Fictitious domain methods for CFD in the context of nuclear power plants

DE LA RECHERCHE À L'INDUSTRIE

Mâtinée Maia, Institut de Mathématiques de Toulouse

June 30, 2022

M. BELLIARD and co-workers

IRESNE/DER/SESI/LEMS

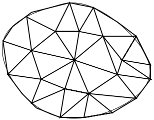
Institut de recherche sur les systèmes nucléaires pour la production d'énergie bas carbone

Summary

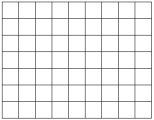
Introduction

Fictitious domain approach

- **Motivations** Computational domain $\leftrightarrow$ Mesh??

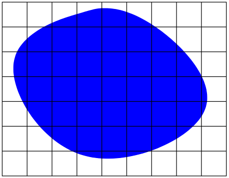


$\leftrightarrow$



Body-fitted mesh Cartesian mesh

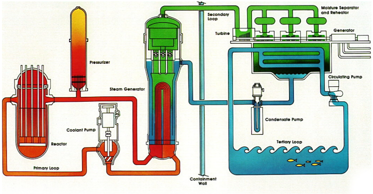
- **Pioneer's work** [Saul'ev, 1963]
  - original domain embedded in a geometrically bigger and simply-shaped domain : fictitious domain
  - computat. domain = fictitious domain
  - Cartesian mesh $\Rightarrow$ fast linear solvers, good numerical properties, easy multiresolution techniques

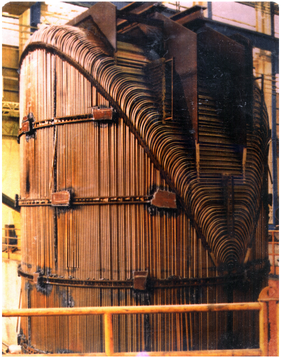


Which FD method? (R&D activity at CEA from 2003 -> today)

- Method with a good accuracy/cost ratio :
  - unique formalism for any immersed B.C. (Dirichlet, Robin, Neumann) or mixed B.C. types
  - moving free surfaces
  - a unique Cartesian mesh, no extra unknown, no numerical scheme modification $\Rightarrow$ first or second order method
  - easy AMR implementation
- Two kinds of immersed boundary approximation :
  - Spread interface
  - Thin interface
- Milestones :
  - Scalar convection-diffusion problem
  - Navier-Stokes problem
  - Fluid-structure problem

Fluid-Structure Interactions in nuclear components



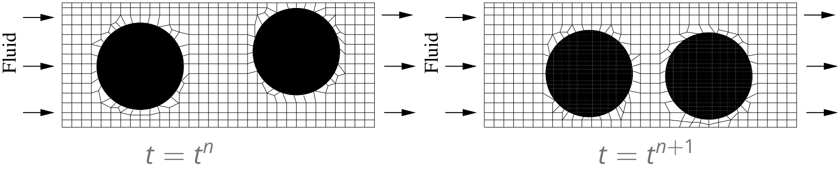


Flow-induced vibrations $\rightarrow$ possible fuel-rod or U-tube damages

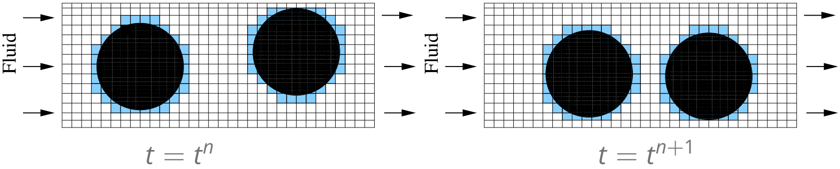
Fast FSI computations with large deformations $\rightarrow$ Immers. Bound. Methods

Fluid-Structure Interactions in nuclear components

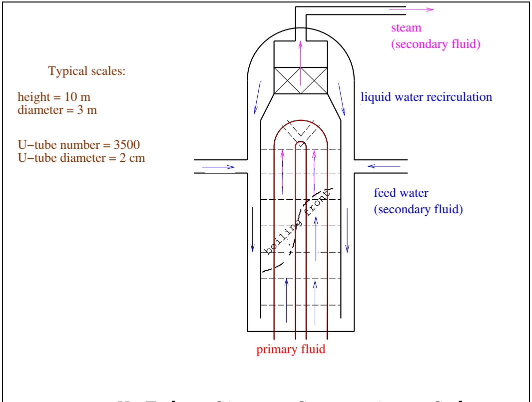
- With re-meshing....



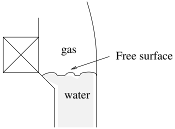
- Without re-meshing....

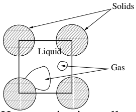


Steam Generator context

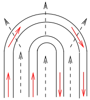


U−Tube Steam Generator Scheme

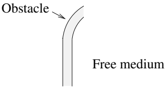




Homogenization cell

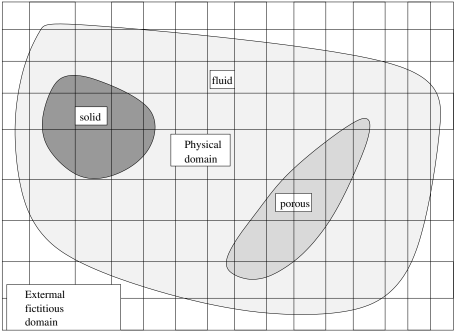


U−Tube bundle (porous)



Steam Generator context

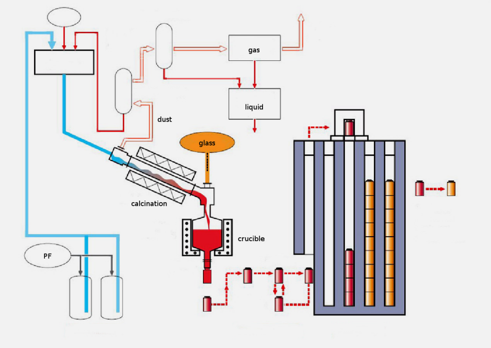
- An (abstract ?) SG model

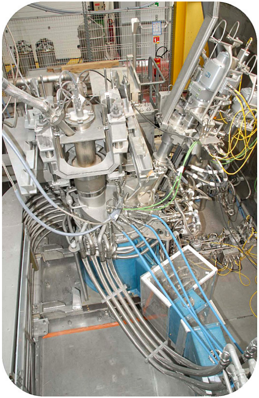


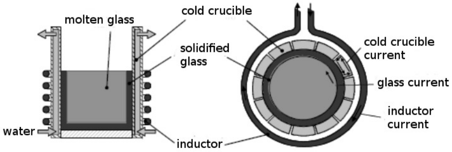
- $\partial_t(\rho \overrightarrow{\vartheta}) + \overrightarrow{\nabla}.(\rho \overrightarrow{\vartheta} \otimes \overrightarrow{\vartheta}) = \overrightarrow{f_v} - \overrightarrow{\nabla} P + \overrightarrow{\nabla}(\mu_T(\overrightarrow{\nabla} \overrightarrow{\vartheta} + \overrightarrow{\nabla}^t \overrightarrow{\vartheta})) - \frac{\mu}{K} \overrightarrow{\vartheta}$
  [Khadra et al., 2000]

