Tropical Cubics of Any Genus

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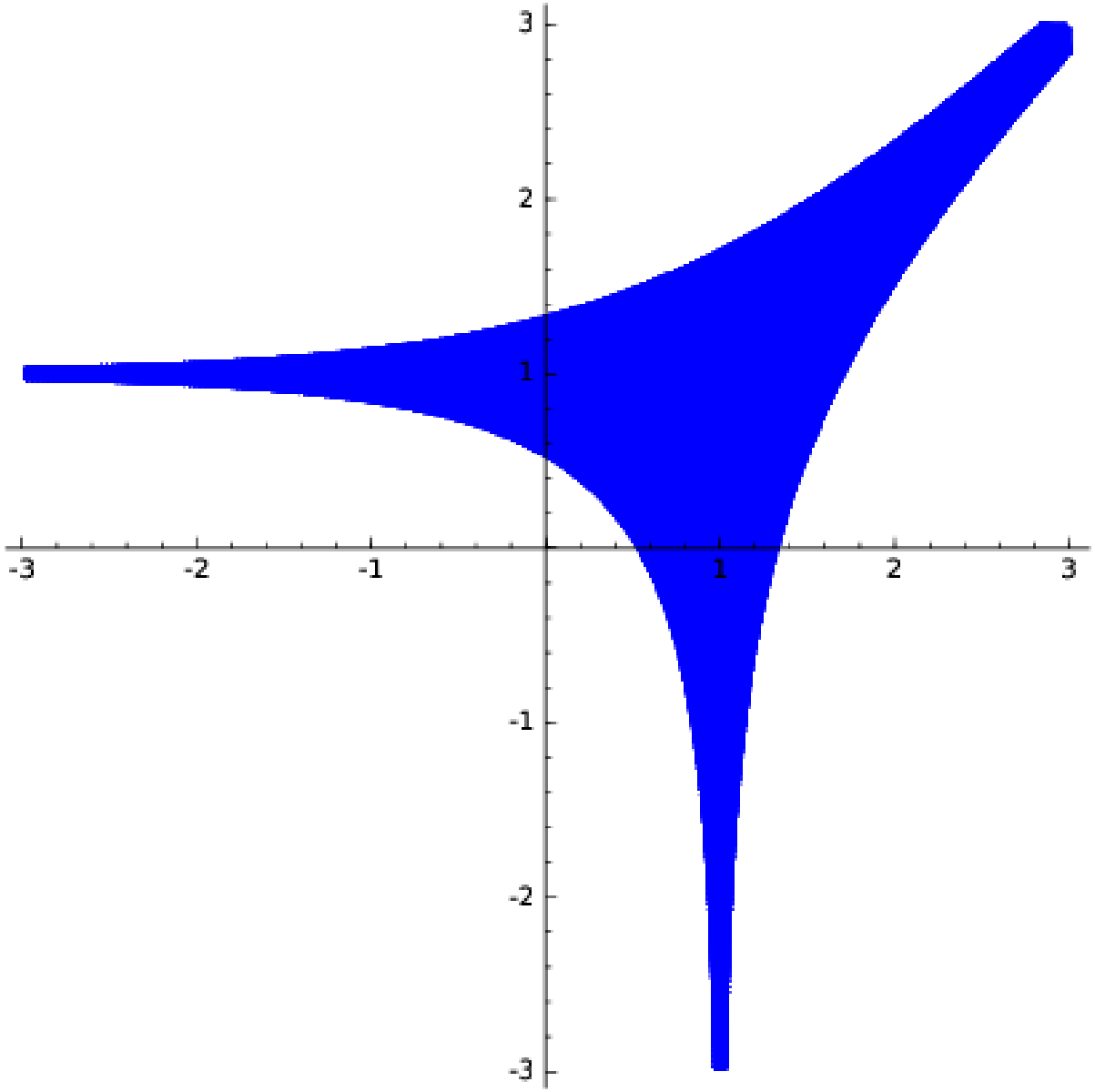
