Tan Lei and Shishikura's example of obstructed polynomial mating without a levy cycle.

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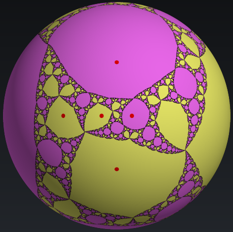
Origins



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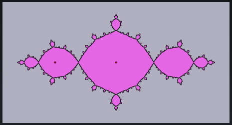

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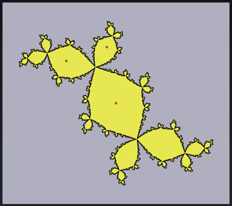

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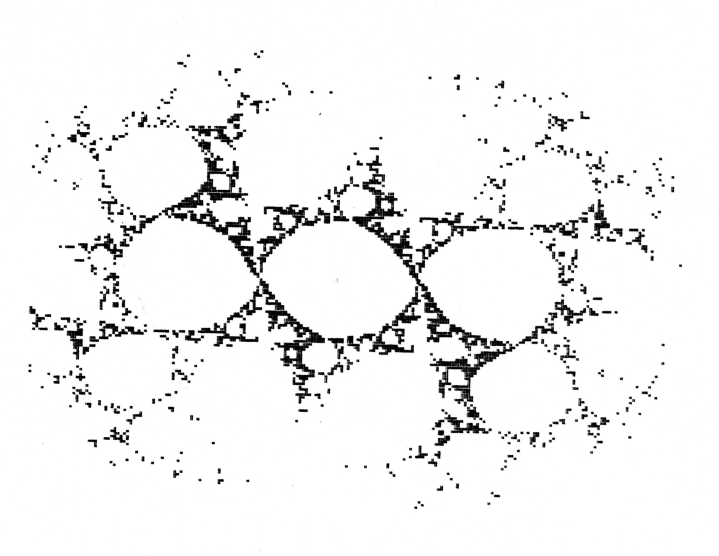
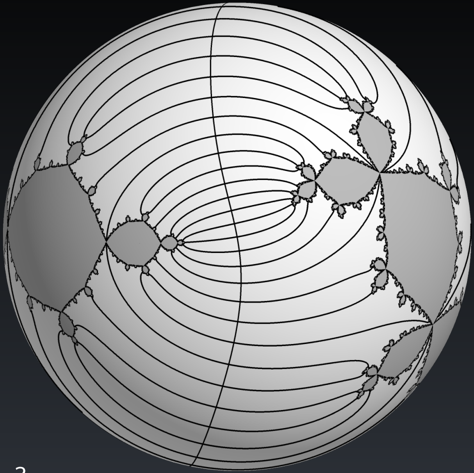


Topological (instant) mating

$K(P_1) \coprod K(P_2) / \sim$
with $\sim$: relation generated by identifying endpoints of external rays. A dynamics is well defined thereon.



When is the quotient a sphere?

When is the dynamics conjugated to a rational map?

Formal mating

Since PCF (post-critically finite) rational maps are characterized by Thurston's theorem, it is tempting to try and guess the Th-equivalence class of a potential mating of P_1 and P_2.

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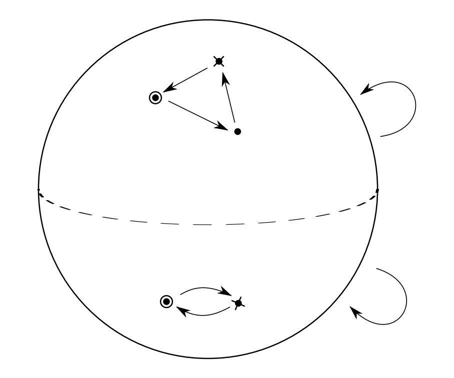

