^*\, \nabla_\theta E_r \rangle \big)$$ poloidal entrainment

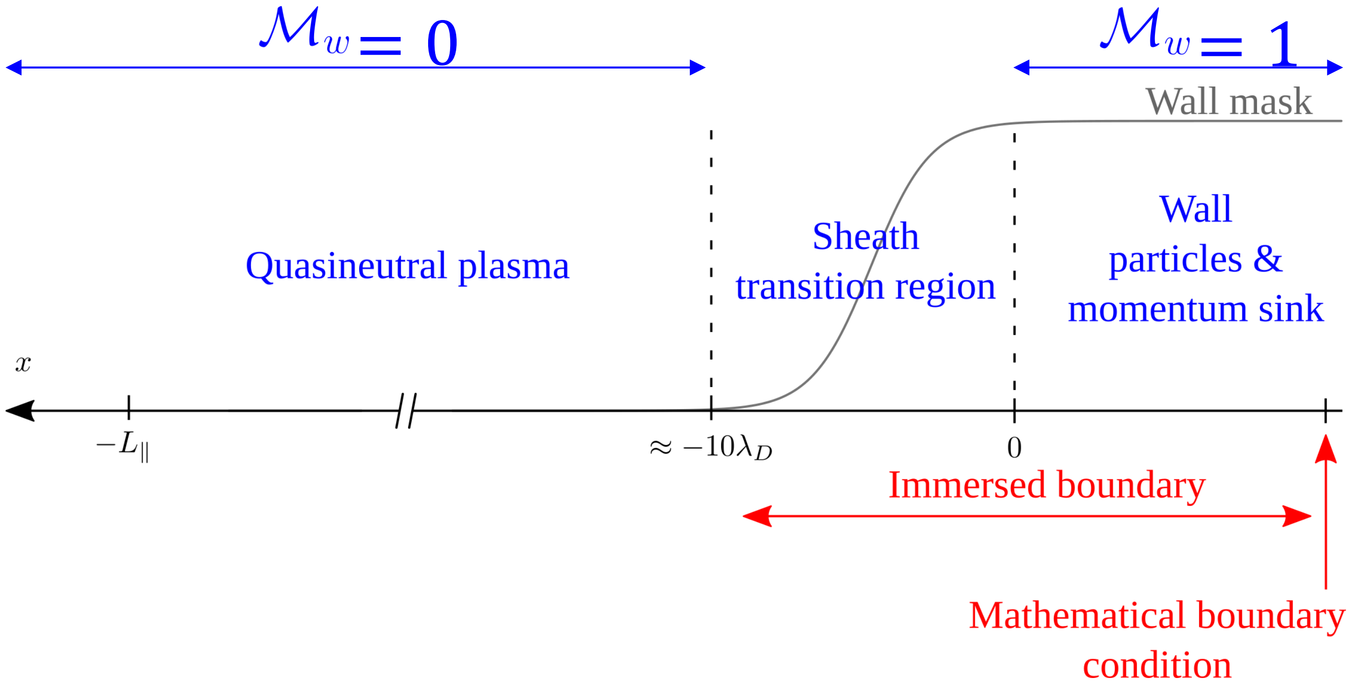


Causality elucidated thanks to "transfer entropy":

[Schreiber 2000; Van Milligen 2014; Nicolau 2018]

> Initially: diamagn. RS → Large E_r at limiter
> Later time: RS dominant downstream

Immersed boundary conditions



- Mathematical boundary condition on edge of the domain (dirichlet, etc.)
- No explicit boundary condition imposed at the plasma boundary
- Immersed boundary condition: wall = plasma with altered dynamic
- Interest : decorrelate the plasma wall transition physics from any arbitrary boundary condition

Immersed boundary conditions

Choice for the nu coefficients : $S_{sink}(f_a) = M(x)\nu_{aw}(f_a - g_w)$

1. constant coefficients: conducting plasma wall transition

Charge conservation from Boltzmann eq.