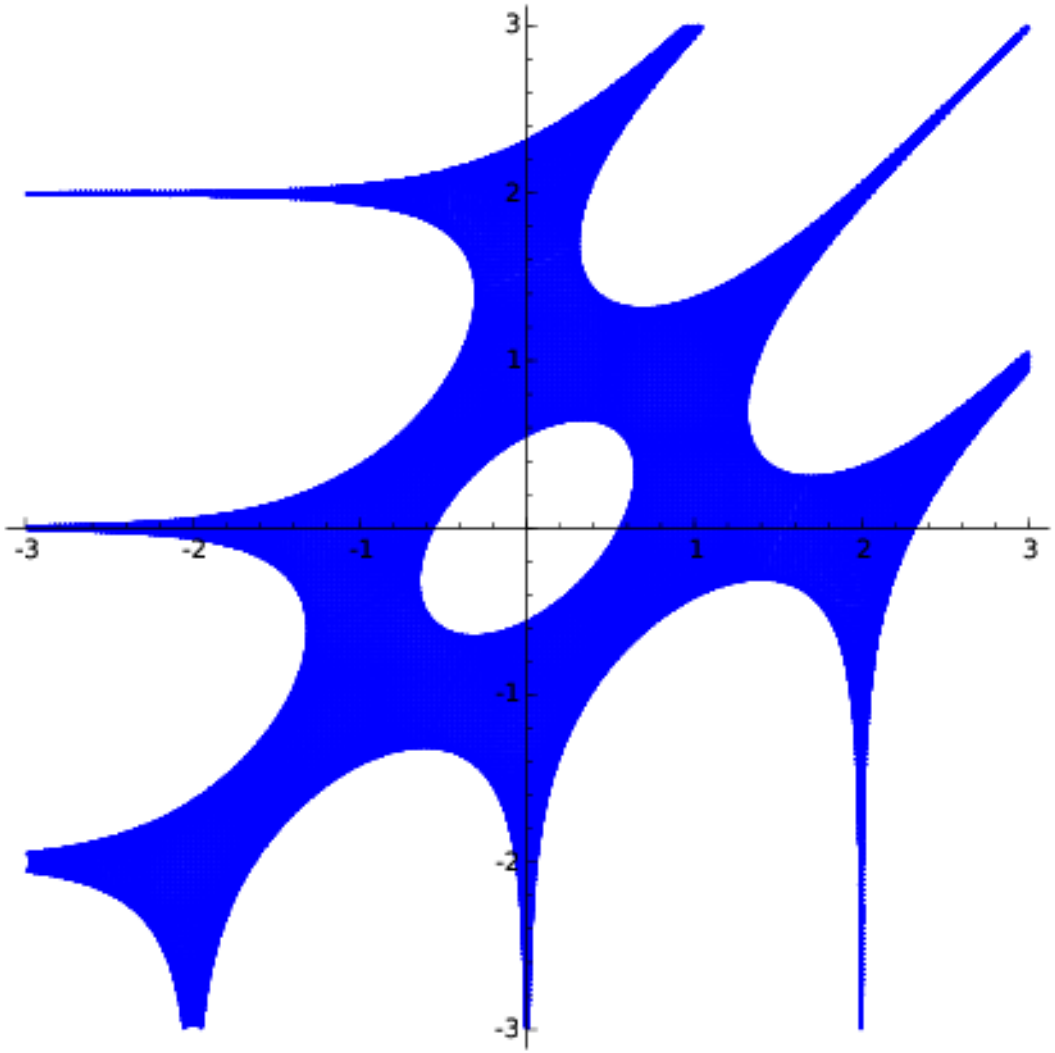
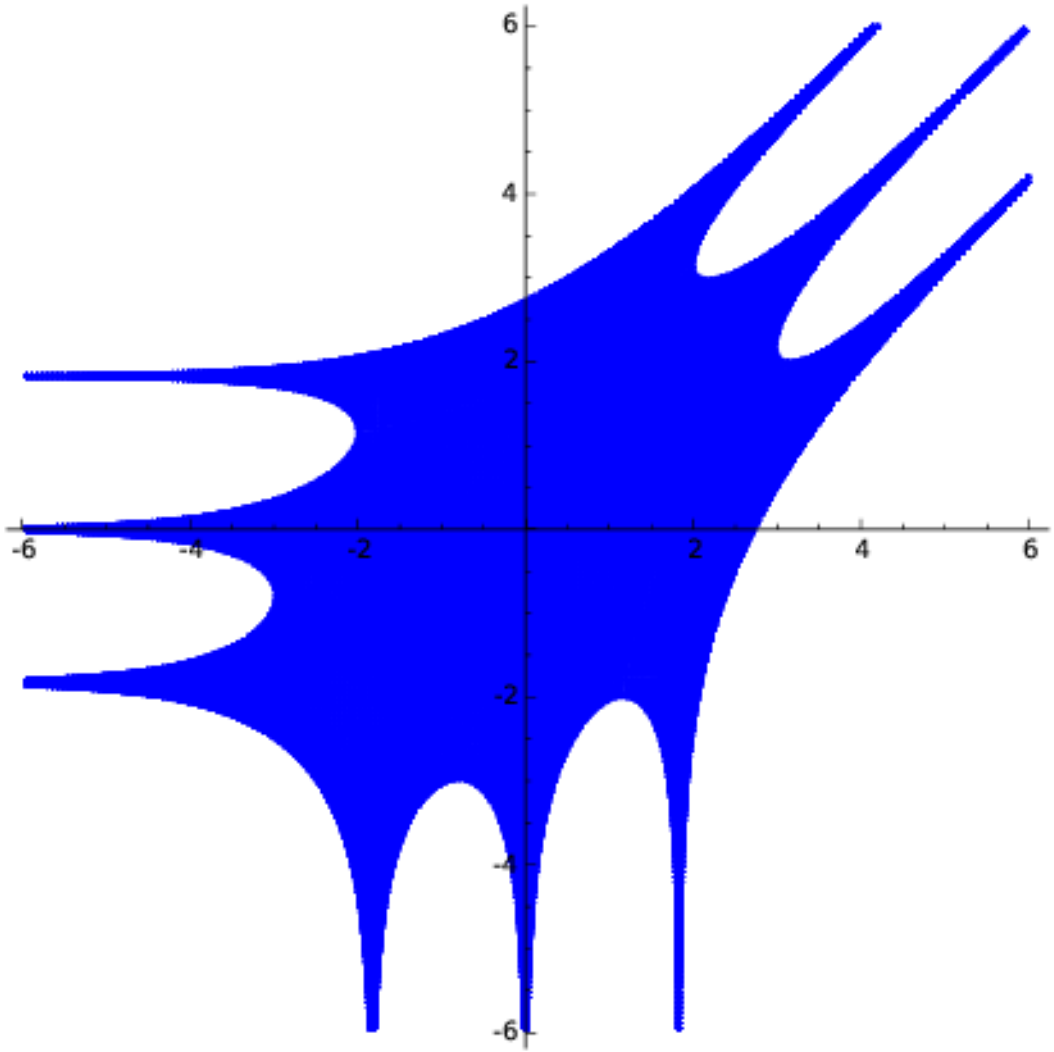
Amœba of a line