  A unique formalism for free, porous or blind, fixed or moving regions

Vitrification process for the radioactive waste storage





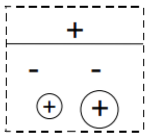


Vitrification process for the radioactive waste storage

- Problems involving complex flow regimes combined with time-varying geometries
  - Multiphase multicomponent flow at high temperature (gas bubbles, molten glass)
  - Static (vessel structure, measuring apparatus), moving (mechanical stirrer) bodies

- From the industrial point of view, the numerical constraints are
  - To develop robust, efficient and easily parallelizable numerical tools
    $\longrightarrow$ use non-boundary conforming methods (several proposed in literature)
  - To get accurate solutions at lower computational cost
    $\longrightarrow$ use high-order methods ($\sim$ second order in nuclear safety context)

A 1-fluid / Front Tracking Model

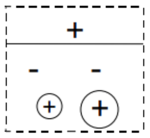


- Lagrangian meshes for the bubbles
- Eulerian phase indicator $\zeta \to \rho$ and μ

$$\rho = \rho_0 + \zeta(\rho_1 - \rho_0) \quad \mu = \mu_0 + \zeta(\mu_1 - \mu_0)$$

$$-\nabla(P + \rho g\mathbf{z}) + (\rho_1 - \rho_0)g\mathbf{z} \cdot \nabla\zeta$$

A 1-fluid / Front Tracking Model

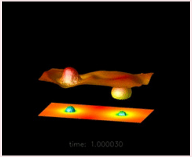


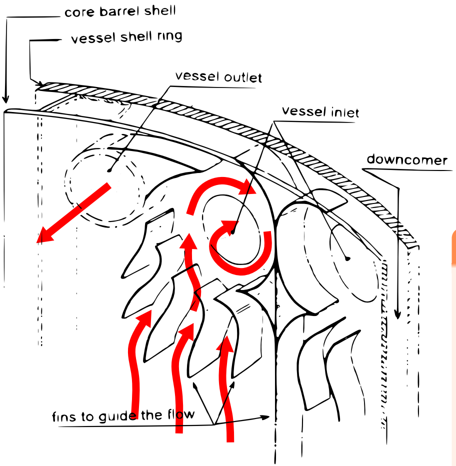
- Lagrangian meshes for the bubbles
- Eulerian phase indicator $\zeta \rightarrow \rho$ and μ

$$\rho = \rho_0 + \zeta(\rho_1 - \rho_0) \quad \mu = \mu_0 + \zeta(\mu_1 - \mu_0)$$

$$-\nabla(P + \rho g\mathbf{z}) + (\rho_1 - \rho_0)g\mathbf{z} \cdot \nabla\zeta$$

- Interfaces move with the continuous-phase velocities : $d_t\mathbf{X}_L = \mathcal{I}(\mathbf{u}, \mathbf{X}_L)$
- Lagrangian remeshing process and fragmentation/collision models





Patented Passive Safety System

- Located in a PWR core
- Composed of thin fins which :
  - Nominal flow not disturbed
  - Vortex in reverse flow
  - ⇒ *Postpone core dewatering*

Preliminary studies

- At a system scale
  - CATHARE
  - ⇒ *Operating limits*
- At a component scale
  - GENEPI2
  - ↪ *Upscaling*
  - ⇒ *Demonstrated interest*

In-vessel flow limiter [Gautier, 1988, Belliard, 2018]

Problem description

- Thermal-hydraulics
  - ↪ *Two-phase flow*
  - ↪ *Compressible*
  - ↪ *Low Mac Number*
- Passive Safety systems
  - ↪ *Design and shape optimization*
  - ⇒ *Large number of simulations*
  - ⇒ *Need of a fast tool*

⇓

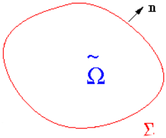
Modeling choices

- Homogeneous Equilibrium Model
- Projection scheme
- ~~“Body-fitted” approach~~
- “Immersed boundary” approach

Numerical methods

- Original problem $(\tilde{\mathcal{P}})$ in $\tilde{\Omega} \subset \mathbb{R}^d$:
  For $\widetilde{\mathbf{a}} \in (L^\infty(\tilde{\Omega}))^{d\times d}$, $\widetilde{\mathbf{v}} \in (L^\infty(\tilde{\Omega}))^{d}$, $\tilde{b} \in L^\infty(\tilde{\Omega})$ and $\tilde{f} \in L^2(\tilde{\Omega})$, find $\tilde{u} \in H^1(\tilde{\Omega})$ solution of :

$$(\tilde{\mathcal{P}}): \quad -div\,(\widetilde{\mathbf{a}}\,.\nabla\tilde{u}) + div\,(\widetilde{\mathbf{v}}\tilde{u}) + \tilde{b}\tilde{u} \;=\; \tilde{f} \quad \text{in } \tilde{\Omega}$$



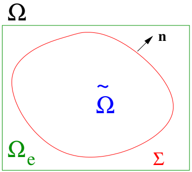
with the B.C. on $\partial\tilde{\Omega}$

$$\begin{cases} \bullet \text{ Dirichlet : } \tilde{u} = u_D,\ u_D \in H^{1/2}(\partial\tilde{\Omega}) \\ \bullet \text{ Robin (or Neumann) : } -(\widetilde{\mathbf{a}}\,.\nabla\tilde{u}).\mathbf{n} = \alpha_R\,(\tilde{u} - u_R) + g_R\,, \\ \qquad 0\leqslant\alpha_R \in L^\infty(\partial\tilde{\Omega}),\ \text{and } g_R \in L^2(\partial\tilde{\Omega}) \end{cases}$$

A scalar convection-diffusion model

- **Original problem** $(\tilde{\mathcal{P}})$ in $\tilde{\Omega} \subset \mathbb{R}^d$:
  For $\widetilde{\mathbf{a}} \in (L^\infty(\tilde{\Omega}))^{d\times d}$, $\widetilde{\mathbf{v}} \in (L^\infty(\tilde{\Omega}))^d$, $\tilde{b} \in L^\infty(\tilde{\Omega})$ and $\tilde{f} \in L^2(\tilde{\Omega})$, find $\tilde{u} \in H^1(\tilde{\Omega})$ solution of :

$$(\tilde{\mathcal{P}}) : \quad -div\,(\widetilde{\mathbf{a}}\,.\nabla\tilde{u}) + div\,(\widetilde{\mathbf{v}}\tilde{u}) + \tilde{b}\tilde{u} = \tilde{f} \quad \text{in } \tilde{\Omega}$$



with the B.C. on $\partial\tilde{\Omega}$

$$\begin{cases} \bullet \text{ Dirichlet : } \tilde{u} = u_D,\ u_D \in H^{1/2}(\partial\tilde{\Omega}) \\ \bullet \text{ Robin (or Neumann) : } -(\widetilde{\mathbf{a}}\,.\nabla\tilde{u}).\mathbf{n} = \alpha_R\,(\tilde{u} - u_R) + g_R\,, \\ \qquad 0 \leqslant \alpha_R \in L^\infty(\partial\tilde{\Omega}),\ \text{and } g_R \in L^2(\partial\tilde{\Omega}) \end{cases}$$

- **Fictitious Domain approach** : $\Omega = \tilde{\Omega} \cup \Sigma \cup \Omega_e$
  - Which fictitious problem $(\mathcal{P})$ solved in Ω?
  - Which B.C. on Σ, coefficients on the external domain Ω_e and B.C. on $\partial\Omega$?