$$\partial_t \rho + \partial_x j_{\rm tot} = \sum_{\rm species} q_a \int dv\, \mathcal{S}_{\rm recomb.}(f_a)$$

$$= \sum_{\rm species} -\mathcal{M}_w \nu_{aw} (n_a - n_w) = \mathcal{S}_\rho$$

$$j_{tot} = \sum_{\rm species} j_a \text{ with } j_a = q_a n_a u_a$$

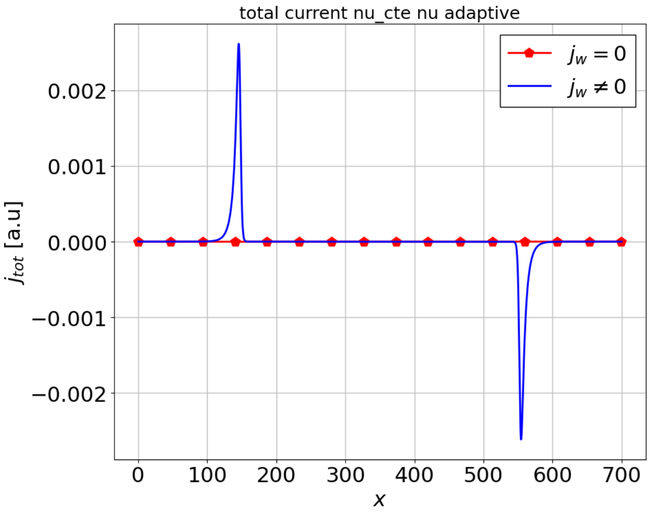
$\mathcal{S}_\rho$ not vanishing in general → currents at the plasma-wall transition

since $j_{tot}(x) = \int_0^{\bar{x}} \mathcal{S}_\rho \, dx$ Steady state

2. adaptive coefficients: ambipolar plasma wall transition

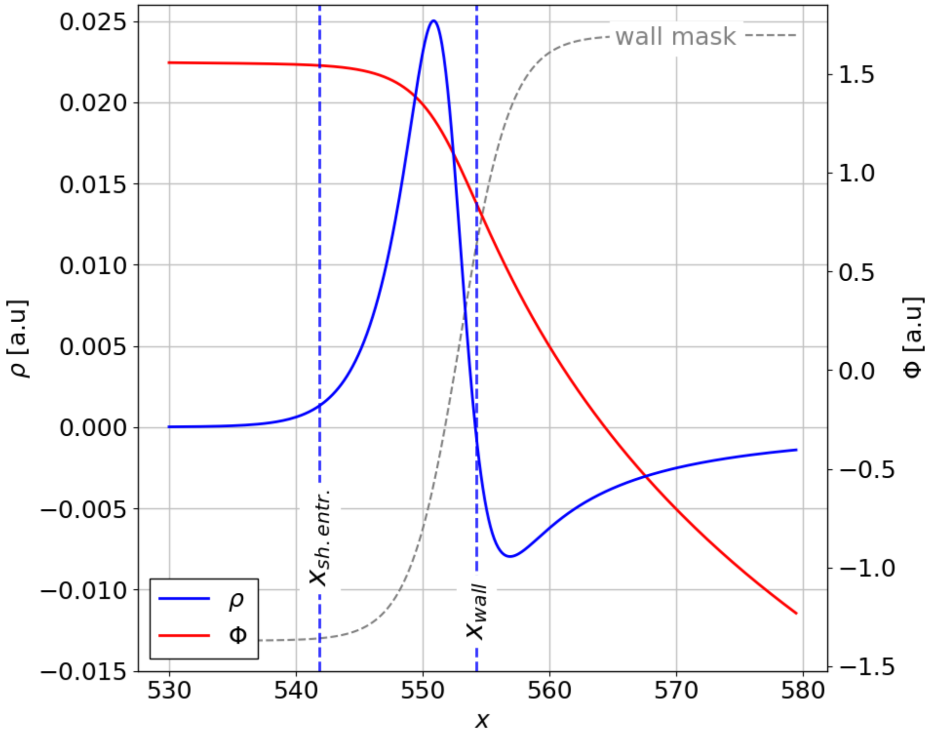
$$\nu_{iw} = \text{constant} \quad \nu_{ew}(x,t) = \nu_{iw} \frac{n_i - n_w}{n_e - n_w}$$