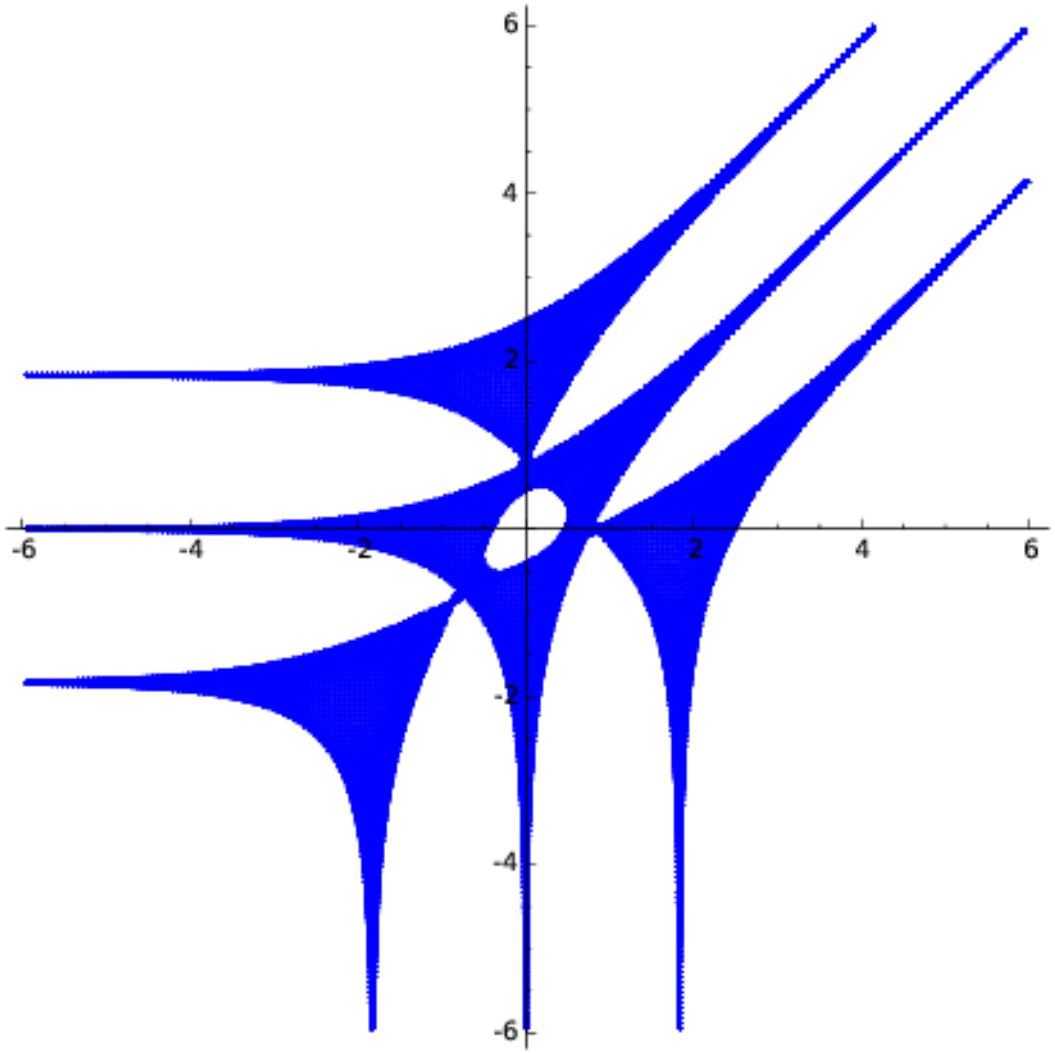
Amœbas



Amœbas

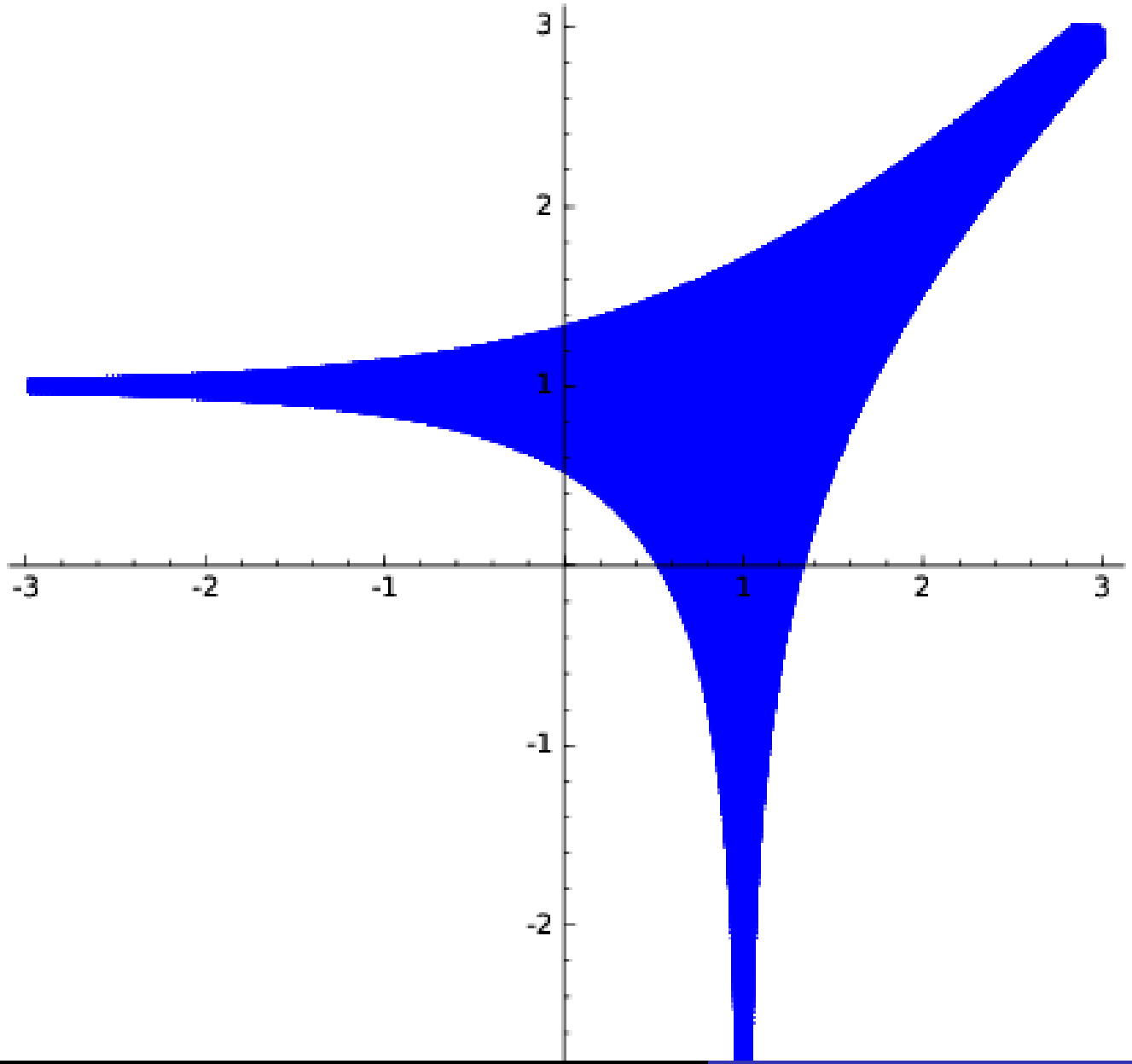


Amœbas



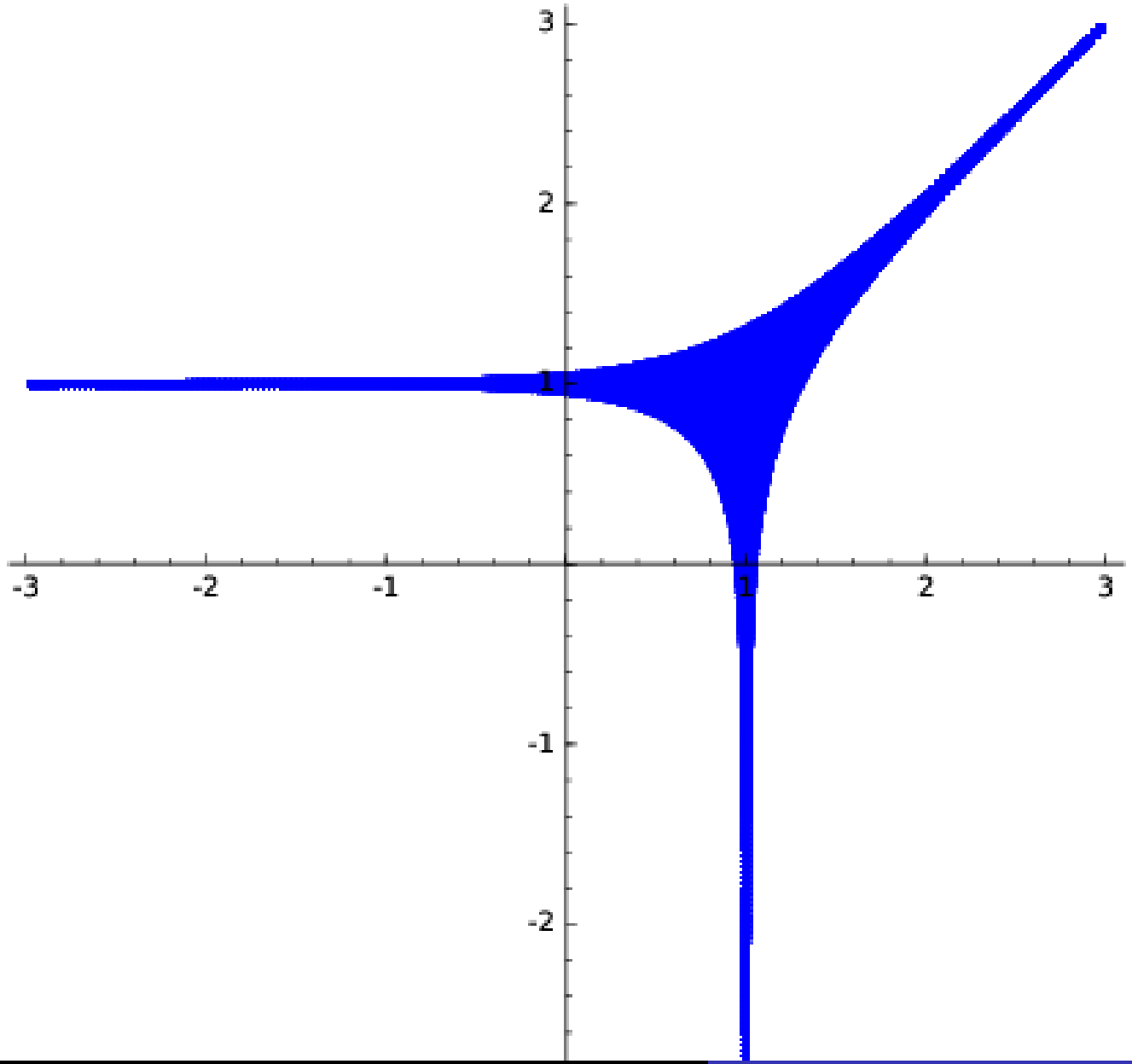
Rescaling: Line

$l : x + y - t = 0$, t=e