The fictitious problem

- Fictitious problem $(\mathcal{P})$ solved in Ω_h :
  Find $u_\eta^h \in H^1(\Omega_h)$ such that

$$(\mathcal{P}) \left\{ \begin{array}{ll} \mathrm{div}(-\mathbf{a}\nabla u_\eta^h + \mathbf{v} u_\eta^h) + b\, u_\eta^h = f & \text{in } \Omega_h \\ (\tilde{\mathcal{P}})\text{'s B.C.} & \text{on } \tilde{\Gamma}_h = \partial\Omega_h \cap \partial\tilde{\Omega}_h \\ \text{suitable B.C. for } u_\eta^h & \text{on } \Gamma_{e,h} = \partial\Omega_h \backslash \partial\tilde{\Omega}_h \end{array} \right.$$

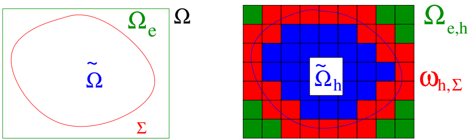
with $\mathbf{a} \in (L^\infty(\Omega_h))^{d\times d},\ \mathbf{v} \in (L^\infty(\Omega_h))^d,\ b \in L^\infty(\Omega_h)$ and $f \in L^2(\Omega_h)$, such that $\mathbf{a}|_{\tilde{\Omega}_h} = \widetilde{\mathbf{a}}|_{\tilde{\Omega}_h},\ \mathbf{v}|_{\tilde{\Omega}_h} = \widetilde{\mathbf{v}}|_{\tilde{\Omega}_h},\ b|_{\tilde{\Omega}_h} = \tilde{b}|_{\tilde{\Omega}_h},\ f|_{\tilde{\Omega}_h} = \tilde{f}|_{\tilde{\Omega}_h}$

$$\mathbf{a}|_{\Omega_{e,h}} = \mathbf{Id} \textit{ or } \ll \mathbf{Id},\ \mathbf{v}|_{\Omega_{e,h}} = \mathbf{0},\ b|_{\Omega_{e,h}} = 0,\ f|_{\Omega_{e,h}} = 0$$

- Robin B.C. : $-(\widetilde{\mathbf{a}}\,.\nabla\tilde{u}).\mathbf{n}|_\Sigma = \alpha_R\,(\tilde{u}|_\Sigma - u_R) + g_R$
- Dirichlet B.C. : $\tilde{u}|_\Sigma = u_D$ by L^2 penalization of Robin B.C. :

$$\alpha_R = \frac{1}{\eta},\ \alpha_R \underset{\eta\to 0}{\to} +\infty,\ u_R = u_D,\ g_R = 0$$

- Spread Approximation $\omega_{h,\Sigma}$ of Σ :



- Robin B.C. : $-(\widetilde{\mathbf{a}}\,.\nabla\tilde{u}).\mathbf{n}|_\Sigma = \alpha_R\,(\tilde{u}|_\Sigma - u_R) + g_R$

  Transmission problem between $\tilde{\Omega}$ and Ω_e :

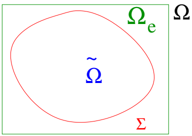
$$\psi = \mathbf{a}, \mathbf{v}, \ldots \quad \psi = \begin{cases} \tilde{\psi} & \text{in } \tilde{\Omega} \\ \psi_e & \text{in } \Omega_e \end{cases} \qquad [\![\psi]\!]_\Sigma = \psi^+ - \psi^-$$

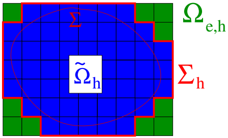
$$\boxed{-div\,(\mathbf{a}.\nabla u) + div(\mathbf{v}\,u) + u = f - [\![(\mathbf{a}.\nabla u).\mathbf{n}]\!]_\Sigma\,\delta_\Sigma + [\![(\mathbf{v}.\mathbf{n})]\!]_\Sigma\,u\,\delta_\Sigma}$$

$$-(\mathbf{a}.\nabla u).\mathbf{n}|_\Sigma^- = \alpha_R\,(u|_\Sigma - u_R) + g_R \quad \text{and} \quad -(\mathbf{a}.\nabla u).\mathbf{n}|_\Sigma^+ \simeq 0 \text{ with } \mathbf{a}|_{\Omega_e} = \eta \mathbf{Id}$$

  Ad-hoc coefficient ϵ_h to approximate δ_Σ on $\omega_{h,\Sigma}$

- Thin Approximation Σ_h of Σ :





- Model of flux and solution jumps [Angot, 2003]

$$\begin{cases} \underline{[\![(\mathbf{a}\nabla u_\eta^h).\mathbf{n}]\!]}_{\Sigma_h} = \alpha\, \overline{u_\eta^h}|_{\Sigma_h} - q & \text{on } \Sigma_h \\ \underline{(\mathbf{a}\nabla u_\eta^h).\mathbf{n}}|_{\Sigma_h} = \beta\, [\![u_\eta^h]\!]_{\Sigma_h} - g & \text{on } \Sigma_h \end{cases}$$

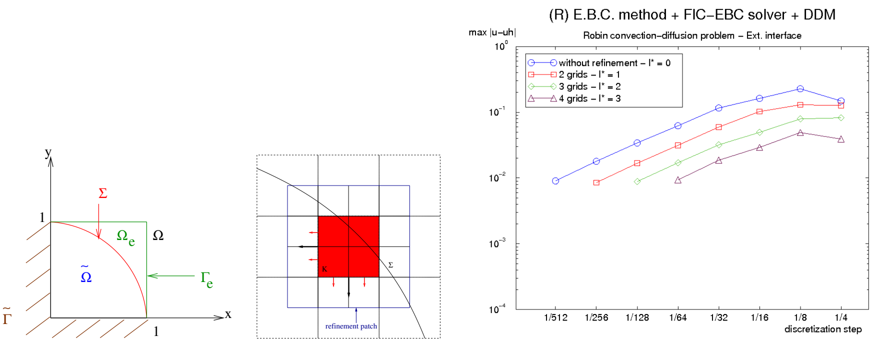
- Immersed transmission conditions
  No exterior control : $\beta = \frac{\alpha}{4}$

$$\begin{cases} \psi_\Sigma^- := -(\mathbf{a}\,.\nabla u)^-.\mathbf{n}|_{\Sigma_h} = \frac{\alpha}{2} u^-|_{\Sigma_h} + g - \frac{q}{2} \\ \psi_\Sigma^- = \alpha_R\, (u^-|_{\Sigma_h} - u_R) + g_R \end{cases}$$

- Ad-hoc coefficient ϵ_h to approximate $\int_\Sigma \psi_\Sigma^- dS$ by $\int_{\Sigma_h} \psi_{\Sigma_h}^- \frac{1}{\epsilon_h} dS$

Multigrid technique (FIC-EBC)

Convergence with the space step (h) - $\eta = 10^{-12}$



(a) Computational domain

(b) A patch of the FIC-EBC method

(c) Robin test case

$\Rightarrow$ Methods in $\mathcal{O}(h_f)$ with h_f : space step of the finest local refinement grid.