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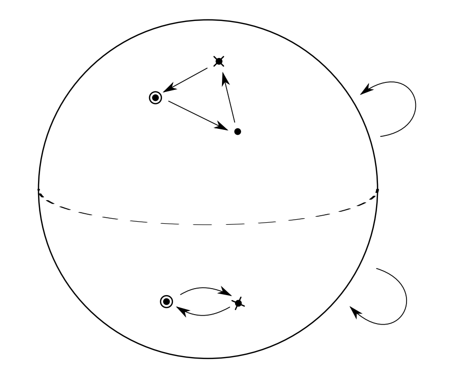
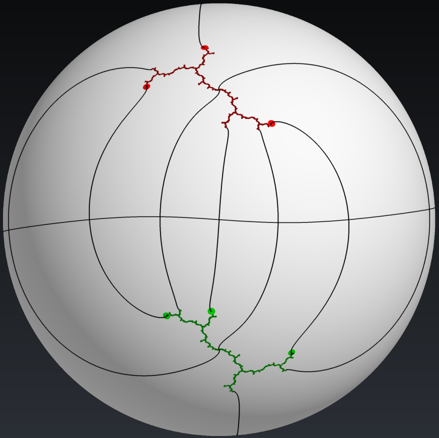


In good cases, it is unobstructed and Th-equivalent to a rational map and to the topological mating.

Degenerate (assisted) mating

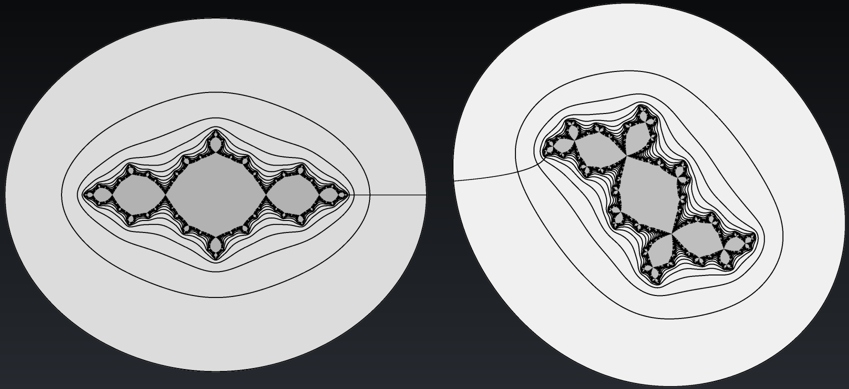
However sometimes the formal mating has a Th-obstruction yet the topological mating is conjugated to a rational map. Rees, Shishikura and Tan Lei have devised a way to detect this on the formal mating and to correct the latter by collapsing some post critical points together, yielding a new ramified cover that is unobstructed, and proved that it is Th-equivalent to a rational map conjugated to the topological mating.



Obstructed matings

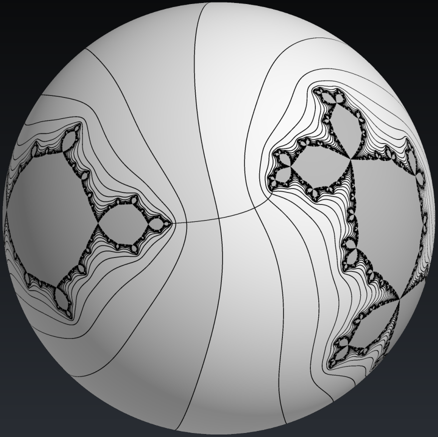
The last case is when the obstruction cannot be removed. Then, the topological mating cannot be equivalent to a rational map (even though the quotient still may be a sphere, or not).

Slow mating



Define a Riemann surface $\mathcal{S}_R$ by cutting & pasting along equipotential e^R, $R > 1$. Glue according to external angle.