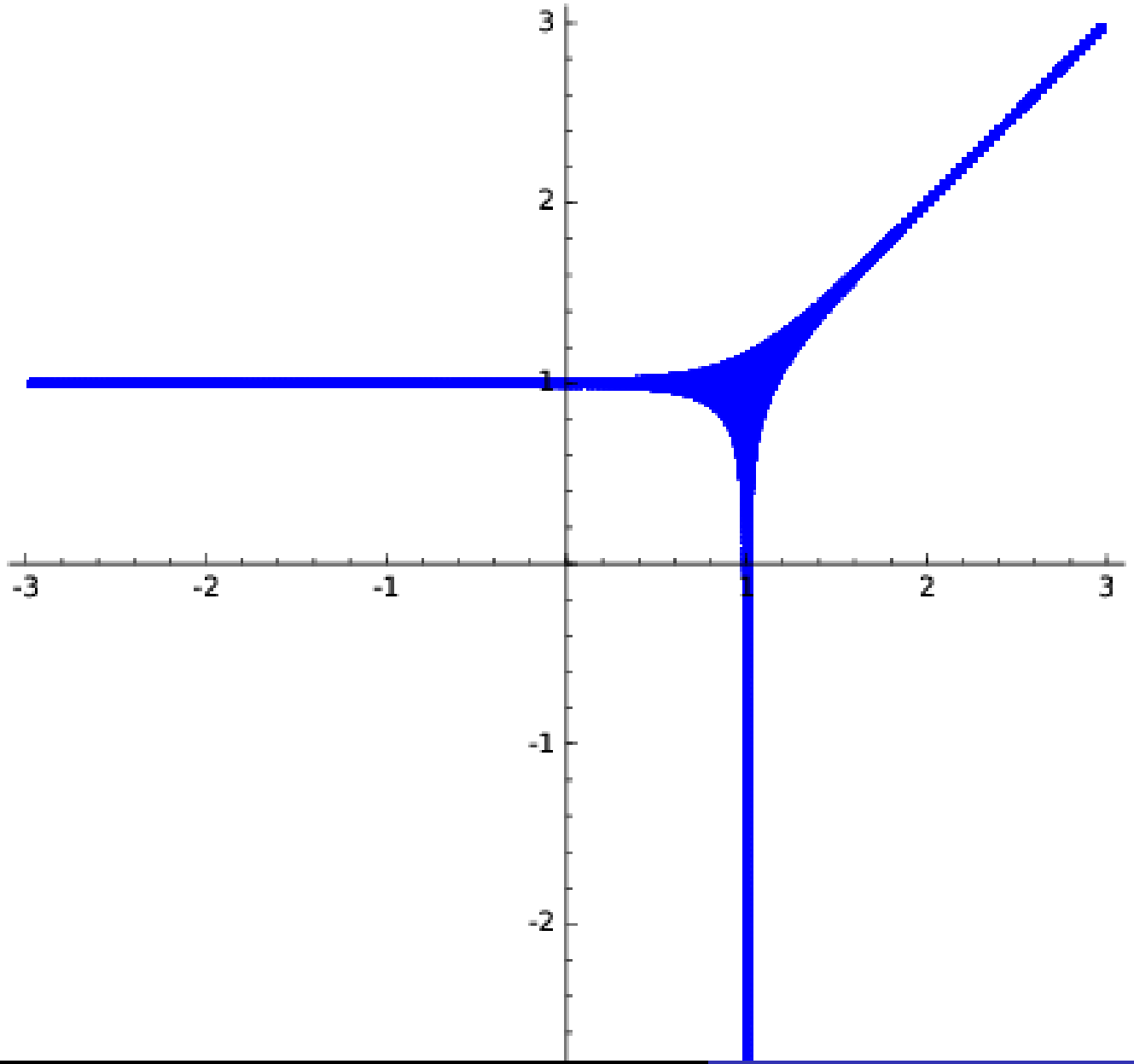
Rescaling: Line

$l : x + y - t = 0$, t=10



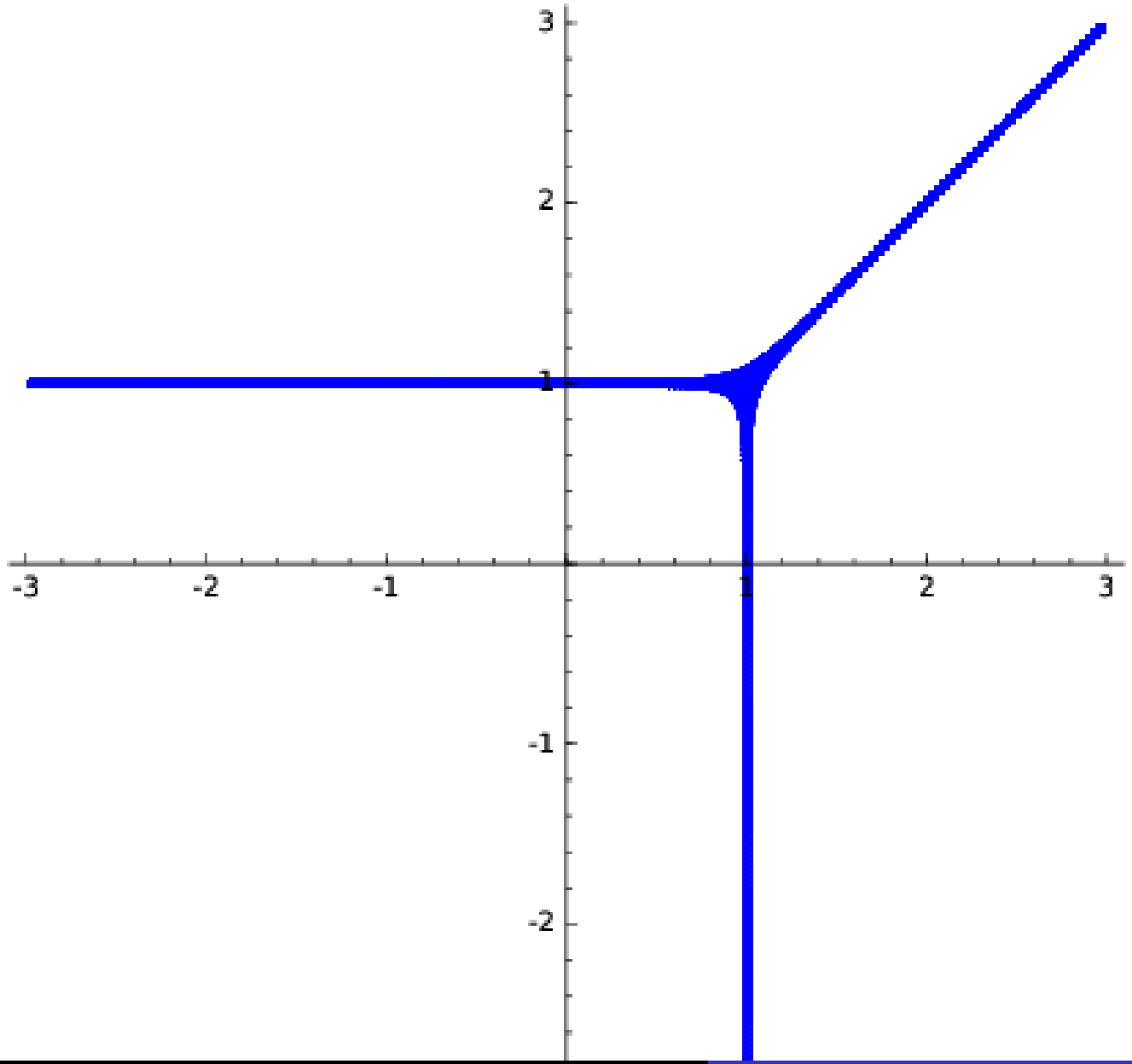
Rescaling: Line

$l : x + y - t = 0,\ t = 100$