PDF method [Introïni et al., 2014, Billo et al., 2021]

Balance equations on an open set Ω (dilatable Navier-Stokes)

$$\begin{cases} \partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u} - \bar{\bar{\sigma}} + \bar{\bar{I}}p) = \rho\mathbf{g} & \text{on } \Omega \\ \nabla\cdot(\rho\mathbf{u}) = 0 & \text{on } \Omega \\ +\text{B.C.} & \text{on } \partial\Omega \\ +\text{I.C.} & \text{on } \Omega \end{cases}$$

- ρ, fluid density
- p, fluid pressure
- $\mathbf{g}$, gravity vector
- $\mathbf{u}$, fluid velocity
- $\bar{\bar{I}}$, identity tensor of $M_3(\mathbb{R})$
- $\bar{\bar{\sigma}}$, deviatoric stress tensor

Projection scheme to solve Navier-Stokes

$$\frac{\rho^n}{\delta t}(\mathbf{u}^* - \mathbf{u}^n) + \nabla \cdot \overline{\overline{\Sigma}}^{n,*} = \rho^n \mathbf{g}$$

$$\frac{\rho^n}{\delta t}\left(\mathbf{u}^{n+1} - \mathbf{u}^*\right) + \nabla \phi^{n+1} = \mathbf{0}$$

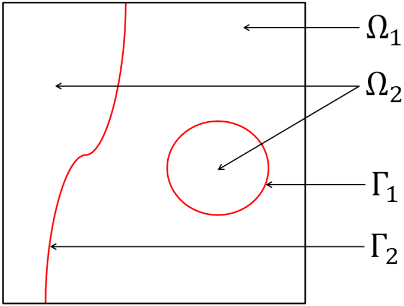
- x^i, approximation of x at time step number $i \in \mathbb{N}$
- δt, time step
- ρ^n, value of density computed a from tabulated equation of state
- $\phi^{n+1} = p^{n+1} - p^n$, pressure corrector
- $\mathbf{u}^*$, predicted velocity
- $\overline{\overline{\Sigma}}^{n,*} = \rho^n \mathbf{u}^n \otimes \mathbf{u}^* - \overline{\overline{\sigma}}^* + \overline{\overline{I}} p^n$

PDF method [Introïni et al., 2014, Billo et al., 2021]

Forcing term addition

$$\mathbf{f}^{n+1} = \frac{\chi}{\eta \delta t} \rho^n \left(\mathbf{u}_\Gamma - \mathbf{u}^{n+1} \right)$$

- $\eta \ll 1$, penalty parameter
- χ, characteristic function of Γ
- $\mathbf{u}_\Gamma$, imposed velocity on Γ



Adaption to projection scheme

$$\mathbf{f}_P^{n+1} := \frac{\chi}{\eta \delta t} \rho^n \left(\mathbf{u}_\Gamma - \mathbf{u}^* \right)$$

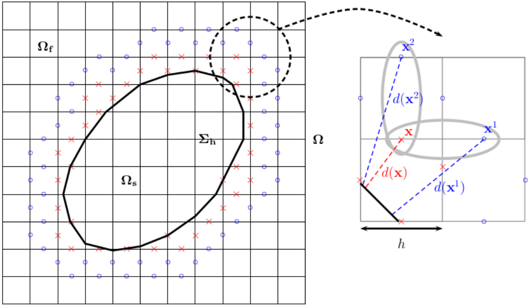
$$\mathbf{f}_C^{n+1} := \frac{\chi}{\eta \delta t} \rho^n \left(\mathbf{u}^* - \mathbf{u}^{n+1} \right)$$

$\Omega = \Omega_1 \cup \Omega_2$, “fluid” domain
$\Gamma = \Gamma_1 \cup \Gamma_2$, immersed boundary

⟹ A robust linear interpolation scheme (second-order)

$$\mathbf{u}_i(\mathbf{x}) := \mathbf{u}_s(\Pi_\Sigma(\mathbf{x})) + \frac{d(\mathbf{x})}{N}\sum_{p=1}^{N}\frac{\mathbf{u}(\mathbf{x}^p) - \mathbf{u}_s(\Pi_\Sigma(\mathbf{x}^p))}{d(\mathbf{x}^p)} + \mathcal{O}\left(h^2\right) \text{ for } \mathbf{x} \in \Omega \text{ s. t. } \chi_s(\mathbf{x}) > 0\,,\ d(\mathbf{x}) > 0$$

- Does not rely on a preferred direction as often done in literature
- Based on an averaged reconstruction of velocity gradients near Σ_h
  - ⇝ the local influence of fluid flow around **x** is fully taken into account



× forcing or penalized nodes

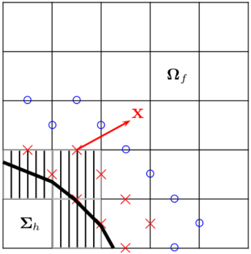
○ associated fluid nodes

- Based on an approximate projection operator (Π_Σ) onto Σ_h

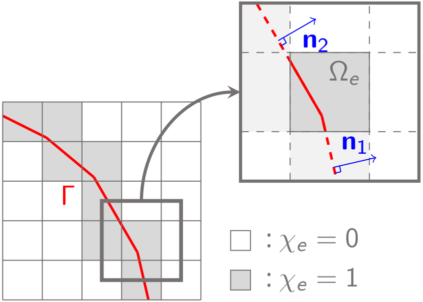
- Π_Σ is defined throughout an algorithm based on this local minimization problem

$$\text{Find } \mathbf{z}\,(=\Pi_\Sigma(\mathbf{x})) \in \mathcal{V}^{\Sigma_h} \text{ s. t. } J(\mathbf{z}) = \inf_{\mathbf{y}\in\mathcal{V}^{\Sigma_h}} J(\mathbf{y}) \quad \text{with } J(\mathbf{y}) = \|\mathbf{y}-\mathbf{x}\|^2$$

  - $\mathcal{V}^{\Sigma_h}$ is an immediate neighborhood of Σ_h
    - ⋆ Σ_h is partially (and locally) reconstructed by the Lagrangian facets contained in the neighboring cells and the "neighbors of neighbors" around **x**
  - $\mathcal{V}^{\Sigma_h}$ is defined by a set of plans passing through each Lagrangian facets of Σ_h
  - $\mathcal{V}^{\Sigma_h}$ tends to Σ_h when the space step h decreases