$$S_\rho = 0 \Rightarrow j_{tot}(x_w) = 0$$



Solution 2 ensures ambipolarity

Fluid properties: charge density



- Positively charged region at plasma
- Wall transition → sheath

- Potential drop : fast electrons confined
- acceleration of ions

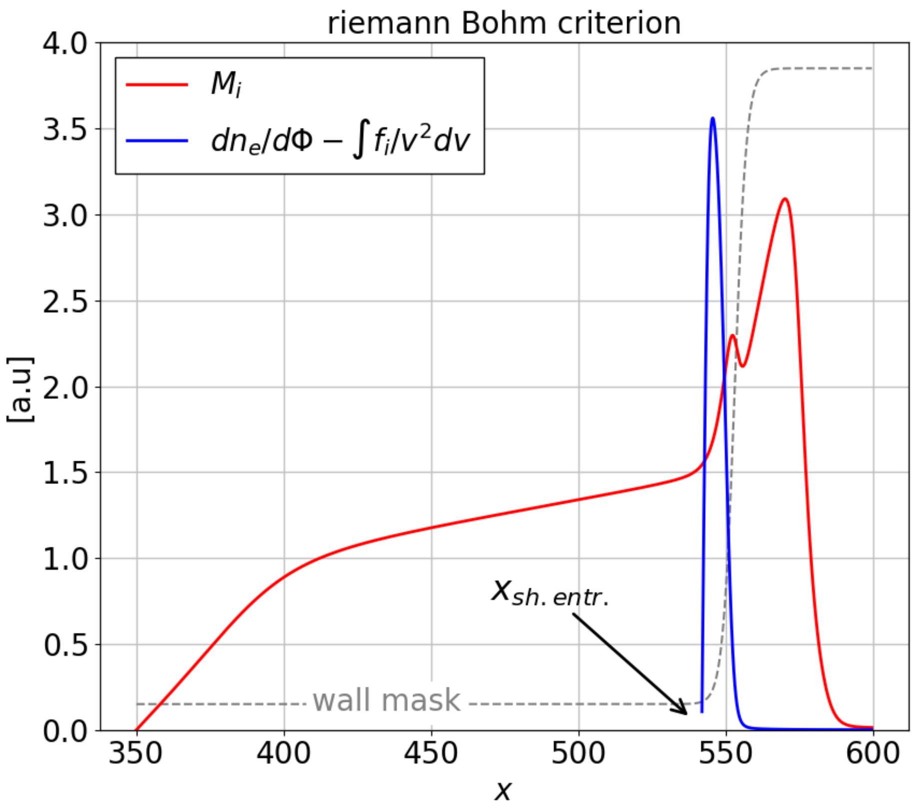
- Negatively charged region in the wall
- consequence of charge conservation
- No charge source, pos. charged region
- somewhere => negatively charged region
- somewhere else

- Wall coordinate defined at the point where charge density becomes negative
- Sheath entrance: riemann criterion cf. later

Fluid properties: fluid velocity

$$M = u_i / c_s$$

$$c_s(x,t) = \sqrt{\frac{T_e(x,t) + T_i(x,t)}{m_i}}$$



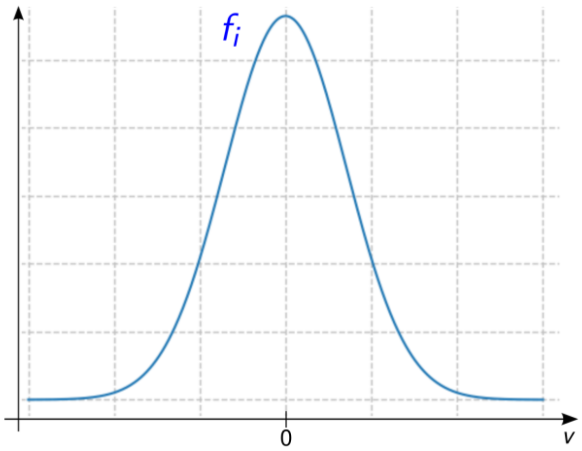
- Mach number increasing towards the wall
- >1 well before the positively charged region

Kinetic physics: riemann criterion

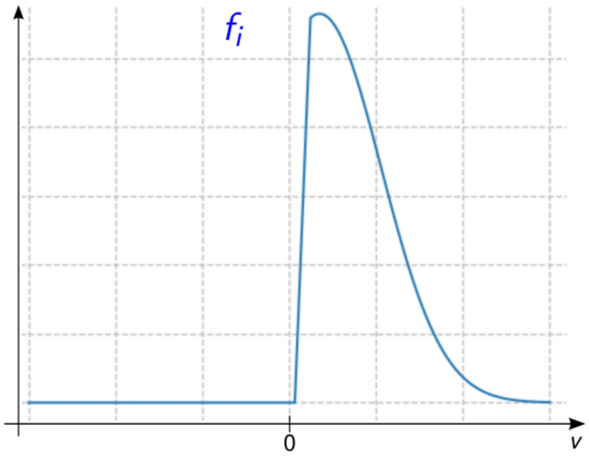
- M > 1 well before the sheath: need for a kinetic pendant of Bohm criterion
- Riemann criterion at sheath entrance

$$\int_0^{+\infty} \frac{1}{v^2} f_0(v)\, dv \leq \left. \frac{dn_e}{d\phi} \right|_{\phi=0}$$