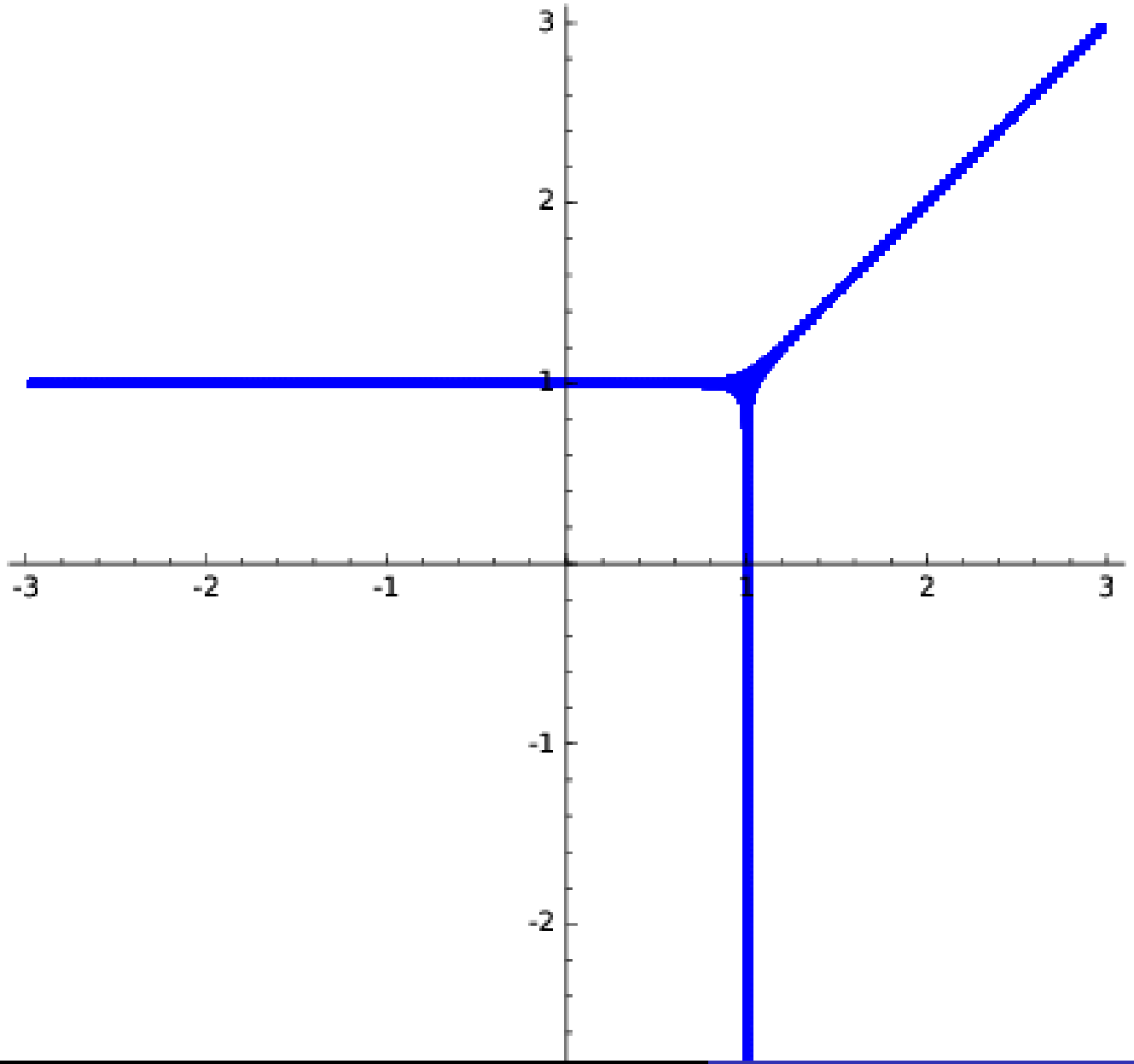
Rescaling: Line

$l : x + y - t = 0,\ t = 10^4$



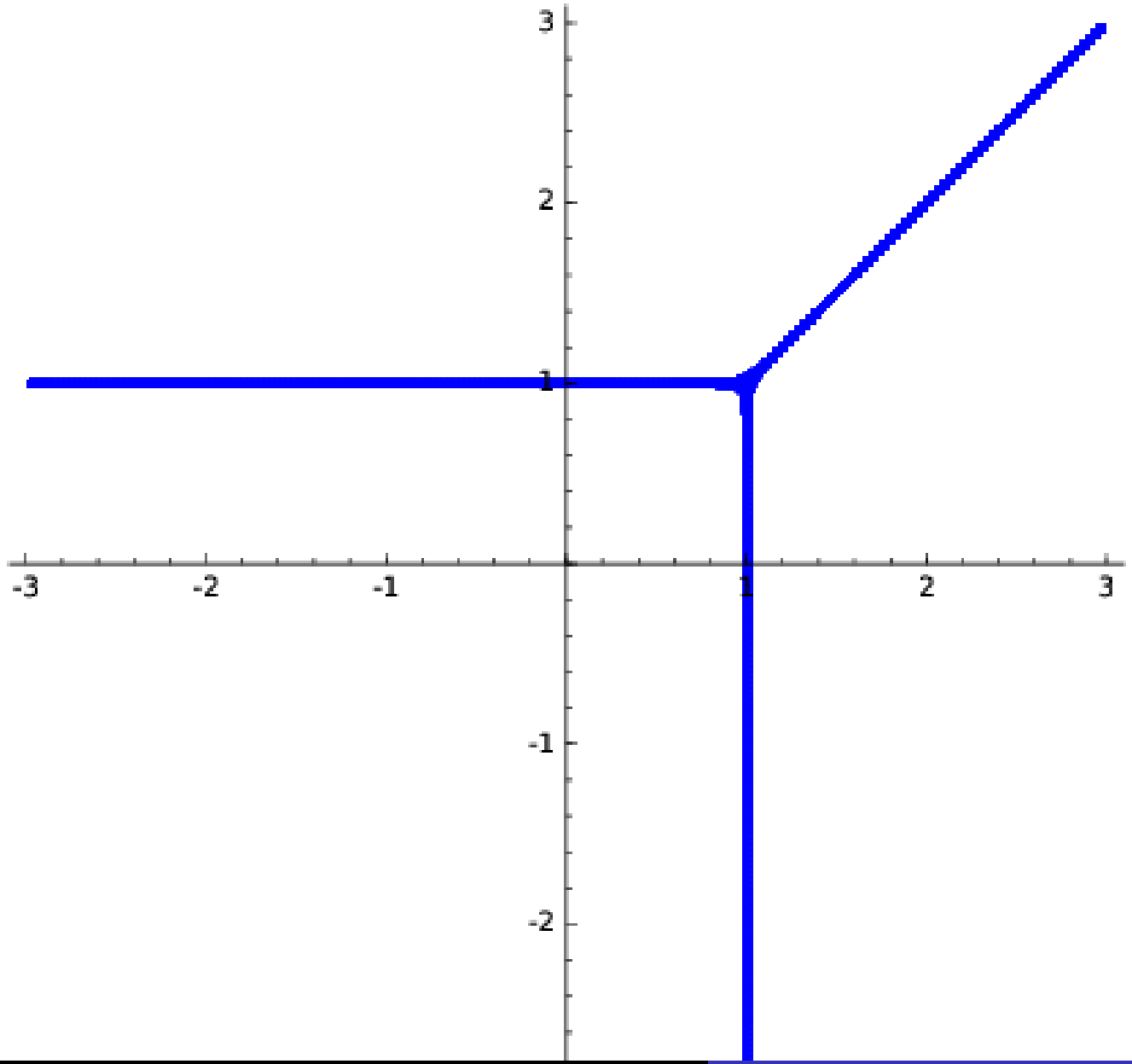
Rescaling: Line

$l : x + y - t = 0,\ t = 10^6$



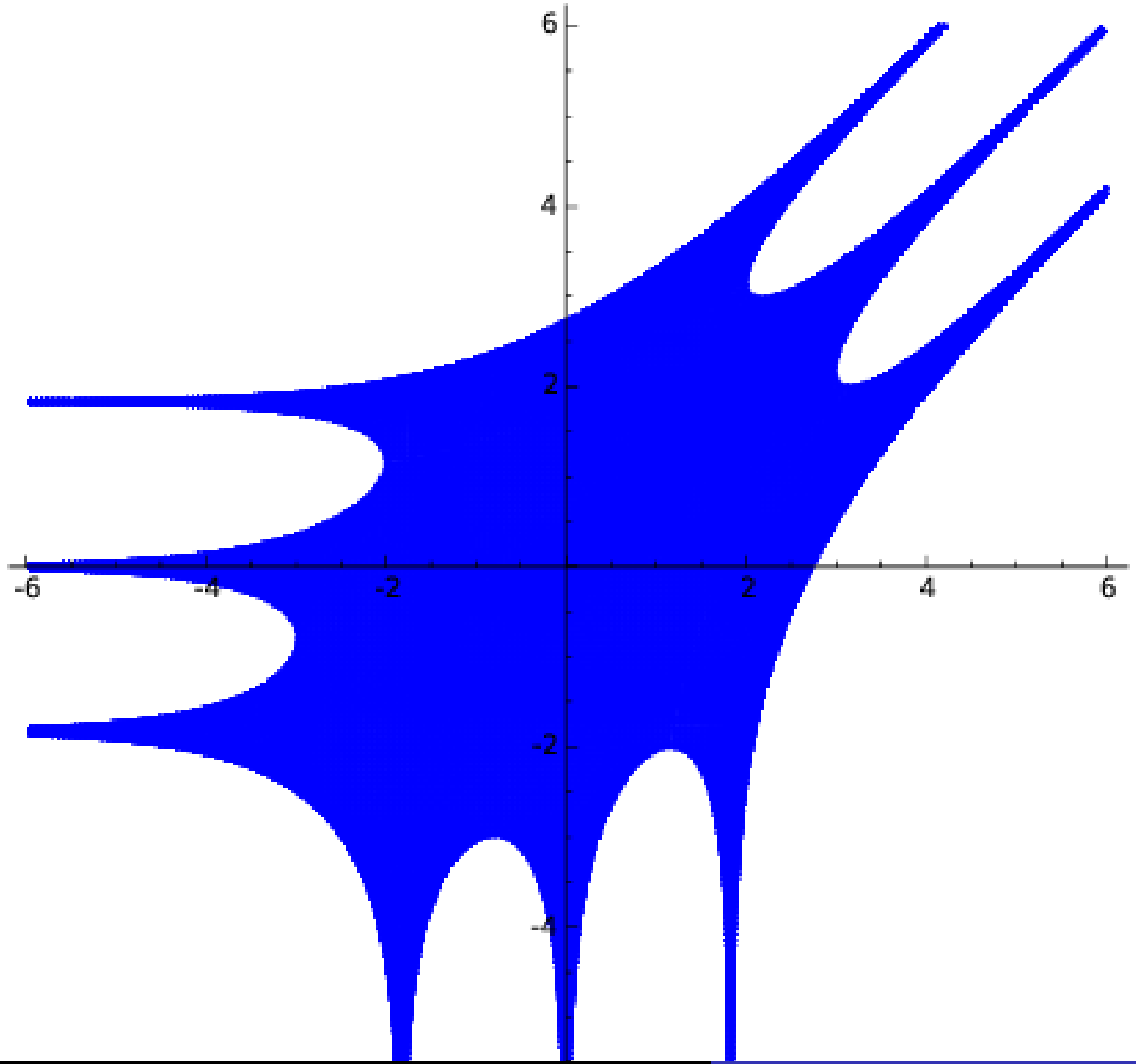