- Resolution of the minimization problem by means of an Uzawa algorithm



Discrete obstacle

- Characteristic χ_e by element
- Normal vector by element

Finite Element Formulation

- Pair of elements ($\mathbb{Q}_1$, $\mathbb{Q}_0$)
- ⇒ *Few degrees of freedom, fast*
- ⇒ *Instabilities soften by diffusion*
- Lumped mass matrix
- ⇒ *Enhance robustness*

Motivations

- Interpolation of $\mathbf{u}_\Gamma$
- ⇒ *Use of $\mathbb{Q}_1$ basis*
- House code of the CEA
- ⇒ *Facilitates developments*

In the context of the Finite Element

[Billo et al., 2021]

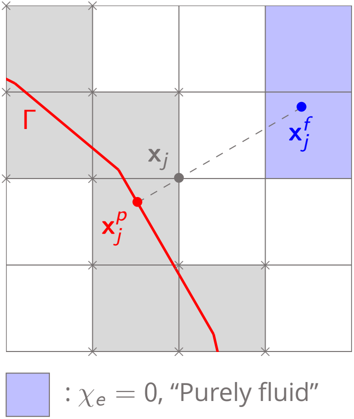
Interpolation of the imposed $\mathbf{u}_\Gamma$

- Projection $\mathbf{x}_j^p$ of a node $\mathbf{x}_j$ on Γ
- Point "purely fluid" $\mathbf{x}_j^f$

$$\mathbf{u}_\Gamma = \mathbf{u}_j^p + \frac{\mathbf{u}_j^f - \mathbf{u}_j^p}{\|\mathbf{x}_j^f - \mathbf{x}_j^p\|} \|\mathbf{x}_j - \mathbf{x}_j^p\|$$

"Purely fluid" element

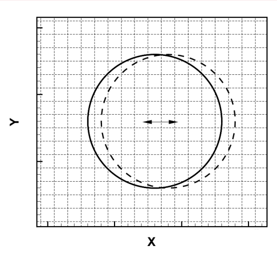
- No immersed Dirichlet node
  i.e. *No node influenced by* Γ



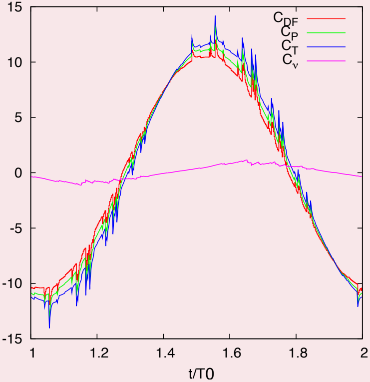
: $\chi_e = 0$, "Purely fluid"

Spurious Force Oscillations . . .

$$x_c(t) = x_c(0) + X_0(1 - cos(2\pi f_0 t))$$
$$y_c(t) = y_c(0)$$



- Drag coefficient $C_T = C_P + C_\nu$
- $C_{DF} = \dfrac{\iiint_{\Omega_S} \rho \underline{F}_{DF} \cdot \underline{e}_x d\Omega}{\underline{F}_{ref} \cdot \underline{e}_x}$



$\Delta x_0 = D/16$ and $\Delta t_0 = 0.002\ T_0$

Regularized Direct forcing [Belliard et al., 2016]

Regularization of $\underline{u}_i$

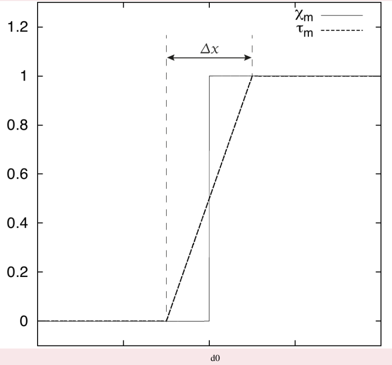
$$\begin{cases} \underline{u}_i & = \xi_m \underline{u}_m + (1 - \xi_m)\underline{u}^\star \\ \underline{F}_{RG} & = \frac{\underline{u}_i - \underline{u}^\star}{\Delta t} \end{cases}$$

$0 \leq \xi_m(\underline{x}) \leq 1$: regular function of $d(\underline{x})$

$$\underline{F}_{RG} = \xi_m \frac{\underline{u}_m - \underline{u}^\star}{\Delta t}$$

An example . . .

$$\xi_m(\underline{x}) = \frac{\Delta x + 2[d(\underline{x}) - d_0^m]}{2\Delta x} \equiv \tau_{m(\underline{x})}$$



Regularization of $\underline{u}_i$

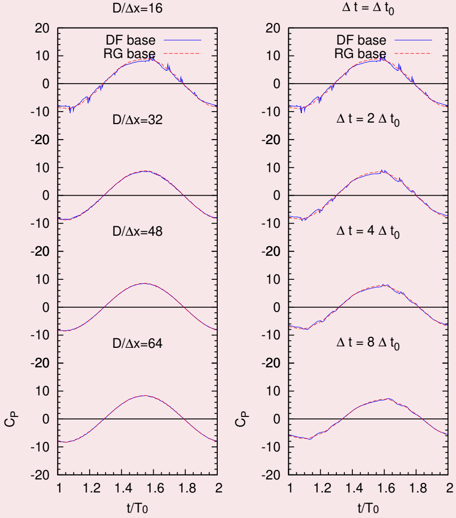
$$\begin{cases} \underline{u}_i & = \xi_m \underline{u}_m + (1-\xi_m)\underline{u}^\star \\ \underline{F}_{RG} & = \frac{\underline{u}_i - \underline{u}^\star}{\Delta t} \end{cases}$$

$0 \leq \xi_m(\underline{x}) \leq 1$: regular function of $d(\underline{x})$

$$\underline{F}_{RG} = \xi_m \frac{\underline{u}_m - \underline{u}^\star}{\Delta t}$$

An example . . .

$$\xi_m(\underline{x}) = \frac{\Delta x + 2[d(\underline{x}) - d_0^m]}{2\Delta x} \equiv \tau_{m(\underline{x})}$$



Validation cases

Thermal-hydraulic [Belliard and Introïni, 2015]

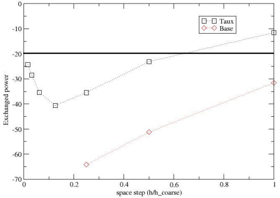
2D molten glass flow over a cold cylinder

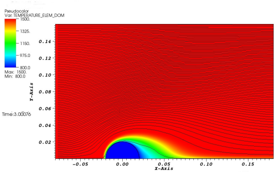
- $\mu(T) \in [1; 10^5]$, $\Lambda(T)$ and Boussinesq approx.
- Fluent benchmark simulation
  (boundary layer : AMR; 14,000 faces)
- Mesh convergence study (2,400 $\rightarrow$ 10 M cells)

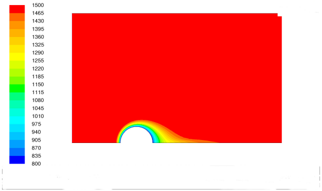
$$\begin{aligned}\partial_t T &= \nabla\left(\frac{\lambda(T)}{\rho_0 C_{P0}} \overrightarrow{\nabla} T\right) \\ &+ (1-\chi_s)\frac{Q}{\rho_0 C_{P0}} \\ &- (1-\chi_s)\overrightarrow{V} \cdot \overrightarrow{\nabla} T + \frac{\chi_s}{\eta}(T_i - T)\end{aligned}$$