Slow mating

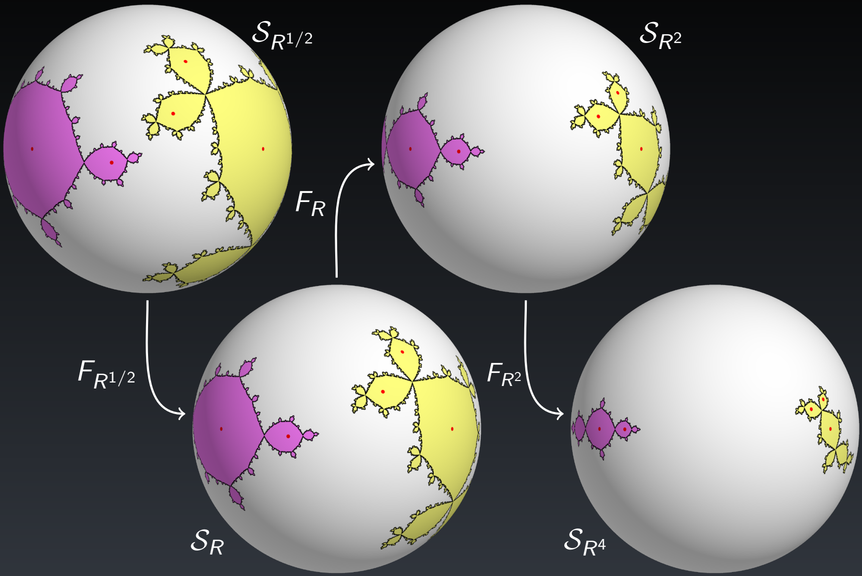


Uniformize to $\widehat{\mathbb{C}}$. Here: stereographic[y] projected to S^2.

There is a natural holomorphic map (rational of degree d after uniformization)

$$F_R : \mathcal{S}_R \to \mathcal{S}_{R^d}.$$

Slow mating



Slow mating

Question: Do the maps F_R converge as $R \longrightarrow 1$ to a rational map of the same degree?

It is then tempting to define the latter as a mating of P_1 and P_2.

Slow mating

In the PCF case, the post-critical set of P_1 and P_2 map to Riemann surfaces $\mathcal{S}_R$, so we get Riemann surfaces with marked points. The sequence of marked $\mathcal{S}_{R^{1/d^n}}$ for $n \in \mathbb{N}$ is an orbit under "Thurston's pull-back map associated to the formal mating".

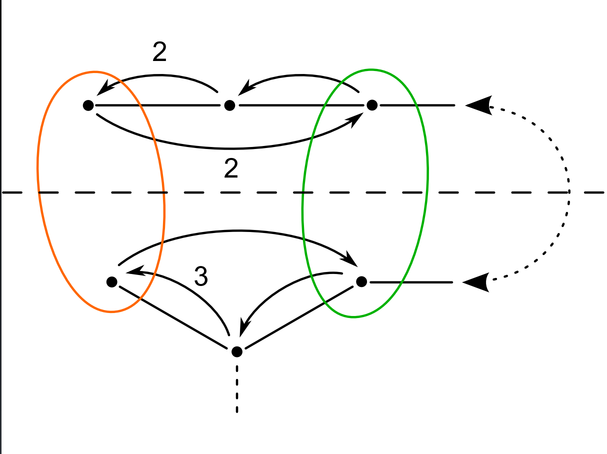
Comparison

$$\begin{tikzcd}
\text{Corrected Formal mating} \arrow[d, dash] & & \\
\text{Th-equiv class of the Formal mating} \arrow[d, dash] & \text{Topological mating} \arrow[d, dash] & \text{Slow mating} \arrow[d, dash] \\
\text{PCF polyn} \arrow[r] & J \text{ connected and locally connected} \arrow[r] & J \text{ connected}
\end{tikzcd}$$

The example

It is a mating of two PCF polynomials of degree 3 whose formal mating has a non removable Th-obstruction.

