Workshop Index Theory

INSTITUT DE MATHEMATIQUES DE TOULOUSE

15-19 October 2018

PROGRAMME

	Monday	Tuesday	Wednesday	Thursday	Friday
9h30-10h30 room	**Mathai 1** Amphi Schwartz 1R3	**Bunke** MIP 1st floor 1R3	**Wang** MIP	**Scott** 106 Bat 1R1	**Yalkinoglu** MIP
10h30-11h00 room	Café A. Schwartz	Café MIP	Café MIP	Café 2nd floor 1R3	Café MIP
11h00-12h00 room	**Azzali** A. Schwartz	**Bunke** MIP	**Hochs** MIP	**Hilsum** 106 Bat 1R1	**Macdonald** MIP
12h00-14h00 Restaurant	Lunch Lesplanade	Lunch Lesplanade	Lunch Lesplanade	Lunch Lesplanade	Lunch Lesplanade
14h00-15h00 room	**Come** A. Schwartz	**Kottke** MIP		**Loizides** MIP	**Guerin** 207 Bat 1R2
15h10-16h10 room	**Yuncken** A. Schwartz	**Mathai 2** MIP		**Androulidakis** MIP	**Carrillo Rouse** 207 Bat 1R2
16h10-16h40 room	Café A. Schwartz	Café MIP		Café MIP	Café 207 Bat 1R2
16h40-17h40 room	**Mougel** A. Schwartz	**Vassout** MIP		**Baldaré** MIP	

Abstracts

1. Iakovos ANDROULIDAKIS (Athens)

   Title: **Riemannian metrics and Laplacians for smooth generalised distributions**

   Abstract: Smooth generalised distributions (not necessarily involutive) appear very often, for instance in Sub-Riemannian Geometry. A tool that might help understand such a distribution is the spectrum of an appropriate Laplacian. In this lecture we report on recent work with Y. Kordyukov, where we give a geometric construction of such a Laplacian. We show that every smooth generalised distribution admits an appropriate Riemannian metric and use it to define our Laplacian. Then we show that it is self-adjoint and hypo-elliptic.

2. Sara AZZALI (Potsdam)

   Title: **Discrete group actions and a weak form of the Baum–Connes conjecture**

   Abstract: We investigate a weak form of the Baum–Connes conjecture, which we formulate in terms of KK-theory with real coefficients. Given a discrete group Γ, we construct a distinguished idempotent $[\tau]$ of the commutative ring $KK^{\Gamma}_{\mathbb{R}}(\mathbb{C},\mathbb{C})$ which is canonically associated to Γ via its standard trace.

   By localizing at $[\tau]$, a natural Baum-Connes type morphism μ_τ is defined between the "tau-parts" of the usual left and right hand sides. We show that the τ form of the Baum-Connes conjecture is weaker than the classical one, but still implies the strong Novikov conjecture. Joint work with Paolo Antonini and Georges Skandalis.

3. Alexander BALDARÉ (Montpellier)

   Title : **Index theory for families of G-transversally elliptic operators**

   Abstract : In this talk, I will introduce the index class of a family of G-transversally elliptic operators. Following Atiyah, I will explain how to reduce the computation of the index map to the case of a trivial fibration $B \times V \to B$, where V is a torus representation. Using the Chern character in bivariant local cyclic homology, I will define the Chern character of the index class as a G-invariant distribution with values in the de Rham cohomology of the base. I will end with the Berline-Vergne formula for families of G-transversally elliptic operators.

4. Ulrich BUNKE (Regensburg) (2 hours talk)

   Title: **Coarse index of Dirac operators and Coarse homotopy theory**

   Abstract: The goal of the talks is to interpret constructions from index theory through coarse homotopy theory. To this end I will introduce the category of bornological coarse spaces and the notion of an equivariant coarse homology theory. The main example for this talk is the equivariant coarse K-homology. I will explain how the coarse index of a Dirac operator can be viewed as a class in this coarse homology theory. I will discuss invariance under suspension and locality as basic properties of the coarse index whose proofs use analysis. I will then support the idea that many index theoretic constructions and theorems can be interpreted in terms of constructions in coarse homotopy theory. In particular, I will consider the symbol class, the ordinary index, or the Piazza-Schick rho invariant from this point of view.