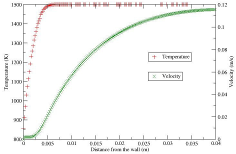
2D molten glass flow over a cold cylinder

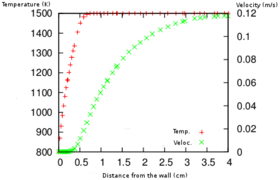
- $\mu(T) \in [1; 10^5]$, $\Lambda(T)$ and Boussinesq approx.
- Fluent benchmark simulation
  (boundary layer : AMR; 14,000 faces)
- Mesh convergence study (2,400 $\rightarrow$ 10 M cells)











Navier-Stokes validation cases [Billo et al., 2021]

Penalty parameter η convergence study

- Laminar 2D Poiseuille flow (channel aligned with the grid)

Mesh convergence studies

- Laminar 2D Poiseuille flow (channel tilted with respect to the grid)
- Taylor-Couette flow
- Laminar flow around a static circular cylinder
- Laminar flow around a rotating circular cylinder

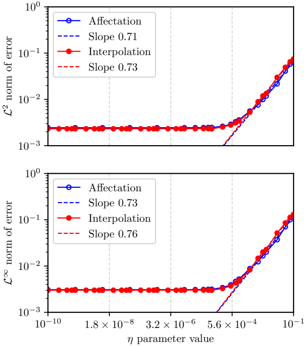
Global quantities studies

- Laminar flow around a static circular cylinder
- Laminar flow around a rotating circular cylinder
- Steady laminar flow past a NACA0012 airfoil

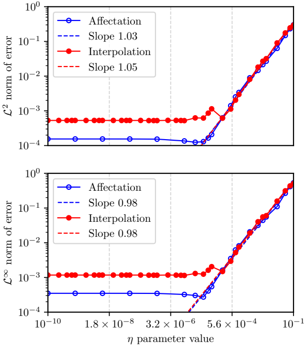
Example of results

Penalty parameter η convergence study (aligned Poiseuille)

Pressure



Velocity



↪ *Order of convergence*

- Consistent with the literature

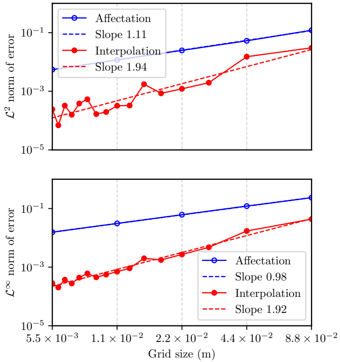
↪ *Linear interpolation*

- No degradation

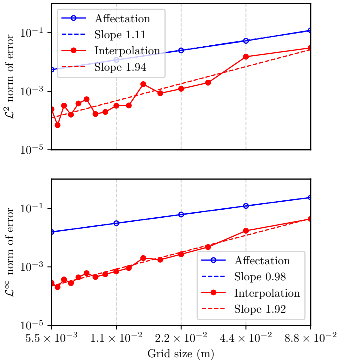
Example of results

Mesh convergence study (45° tilt Poiseuille)

Velocity along x axis



Velocity along y axis



↪ *Order of convergence*

- Consistent with literature

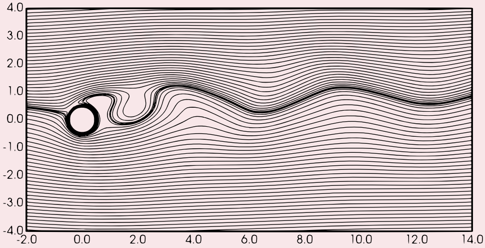
↪ *Linear interpolation*

- Enhances convergence

Example of results

Aerodynamic coefficients for the flow around a rotating circular cylinder (Re=100)

	$2r/h$ = 40	References in Literature
$\overline{C_d}$	1.102	[1.098-1.189]
C_d'	0.091	[0.099-1.120]
$\overline{C_l}$	2.562	[2.405-2.510]
C_l'	0.301	[0.360-0.443]
St	0.166	[0.165-0.173]



Industrial cases

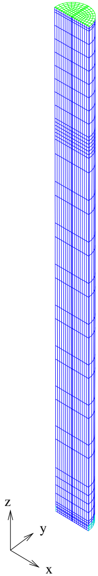
Steam generator thermal balance

[Belliard and Ramière, 2012]

- The energy balance equation

$$\begin{aligned}\beta\rho\partial_t H \; &+ \; \beta\mathbf{G}.\nabla H - \mathrm{div}(\beta\chi_T \nabla H) \\ &= \; \beta Q - \mathrm{div}(\beta x(H)\,(1 - x(H))\,\rho\mathcal{L}\mathbf{V}_R),\end{aligned}$$

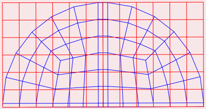
- $\mathbf{G} = \rho\mathbf{V}$, $x(H,P) = \dfrac{H - H_{ls}}{\mathcal{L}}$ static quality, $\mathbf{V}_R$ relative velocity (drift flux)
- $\chi_T = a|\mathbf{G}|L$: Schlichting's turbulent diffusion coefficient
- B.C.
  - $H_D = 119,3\ KJ.kg^{-1}$ (hot leg),$H_D = 118,5\ KJ.kg^{-1}$ (cold leg),
  - $\nabla H.\mathbf{n} = 0$ (wall and out-flow)
- Neptune/pyGene code (F.E. method)



Steam generator thermal balance

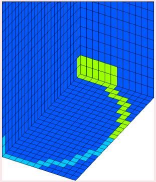
[Belliard and Ramière, 2012]

The ISI formulation



- $$\sum_e \rho_e \int_{\omega_e} \xi^i \beta \partial_t H + \sum_e \mathbf{G}_e . \int_{\omega_e} \xi^i \beta \nabla H$$
$$+ \sum_e \chi_{T_e} \int_{\omega_e} \beta \nabla H . \nabla \xi^i - \sum_e \chi_{T_e} \int_{\partial\omega_e \cap \partial\Omega_h} \xi^i \beta \nabla H . \mathbf{n} + \sum_e (\alpha_R)_e \sum_j H_j \int_{\omega_e} \xi^i \xi^j$$
$$= \sum_e Q_e \int_{\omega_e} \xi^i \beta - \sum_e \int_{\omega_e} \xi^i div(\beta x(1-x)\rho L \mathbf{V}_R) + \sum_e [(\alpha_R)_e (H_D)_e + \frac{(g_R)_e}{\epsilon_e}] \int_{\omega_e} \xi^i$$

