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THE POLYNOMIAL PROJECTORS THAT PRESERVE HOMOGENEOUS DIFFERENTIAL RELATIONS: A NEW CHARACTERIZATION OF KERGIN INTERPOLATION

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We study the polynomial projectors that preserve homogeneous differential relations (i.e., $q(D)f = 0 \implies q(D)(\mathbf{\Pi}(f)) = 0$). We show that this property is equivalent to the preservation of ridge functions and find a characterization in term of the space of interpolation conditions. Several applications are given to the study of the structure of these projectors and, more particularly, to projectors related to Kergin interpolation.

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1. Introduction

Let $H(\mathbb{C}^n)$ be the space of entire functions on $\mathbb{C}^n$ (endowed with its standard compact convergence topology) and $\mathcal{P}_d(\mathbb{C}^n)$ the space of polynomials of n complex variables of (total) degree at most d. A *polynomial projector* of degree d is a continuous linear map $\mathbf{\Pi}$ from $H(\mathbb{C}^n)$ to $\mathcal{P}_d(\mathbb{C}^n)$ such that $\mathbf{\Pi}$ is equal to the identity on $\mathcal{P}_d(\mathbb{C}^n)$, that is $\mathbf{\Pi}(p) = p$ for every $p \in \mathcal{P}_d(\mathbb{C}^n)$ (hence $\mathbf{\Pi}$ is onto and $\mathbf{\Pi}^2 = \mathbf{\Pi}$). Such a projector $\mathbf{\Pi}$ is said to *preserve homogeneous differential relations* when, for every $f \in H(\mathbb{C}^n)$ and every *homogeneous* polynomial $q(z) = \sum_{|\alpha|=k} a_\alpha z^\alpha$ one has

$$q(D)f = 0 \implies q(D)\mathbf{\Pi}(f) = 0, \tag{1}$$

where as usual $q(D) = \sum_{|\alpha|=k} a_\alpha D^\alpha$ and $D^\alpha = \partial^k/\partial z_1^{\alpha_1} \dots \partial z_n^{\alpha_n}$. (We use the standard multinomial notation, in particular, $|\alpha|$ denotes the length of