The example



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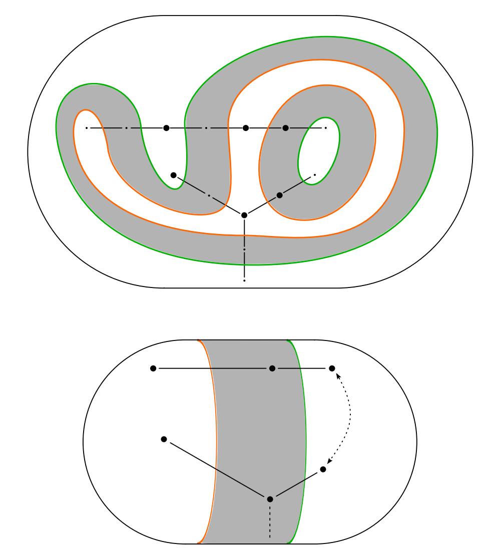


The example

Matrix of the multicurve {orange,green}:

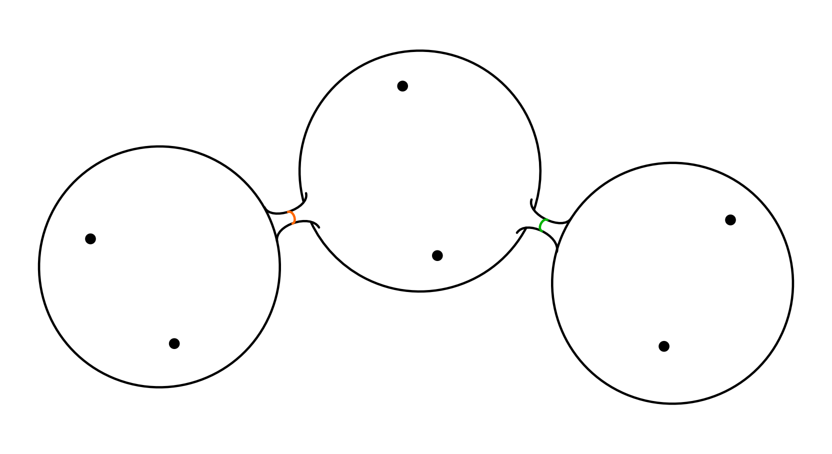
$$\begin{bmatrix} 1/2 & 1/2 \\ 1 & 0 \end{bmatrix}$$

Spectrum: $\{1, 1/2\}$.

The example

Remark: Shishikura and Tan Lei have proved that the ray equivalence relation is closed and that classes are trees with a bounded number of equator crossing: thus the topological mating gives a sphere. Aslo, the topol mating is Th-equivalent to the formal mating (and thus not to a rational map).