Meaning : no slow ions at sheath entrance



Not ok



ok

Standard results recovered & kinetic properties found

- **Riemann criterion** (kinetic analogue of Bohm) defines **sheath entrance**

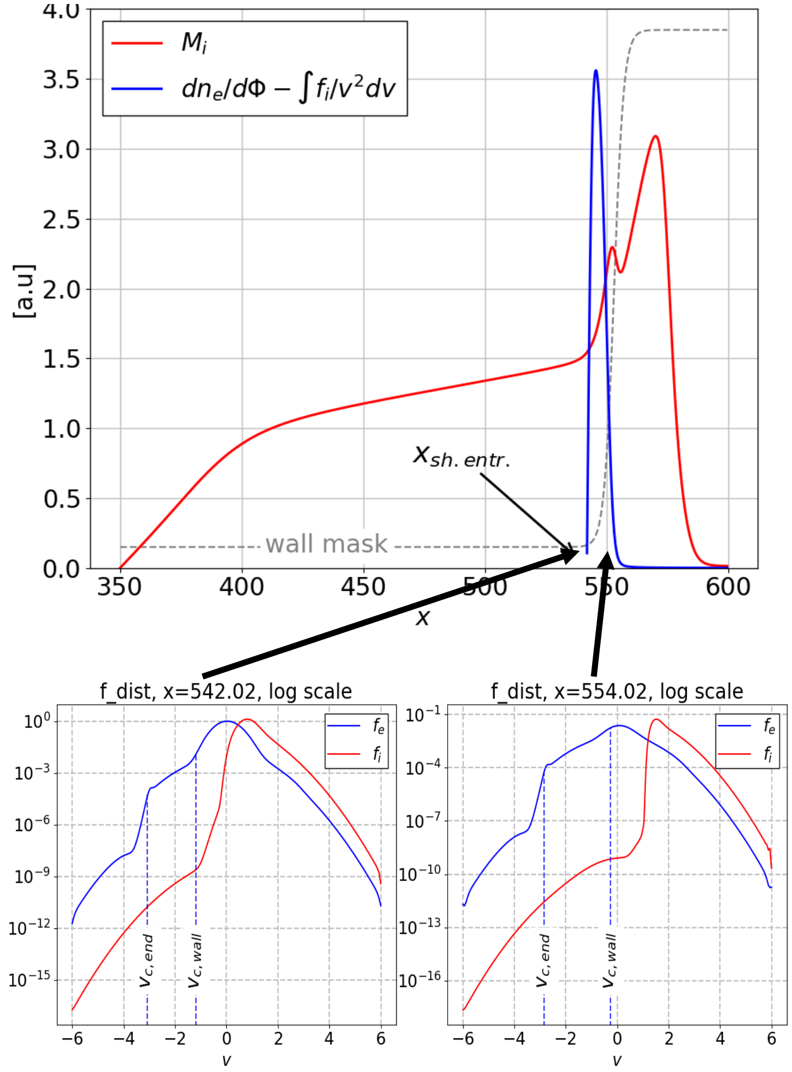
$$\int_0^{+\infty} \frac{1}{v^2} f_0(v)\, dv \leq \left.\frac{dn_e}{d\phi}\right|_{\phi=0}$$

[Riemann (1991)]

- Ion distribution function depleted in half the velocity domain in the sheath

- Collisions ensure convergence / positivity

[Munschy et al., in preparation (2022)]



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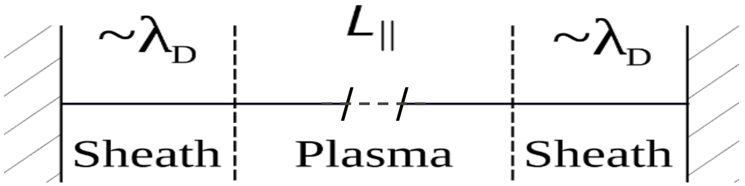