Rescaling: Line

$l : x + y - t = 0,\ t = 10^9$



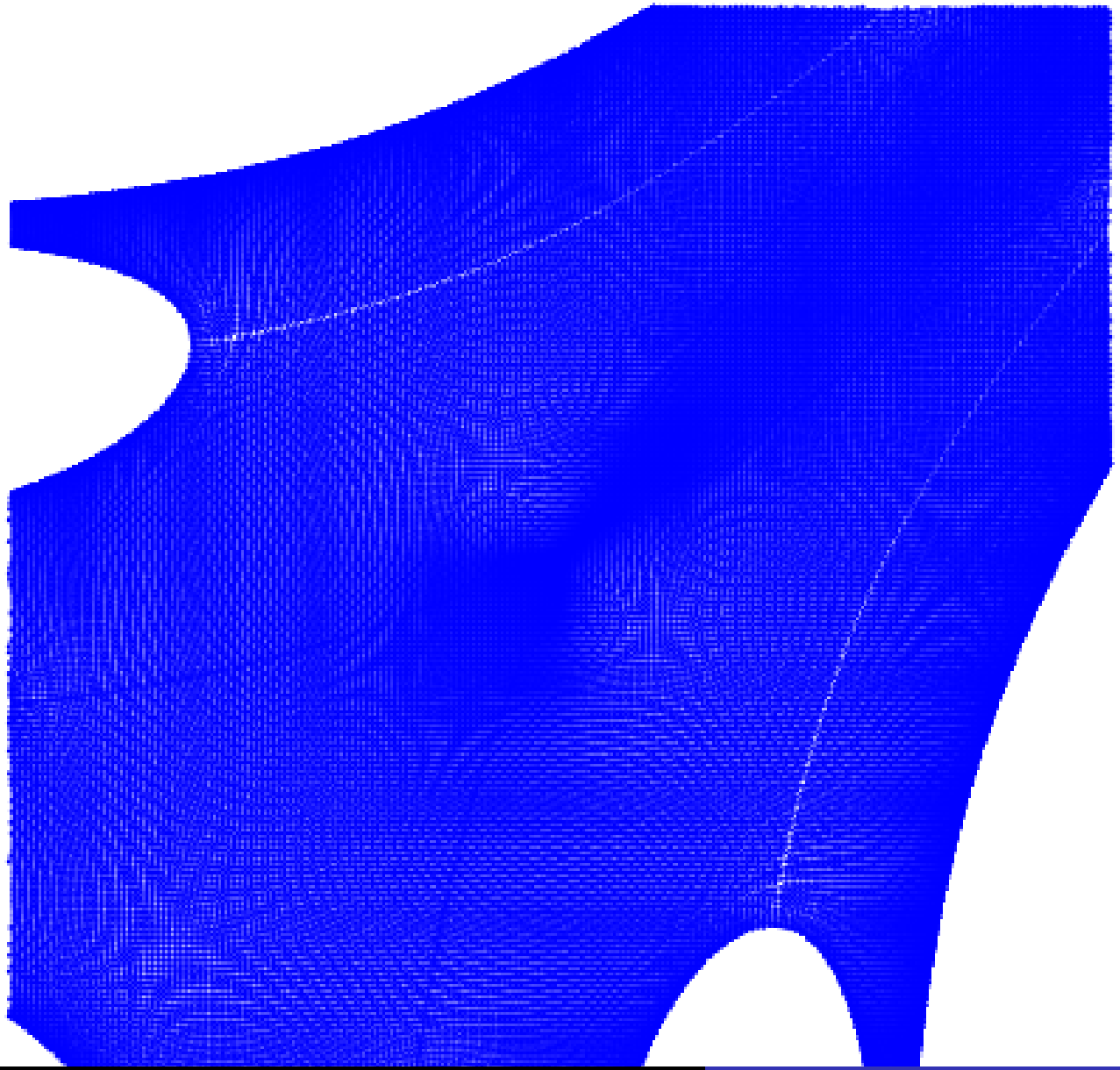
Towards the Tropical Limit

$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = e$$