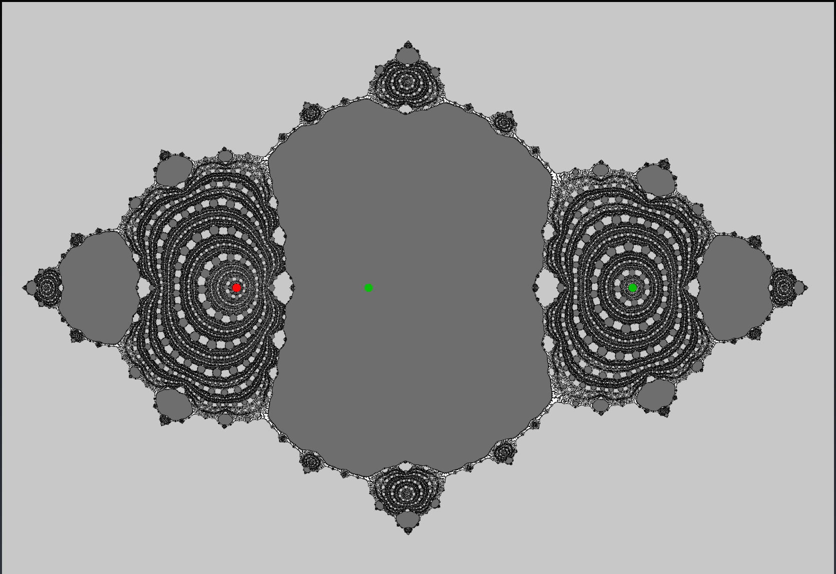
Pinching curves



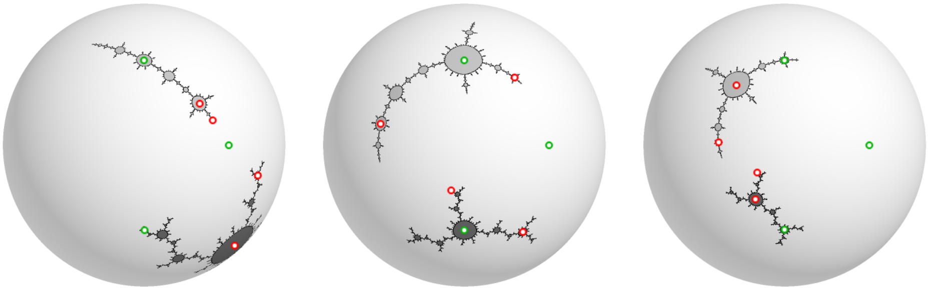
Showtime

Show movie.

Flat view



Three normalizations



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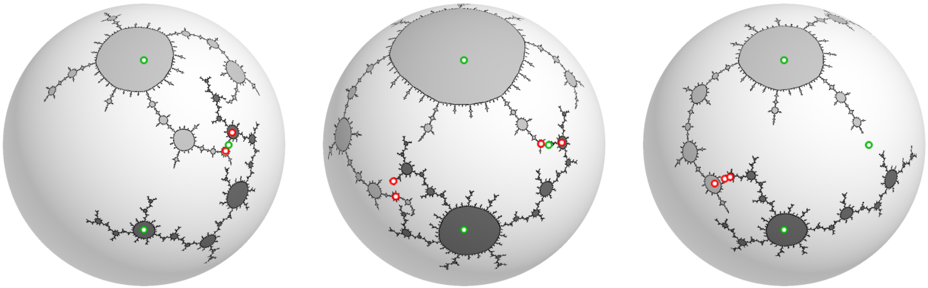

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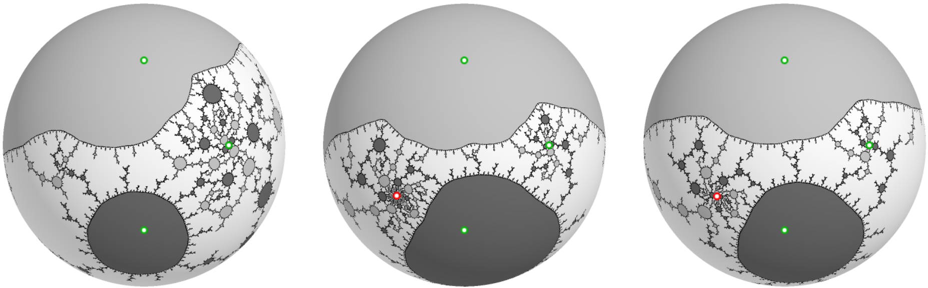

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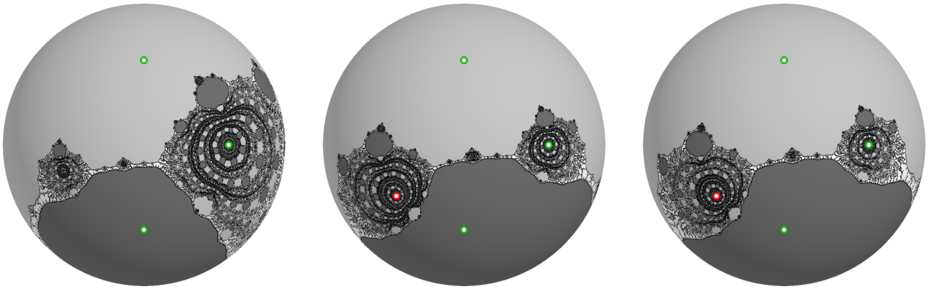
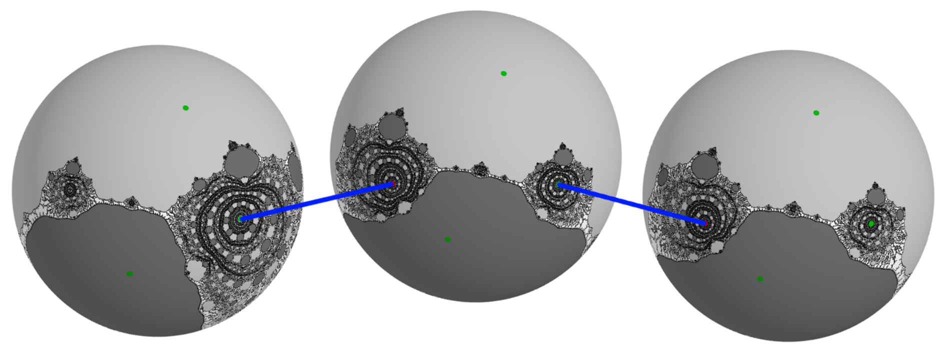