Ion distribution function depleted in half the velocity domain in the sheath

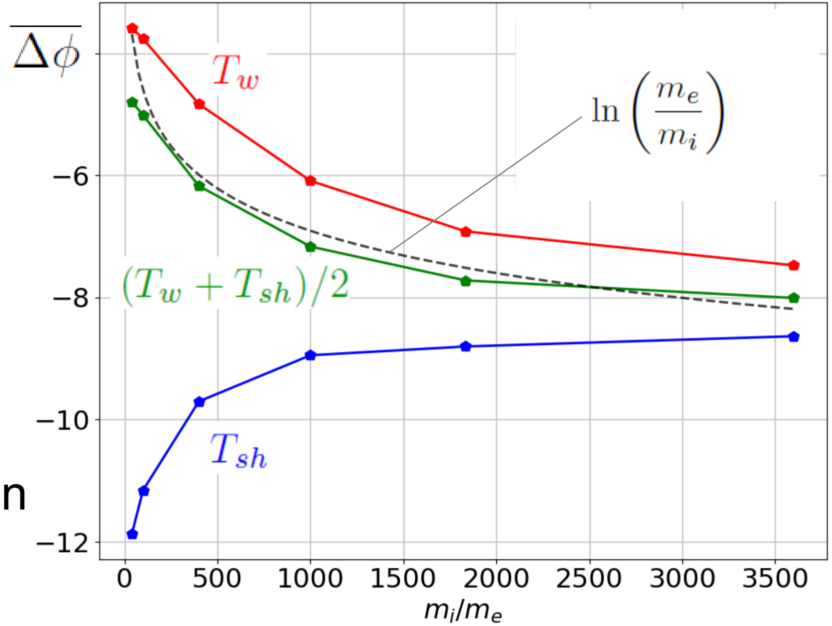
Collisions ensure convergence / positivity

Ongoing scans: m_i/m_e, $L_{//}/\lambda_D$, T_e, …

Getting ready for GYSELA implementation

Focus on sheath potential drop

[Munschy et al., in preparation (2022)]

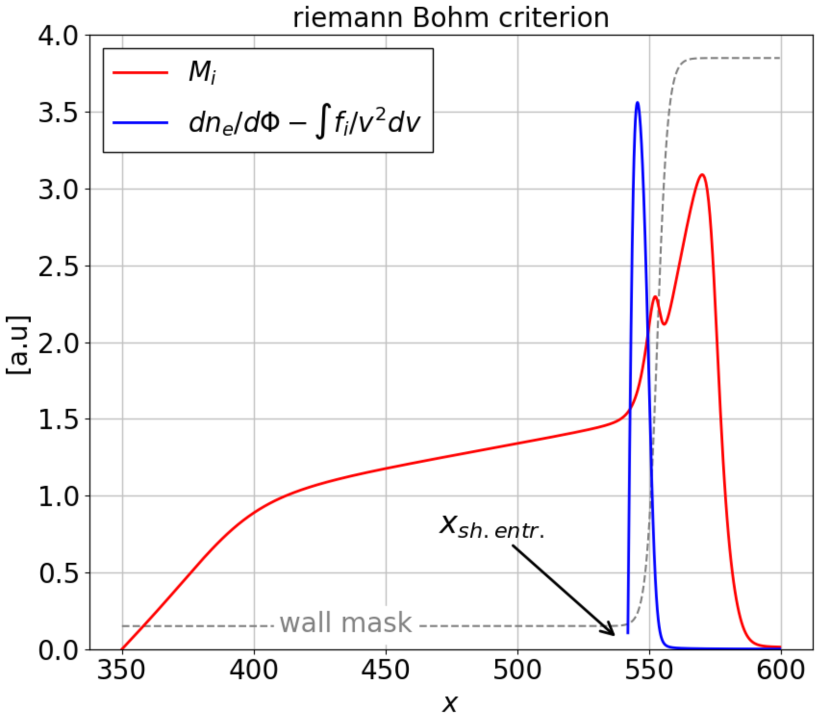


$$\overline{\Delta\phi_{th}} = \frac{2}{T_e}\Delta\phi_{th} - \ln\left\{2\pi\left(1+\frac{T_i}{T_e}\right)\right\} = \ln\left(\frac{m_e}{m_i}\right)$$