5. Paulo CARRILLO ROUSE (Toulouse)

   Title: **Conormal homology and Topological obstructions for Fredholm boundary conditions on manifolds with corners**

   Abstract : If you are still motivated by the end of the week I will tell you about my joint work with Jean-Marie Lescure (Clermont) and Mario Velasquez (Bogota). Sometime ago with Jean-Marie Lescure we gave some geometric obstructions for manifolds with corners of codimension 2, 3 and products of these to have the Stable Fredholm perturbation property (i.e. every b-elliptic operator can be perturbed up to stable homotopy to become Fredholm), these obstructions were given already in terms of a very simple homology complex (already used by Bunke in his related work for Dirac operators) depending on faces of the manifold with corners. In this talk I will talk about how to extend our previous results to general codimensions and how to compute (topologically) the obstructions in some cases.

6. Rémi COME (Metz-Paris)

   Title: **The Fredholm property for groupoids is local**

   Abstract : In many cases, the study of linear partial differential equations on a singular manifold can be related to that of a Lie groupoid whose action generates the (pseudo)differential operators of interest. Obtaining Fredholm conditions for these operators leads to the definition of *Fredholm groupoids* as recently introduced by Carvalho, Nistor and Qiao. We will explain this notion and show that Fredholm groupoids can be characterized locally, i.e. by looking at their reductions to open subsets. Several concrete examples will be provided to show that the local structure of such groupoids is usually very simple to understand.

7. Samuel Guerin (Lyon)

   Title: **Karoubi-Wood exact sequence for bivariant K theory and $\mathbb{Z}/2\mathbb{Z}$ index pairing**

   Abstract: The study of the index theory of real pseudo-differential elliptic operators led Atiyah to define real K theory KO. The bivariant non-commutative version of real K theory theory was given by Kasparov in 1981. We show how the real bivariant K theory can be related to the usual complex bivariant theory. More specifically we exhibit a long exact sequence, first described by Atiyah. We use this exact sequence to give formulas for the real index pairing between a real K theory class in $KO_i(A)$ and a real K-homology class in $KO^{i+2}(A)$. This takes value in $KO_2(\mathbb{R}) \simeq \mathbb{Z}/2\mathbb{Z}$.

8. Michel HILSUM (Paris)

9. Peter HOCHS (Adelaide)

   Title: **Orbital integrals in index theory and K-theory**

   Abstract: An orbital integral of a function on a group G is its integral over a conjugacy class in G. If such an orbital integral defines a continuous functional on a convolution algebra A(G) of functions on G, then it is a trace on that algebra. If the conjugacy class consists of just the identity element, this is the classical von Neumann trace. In general, such a trace induces a map on the K-theory of A(G) with values in the complex numbers. If A(G) is dense in the reduced or full group C*-algebra of G and closed under holomorphic functional calculus, then this gives a map on the K-theory of that group C*-algebra. It has turned out in recent years that such maps are useful tools for studying elements of these K-theory groups. This is true in particular for K-theoretic indices of G-equivariant elliptic operators. Index formulas for the numbers obtained in this way have turned out to have implications to representation theory and geometry. In this talk, I will discuss this development, including joint work with Hang Wang.

10. Chris KOTTKE (Florida)

    Title: **A Callias-type index theorem with degenerate potentials**

    Abstract: The classical Callias index theorem (due to Callias on R^n and generalized by others to complete Riemannian manifolds) gives a formula for the index of an operator of the form $D + \Phi$, where D is a self-adjoint Dirac operator and Φ a skew-adjoint endomorphism which is nondegenerate outside a compact set. The deformation theory of non-abelian magnetic monopoles in 3 dimensions gives rise to such an operator, except that Φ has nontrivial kernel at infinity. I will describe a generalization of the Callias index theorem in suitable geometries to this degenerate setting, in which the index is computed with respect to certain weighted Sobolev spaces and wall crossing phenomena arise with respect to the chosen weights. As an application, we compute the virtual dimensions of the magnetic monopole moduli spaces over an asymptotically conic 3-manifold.

11. Yiannis LOIZIDES (Penn State)

    Title: **Witten deformation for Hamiltonian loop group spaces**

    Abstract: I will describe an approach to the quantization problem for Hamiltonian loop group spaces, how to do 'Witten deformation' in this context, and the relation with the quantization-commutes-with-reduction theorem. This is joint work with Yanli Song and Eckhard Meinrenken.

12. Lachlan MACDONALD (Montpellier)

    Title: **The Godbillon-Vey invariant in equivariant KK-theory**

Abstract: The Godbillon-Vey invariant is a de Rham cohomology class associated to any transversely orientable foliated manifold, which can be explicitly constructed at the level of differential forms. In 1983 Connes showed that the Godbillon-Vey invariant of a codimension 1 foliation could be realised as a cyclic cocycle on the convolution algebra associated to the foliation. The cyclic cohomology point of view has since been shown to be closely linked to Hopf symmetry. In this talk I will realise Connes' cocycle as the Chern character of a semifinite spectral triple built using groupoid equivariant KK-theory, and show how the construction generalises to foliations of arbitrary codimension.