Interpretation: limit dynamical system.

There is a limit dynamical system on a tree of spheres: the tree of three spheres obtained when the canonical obstruction gets completely pinched.

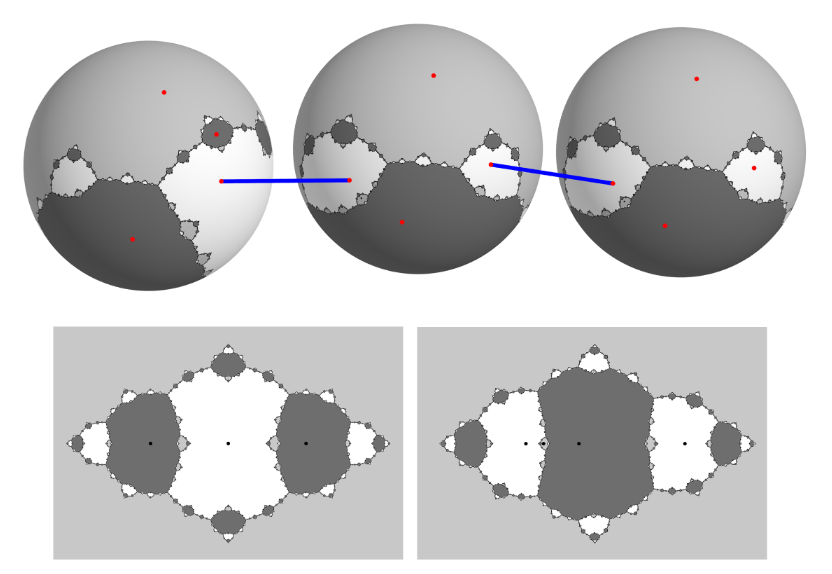
Interpretation: limit dynamical system.



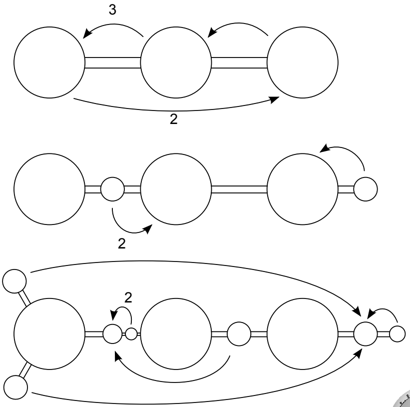
Interpretation: limit dynamical system.

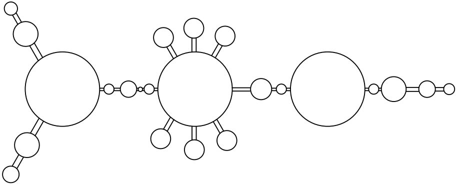
The third iterate of the limit maps each sphere to itself, by three semi-conjugated degree 6 rational maps.

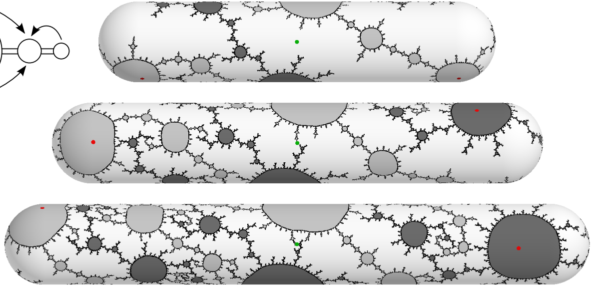
Interpretation: limit dynamical system.



