ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Journal of Approximation Theory ■■■ (■■■■) ■■■–■■■

Journal of Approximation Theory

www.elsevier.com/locate/jat

Uniform approximation by discrete least squares polynomials

Jean-Paul Calvi[a,*], Norman Levenberg[b]

[a]*Institut de Mathématiques de Toulouse, Université Paul Sabatier, 32062, Toulouse Cedex 9, France*
[b]*Department of Mathematics, Indiana University, Bloomington, Indiana, IN 47405, USA*

Received 24 August 2006; received in revised form 11 May 2007; accepted 23 May 2007

Communicated by András Kroó

Abstract

We study uniform approximation of differentiable or analytic functions of one or several variables on a compact set K by a sequence of discrete least squares polynomials. In particular, if K satisfies a Markov inequality and we use point evaluations on standard discretization grids with the number of points growing polynomially in the degree, these polynomials provide nearly optimal approximants. For analytic functions, similar results may be achieved on more general K by allowing the number of points to grow at a slightly larger rate.

MSC: 41A10; 41A63; 65D05

Keywords: Polynomial approximation; Discrete least squares; Markov's inequality

1. Introduction

We are interested in constructive approximation of analytic or sufficiently smooth functions on a compact set $K \subset \mathbb{C}^N$ by polynomials. One way to approximate a function f by a polynomial is to use Lagrange interpolation. In the univariate case, the efficiency and accuracy of this classical method generally depends on the location of the interpolation points. From a theoretical point of view, this has been known for more than 80 years. Very few results are available on multivariate Lagrange interpolation. The so-called Fekete points (see Section 4.2) work well but are difficult to locate. Indeed, aside from the beautiful *Padua points* recently discovered by Caliari et al. in a square

* Corresponding author. Fax: +33 561030497.
E-mail addresses: jean-paul.calvi@math.ups-tlse.fr (J.-P. Calvi), nlevenbe@indiana.edu (N. Levenberg).

0021-9045/$ - see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.jat.2007.05.005

Please cite this article as: J.-P. Calvi, N. Levenberg, Uniform approximation by discrete least squares polynomials Journal of Approximation Theory (2007), doi: 10.1016/j.jat.2007.05.005