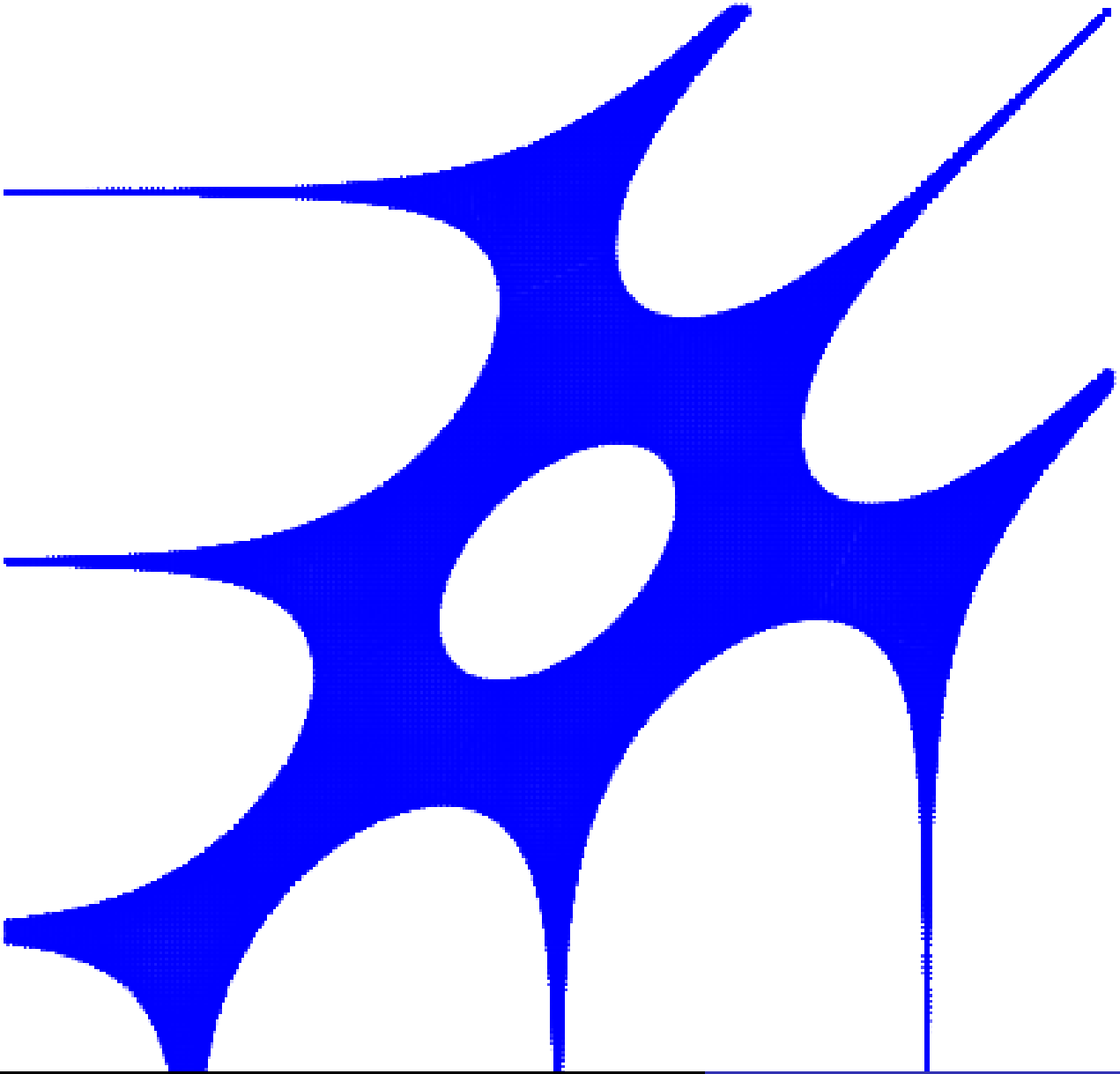
Towards the Tropical Limit

$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = e$$



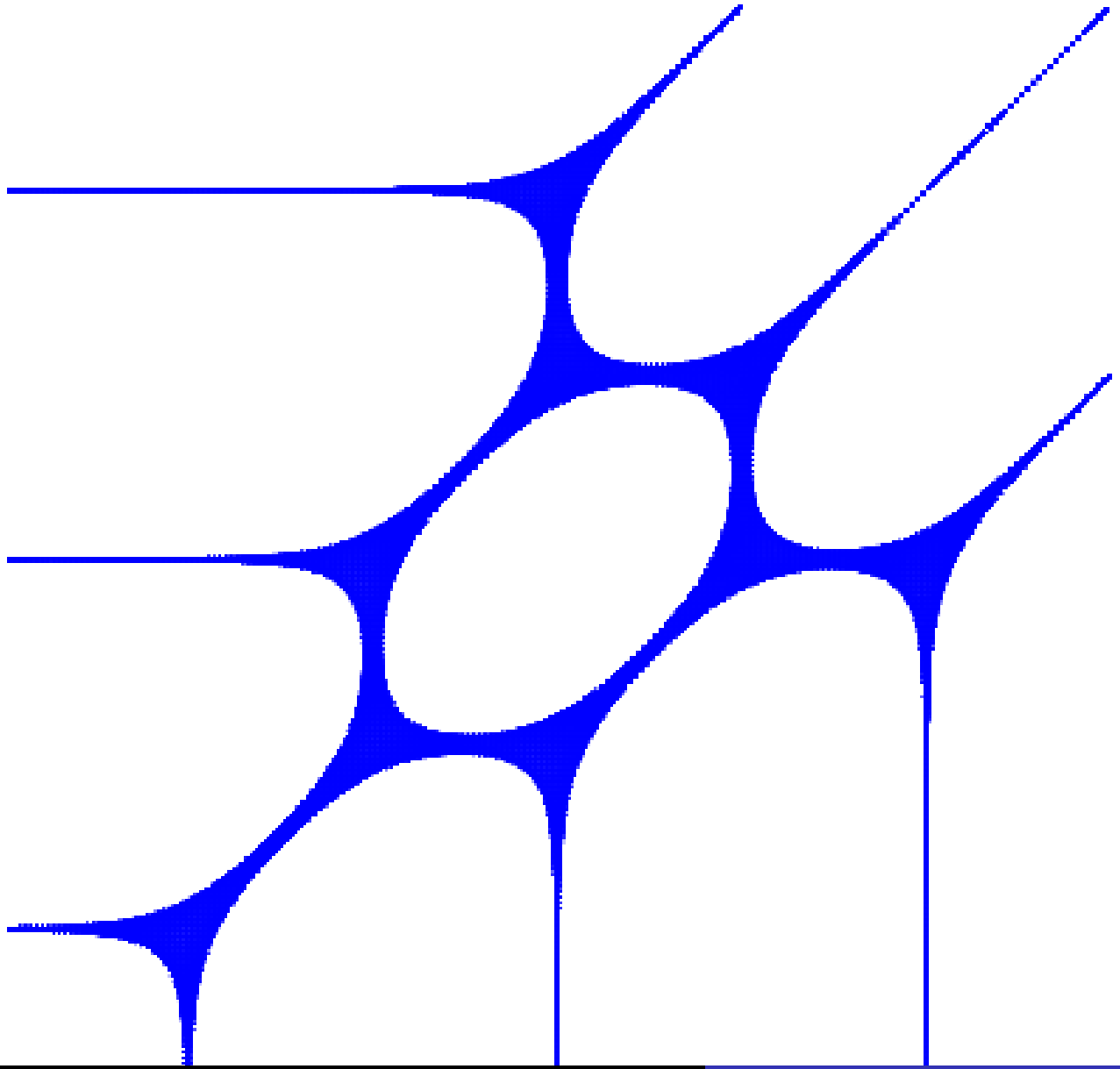
Towards the Tropical Limit