Kinetic physics: sheath entrance

$$\int_0^{+\infty} \frac{1}{v^2} f_0(v)\, dv \leq \left.\frac{dn_e}{d\phi}\right|_{\phi=0}$$

rhs-lhs as a function of space



Becomes positive at the plasma-wall Transition → sheath entrance definition

Independent of velocity discretization

Code verification : fluid conservations

Conservation of density, momentum and pressure

Moments of Boltzmann equation

$$\partial_t n_a + \sqrt{A_a}\partial_x(n_a u_a) = S_{na}$$

$$n_a(\partial_t u_a + \sqrt{A_a} u_a \partial_x u_a + \sqrt{A_a}\partial_x(n_a T_a)) = n_a Z E + S_{ua}$$

$$\partial_t(n_a T_a) + \sqrt{A_a} u_a \partial_x(n_a T_a) + \sqrt{A_a}(\partial_x Q_a + 3 n_a T_a \partial_x u_a) = S_{ha}$$

- We compute the fluid quantities from the code output (distribution functions)
- lhs – rhs for each fluid equation gives an idea of the error

Kinetic physics: mass ratio scan

Theoretical prediction

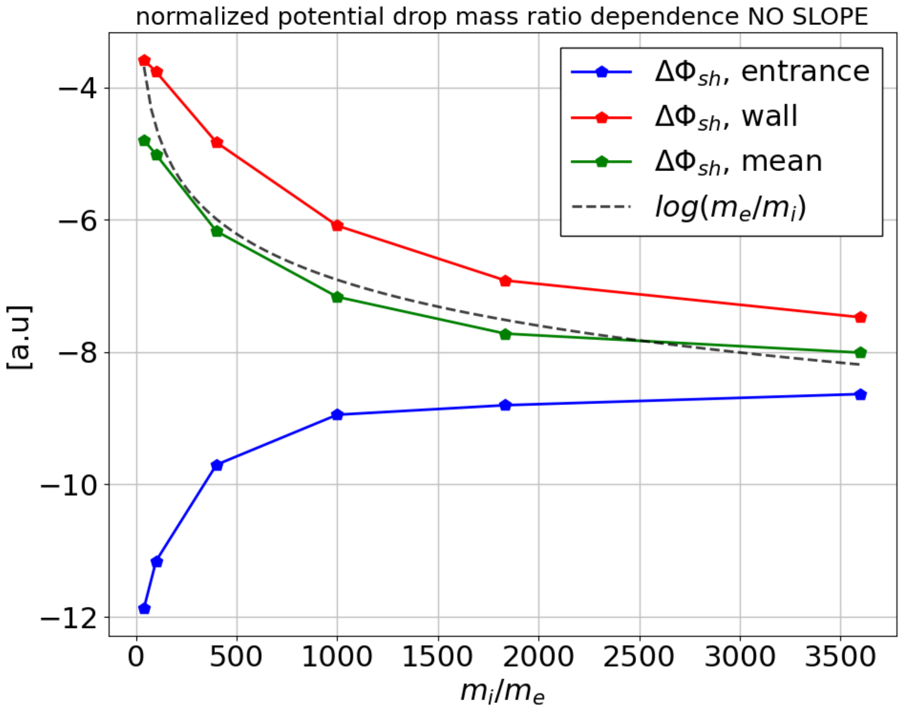
$$\Delta\phi_{sh.th.} = \frac{T_e}{2}\log\left\{2\pi A_i\left(1+\frac{T_i}{T_e}\right)\right\}$$

Normalized

$$\Delta\phi^{\text{eff.}}_{sh.} = \frac{2}{T_e}\Delta\phi_{sh.} - \log\left\{2\pi\left(1+\frac{T_i}{T_e}\right)\right\}$$