13. Jeremy MOUGEL (Metz)

    Title: **C*-algebra, exhaustive families and the essential spectrum of the N-body problems**

    Abstract: I will begin by reviewing a general method to determine the essential spectrum of Schrodinger-type operators. The method is based first on the fact that an operator is Fredholm if, and only if, it is inversible modulo the compacts (Atkinson's theorem). This reduces the study of quotient of a crossed-product C*-algebra by the compact operators. To study the invertibility in these quotients, one uses, following Georgescu, Mantoiu, and others, a determination of the primitive spectrum of a crossed-product C*-algebra or equivalently the quasi-orbits associed of the crossed-product. I will give an example of how this method works using a natural C*-algebra associated to the N-body problem. This is a joint work with Victor Nistor and Nicolas Prudhon.

14. Simon SCOTT (London)

    Title: **Vertical genera and fibred bordism homology**

    Abstract: A vertical genus is a ring homomorphism from the bordism ring of fibrations over a closed manifold X to the cohomology ring of X. It turns out that such objects admit a stable homotopy characterisation along the lines of the classical case (i.e. X=point). In this talk we explain these constructions and some applications to families of Dirac operators.

15. Mathai VARGHESE (Adelaide) (talk 1)

    Title: **Pseudodifferential Algebra Bundles and Index Theory**

    Abstract: I will discuss the classification of algebra bundles over a smooth base manifold, with typical fibre the algebra of classical pseudodifferential operators acting on smooth sections of a vector bundle over a compact manifold and of integral order, using Fourier Integral Operators.

    If time permits, I will also discuss the Index Theorem in this context.

    This is joint work with Richard Melrose.

16. Mathai VARGHESE (Adelaide) (talk 2)

    Title: **The magnetic spectral gap-labelling conjecture and recent progress**

    Abstract: Given a constant magnetic field on Euclidean space $\mathbb{R}^p$ determined by a skew-symmetric $(p \times p)$ matrix Θ, and a $\mathbb{Z}^p$-invariant probability measure μ on the disorder set Σ which is by hypothesis a Cantor set, where the action is assumed to be minimal, the corresponding Integrated Density of States of any self-adjoint operator affiliated to the twisted crossed product algebra $C(\Sigma) \rtimes_\sigma \mathbb{Z}^p$, where σ is the multiplier on $\mathbb{Z}^p$ associated to Θ, takes on values on spectral gaps in the magnetic gap-labelling group. The magnetic frequency group is defined as an explicit countable subgroup of $\mathbb{R}$ involving Pfaffians of Θ and its sub-matrices. We conjecture that the magnetic gap labelling group is a subgroup of the magnetic frequency group. We give evidence for the validity of our conjecture in 2D, 3D, the Jordan block diagonal case and the periodic case in all dimensions. Recently we also established the MGL conjecture for principal solenoidal tori in all dimensions. This is joint work with Moulay Benameur.

17. StÃ©phane VASSOUT (Paris)

    Title: Fourier integral operators on Lie groupoids

18. Hang WANG (Shangai)

    Title: **A fixed point theorem for proper actions by discrete groups**

    Abstract: I will talk about an extension of Connes-Moscovici's index formula in their proof of the Novikov conjecture for hyperbolic groups to the case of proper actions, by pairing the higher index

of an invariant elliptic operator with a cocycle coming from the cohomology of the centraliser of an element in the discrete group. A special case is a fixed point theorem which coming from two different pairings, one from convolution algebras of the group and one involves crossed product algebras associated to the proper action.

19. Bora YALKINOGLU (Strasbourg)

    Title: **Arithmetic solitons and prime numbers**

20. Robert YUNCKEN (Clermont-Ferrand)

    Title : **Twisted spectral triples and pseudodifferential calculus on quantum projective spaces**

    Abstract: This talk concerns the goal of incorporating quantum groups and their homogeneous spaces into Connes-style noncommutative geometry, which turns out to be more difficult than one might have hoped. For instance, amongst the quantum flag varieties, we have a local index formula only in the simplest case of the Podles sphere (Neshveyev-Tuset). As shown by Connes-Moscovici, one of the key points is to construct algebras of pseudodifferential operators on noncommutative spaces. To apply these ideas to Krahmer's Dirac operator on a quantum projective space, we need to generalize results of Higson and Uuye to a new class of twisted spectral triples. (Joint work with M. Matassa)