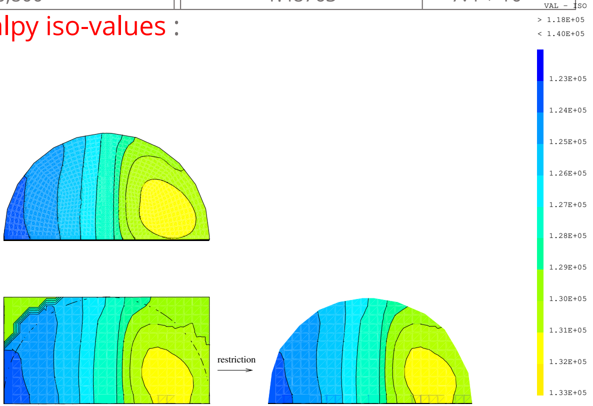
- Immersed B. C. :
  - In-flow, Dirichlet : L^2 penalization $\alpha_R = \frac{1}{\eta}$ $0 < \eta \ll 1, g_R = 0$
  - Wall, Neumann : no flux $\alpha_R = 0, g_R = 0$



- **Primary/secondary fluid exchanged power :**

Cells	Exchanged power (MW)	Relative error
143,360 (body-fitted)	1.48873	-
7,200	1.49054	$1.1 * 10^{-3}$
57,600	1.48851	$1.5 * 10^{-4}$
460,800	1.48763	$7.4 * 10^{-4}$

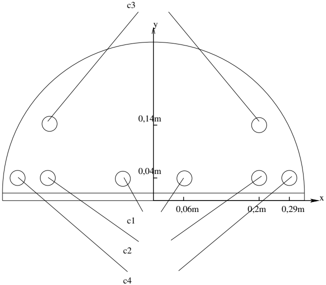
- **Specific enthalpy iso-values :**
  57,600 cells

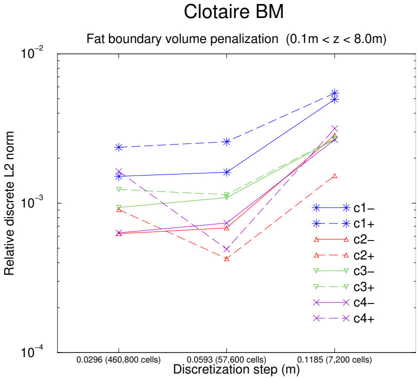


- **Primary/secondary fluid exchanged power :**

Cells	Exchanged power (MW)	Relative error
143,360 (body-fitted)	1.48873	-
7,200	1.49054	$1.1 * 10^{-3}$
57,600	1.48851	$1.5 * 10^{-4}$
460,800	1.48763	$7.4 * 10^{-4}$

- **Mesh convergence (L^2 norm) :**

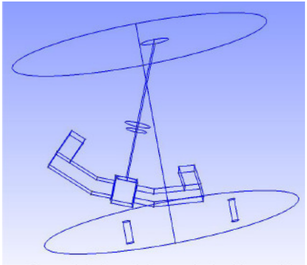


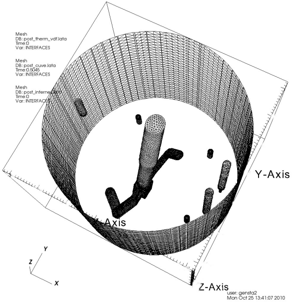


Nuclear waste vitrification process

[Belliard and Introïni, 2015]

- Hydraulic validations of flows induced by a stirrer
- Thermal-hydraulic validations of flows around a rod
- Cold crucible full-scale simulation involving bubbling

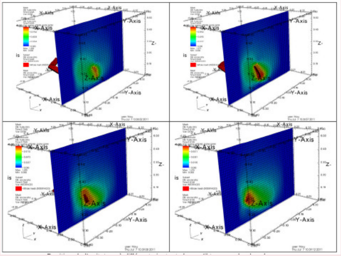


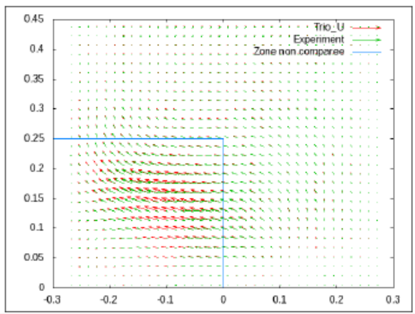


- IBC Lagrangian meshes from CAD
- Regular Eulerian meshes

Hydraulic validations

- Experimental device filled with viscous oil ($\mathrm{Re} \approx 100$) - PIV -
- Fluent body-fitted sliding-mesh simulations (1 M cells with AMR)
- Velocity-profile comparisons on a cut-plane and lines

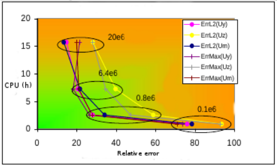


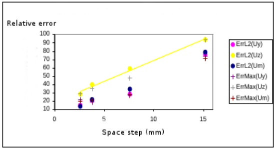


Nuclear waste vitrification process

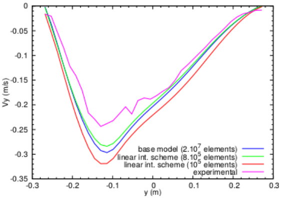
[Belliard and Introïni, 2015]

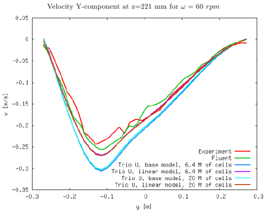
- Base model : 0.05 → 20 M cells on 2 to 400 CPUs : rate of convergence about 0.6

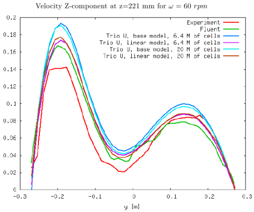




- Linear vs base model : cost ↘ for a given accuracy (1 M cells is quasi-optimal) : rate of convergence about 1.0





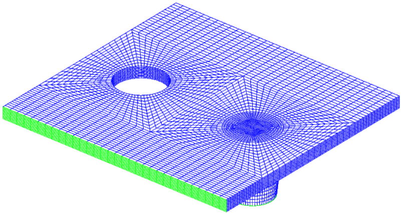


- Trio_U results compare well with experiment and Fluent ones

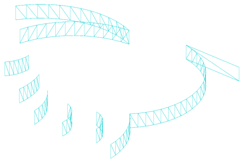
Description of the case [Belliard, 2018]

Flow in a hydraulic diode

- Completely 3D case
- Water (tabulated EOS)
- Pressure of 50 bar
- inlet flow of 4.680 $kg.s^{-1}$



+

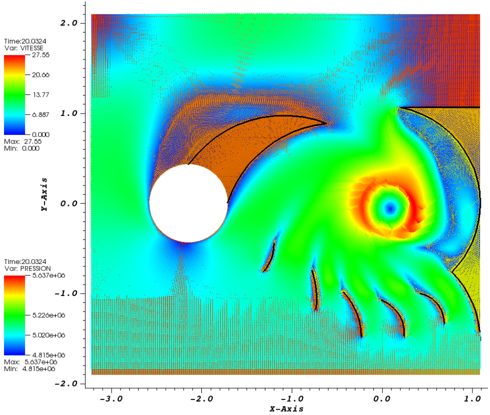


Studies

- Mesh convergence
- ↪ *Pressure drop coefficient*
- ↪ *Finest grid = reference*

Results [Billo et al., 2021, Billo et al., sub]