Interpretation: tubes and mess.







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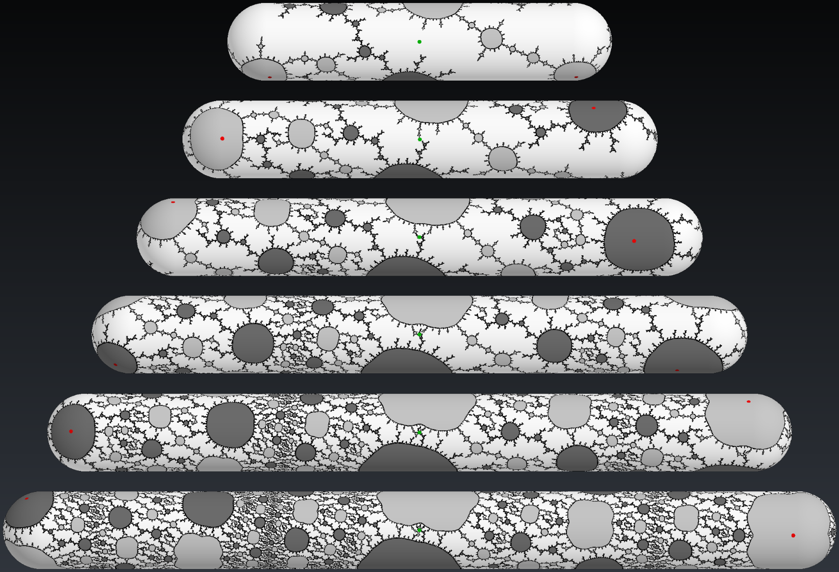

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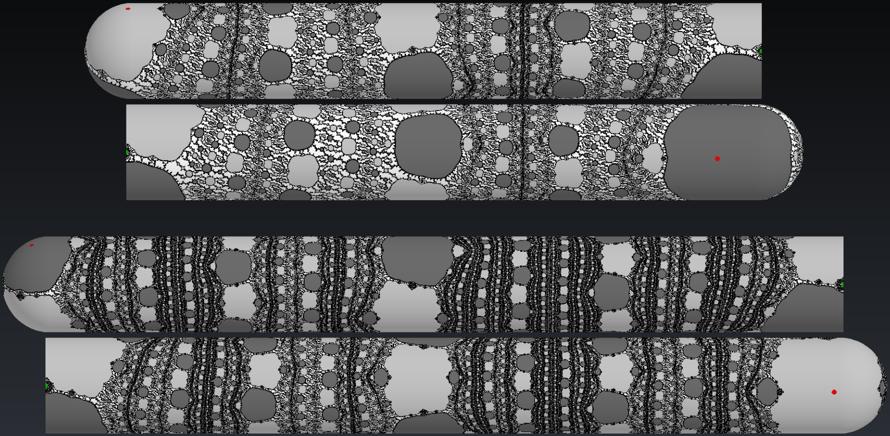

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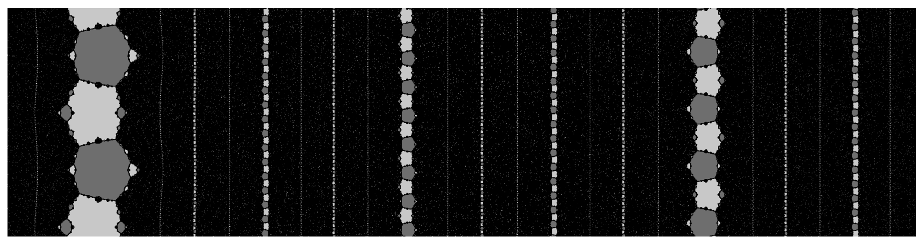

Interpretation: tubes and mess.