Normalized theoretical value

$$\Delta\phi^{\text{eff.}}_{sh.th.} = \log(A_i)$$

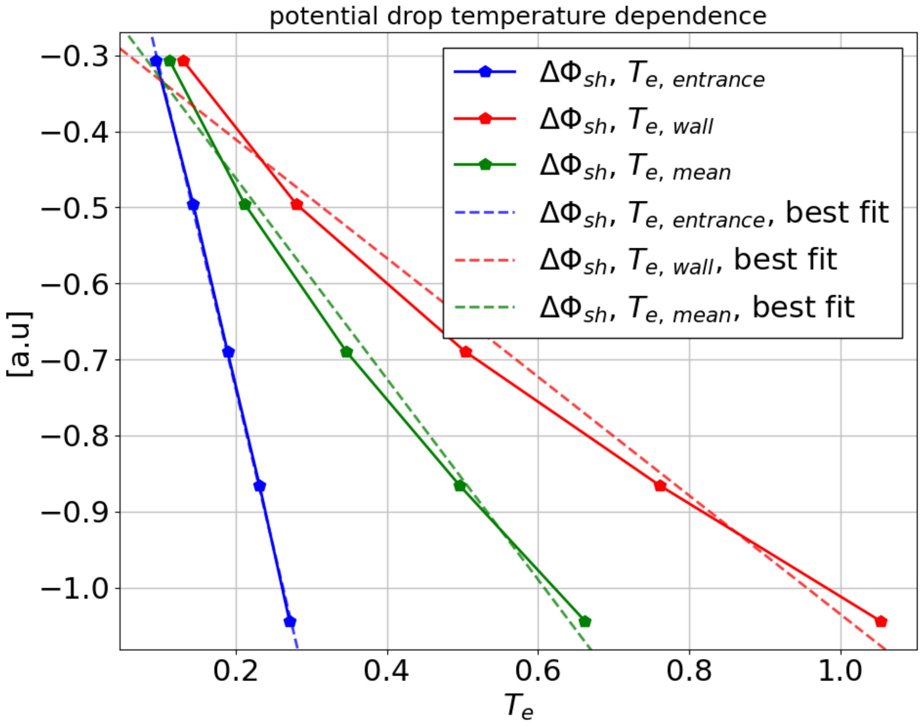


- Huge impact of spatial location of
- temperature :
- Sheath entrance, wall, mean
- Mean seem to fit perfectly with theoretical
- prediction (coincidence?)
- At high m_i/m_e less impact of the temper
- ature location choice

Kinetic physics: electronic temperature scan

Theoretical prediction

$$\Delta\phi_{sh.th.} = \frac{T_e}{2}\log\left\{2\pi A_i\left(1+\frac{T_i}{T_e}\right)\right\}$$

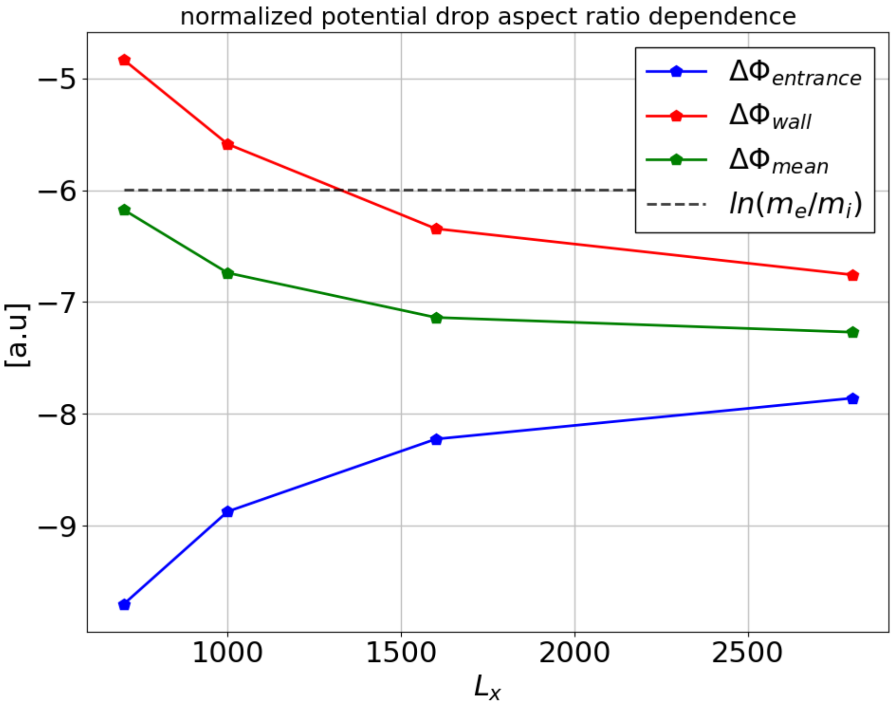


- Linear dependence of potential drop
- with electronic temperature
- approximately ok

- Very ok for sh. entrance temperature

Kinetic physics: aspect ratio scan

- Normalized potential drop against simulation box length (aspect ratio scan)



Again less impact of temperature
Location choice when going towards
Large boxes

commentaire