Example of obtained pressure and velocity maps



Vectors

Velocity in $m.s^{-1}$

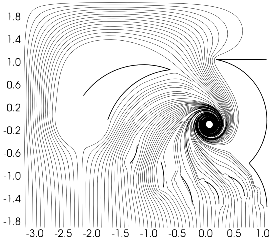
Colors

Pressure in bar

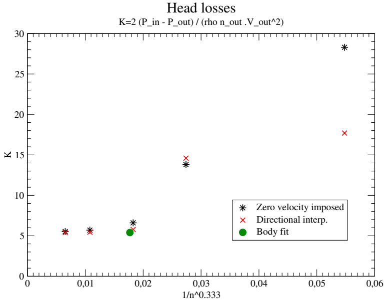
Analysis

- Expected behavior
- Consistent with [Bel18]

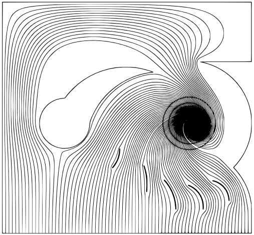
Results [Billo et al., 2021, Billo et al., sub]



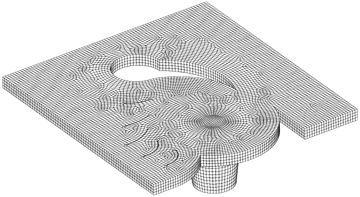
PDF



Head loss

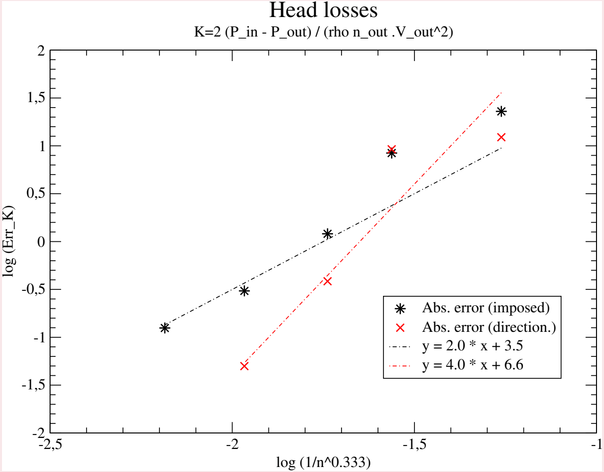


« Body-fitted »



« Body-fitted »mesh

Mesh convergence



⇝ Convergence enhanced by the interpolation of the imposed velocity

Conclusions and perspectives

Conclusions

- 20 years of R&D in fictitious domain methods for nuclear-industry applications
  - ISI, JEBC, DF and PDF methods for thermal-hydraulic solvers
  - FE, FD and FV discretizations
  - Various linear interpolations near the immersed boundary
  - Validation on academic test cases
- Industrial applications : IFS, waste vitrification process, hydraulic diodes

Conclusions and perspectives

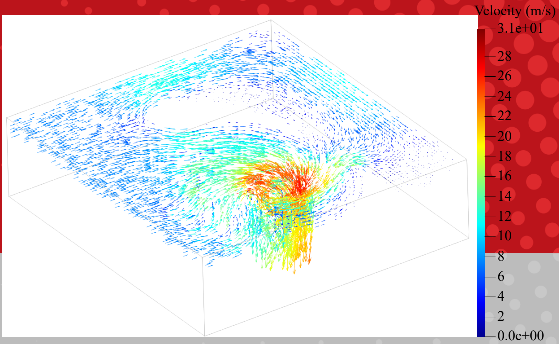
Conclusions

- 20 years of R&D in fictitious domain methods for nuclear-industry applications
  - ISI, JEBC, DF and PDF methods for thermal-hydraulic solvers
  - FE, FD and FV discretizations
  - Various linear interpolations near the immersed boundary
  - Validation on academic test cases
- Industrial applications : IFS, waste vitrification process, hydraulic diodes

Perspectives

- Immersed turbulent wall laws
- Shape optimization during computations
- AMR

Thank you for listening . . .



References

[Angot, 2003] Angot, P. (2003).
A model of fracture for elliptic problems with flux and solution jumps.
C.R. Acad. Sci. Paris, Ser I Math., 337 (6) 425-430.

[Belliard, 2018] Belliard, M. (2018).
Numerical Modeling of an in-vessel flow limiter using an Immersed Boundary Approach.
Nuclear Engineering and Design, 330 :437–449.

[Belliard et al., 2016] Belliard, M., Chandesris, M., Dumas, J., Gorsse, Y., Jamet, D., Josserand, C., and Mathieu, B. (2016).
An analysis and an affordable regularization technique for the spurious force oscillations in the context of direct-forcing immersed boundary methods.
Computers and Mathematics with Applications, 71 :1089–1113.

[Belliard and Introïni, 2015] Belliard, M. and Introïni, C. (2015).
Trio_u thermal-hydraulic simulations of flows induced by a stirrer and bubbling in a molten glass bath.
In *Joint EUROMECH/ERCOFTAC Colloquium 549, Immersed Boundary Methods : Current Status and Future Research Directions*, Leiden, The Netherlands, June 17-19.

[Belliard and Ramière, 2012] Belliard, M. and Ramière, I. (2012).
Fictitious domain methods for two-phase flow energy balance computations in nuclear components.
International Journal for Numerical Methods in Fluids, 68(8) :939–957.

[Billo et al., 2021] Billo, G., Belliard, M., and Sagaut, P. (2021).
A finite element penalized direct forcing immersed boundary method for infinitely thin obstacles in a dilatable flow.
Computers & Mathematics with Applications, 99 :292–304.

[Billo et al., sub] Billo, G., Belliard, M., and Sagaut, P. (sub.).
Comparison of several interpolation methods to reconstruct field data in the vicinity of a finite element immersed boundary.
submited to *Computers & Mathematics with Applications*.

[Gautier, 1988] Gautier, G. (1988).
Patent 88 12665.

References

[Introïni et al., 2014]   Introïni, C., Belliard, M., and Fournier, C. (2014).
A second order penalized direct forcing for hybrid Cartesian/immersed boundary flow simulations.
Computers & Fluids, 90 :21–41.

[Khadra et al., 2000]   Khadra, K., Angot, P., Parneix, S., and Caltagirone, J. (2000).
Fictitious Domain Approach for Numerical Modelling of Navier-Stokes equations.
International Journal for Numerical Methods in Fluids, 34(8) :651–684.

[Ramière et al., 2007a]   Ramière, I., Angot, P., and Belliard, M. (2007a).
A fictitious domain approach with spread interface for elliptic problems with general boundary conditions.
Computer methods in applied mechanics and engineering, 196 :766–781.

[Ramière et al., 2007b]   Ramière, I., Angot, P., and Belliard, M. (2007b).
A general fictitious domain method with immersed jumps and non-conforming structured mesh.
Journal of Computational Physics, 225(2) :1347–1387.

[Saul'ev, 1963]   Saul'ev, V. (1963).
On the solution of some boundary value problems on high performance computers by fictitious domain method.
Siberian Math. Journal, 4(4) :912–925.
(in Russian).