$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = 10$$



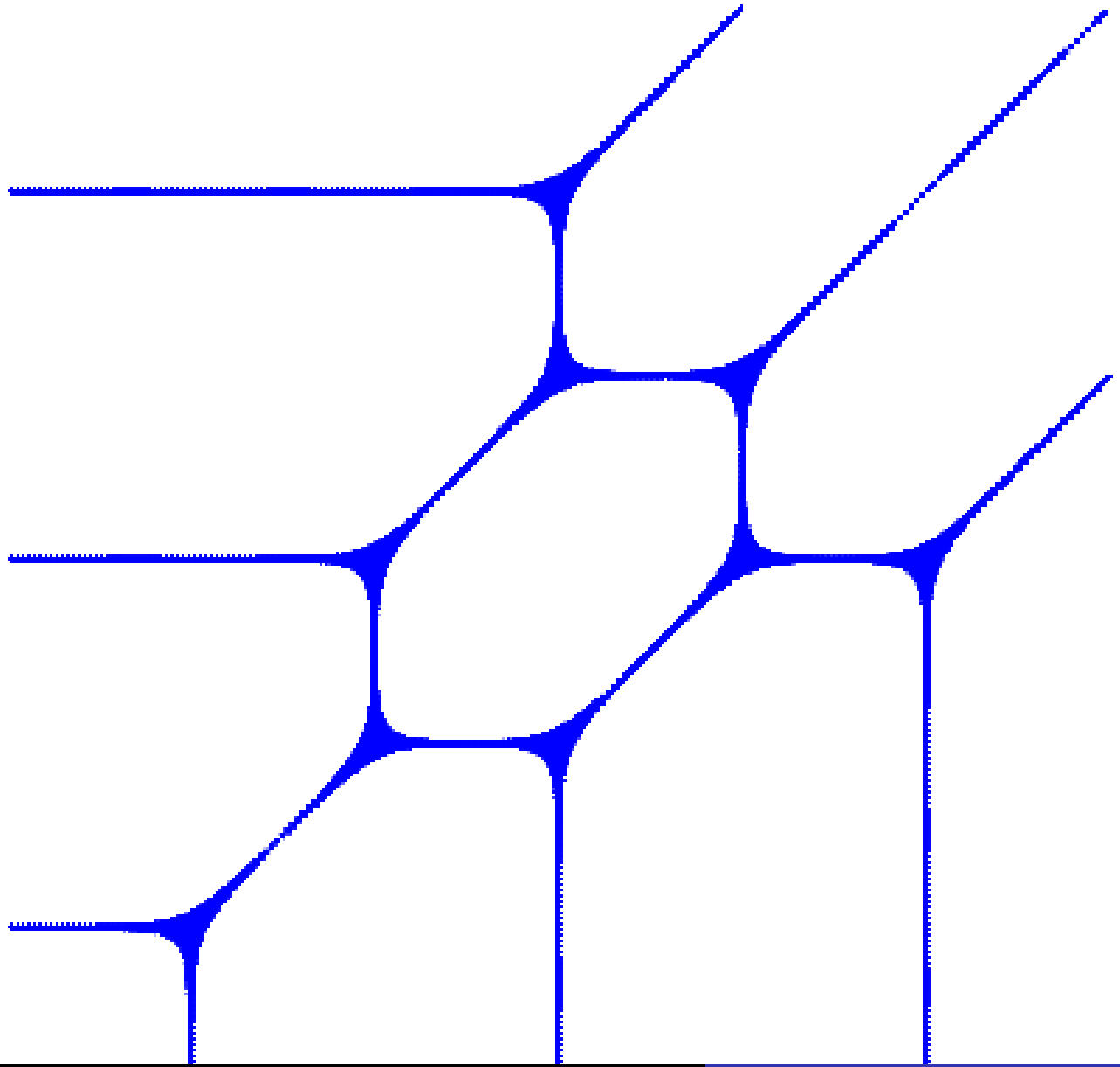
Towards the Tropical Limit

$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = 10^2$$



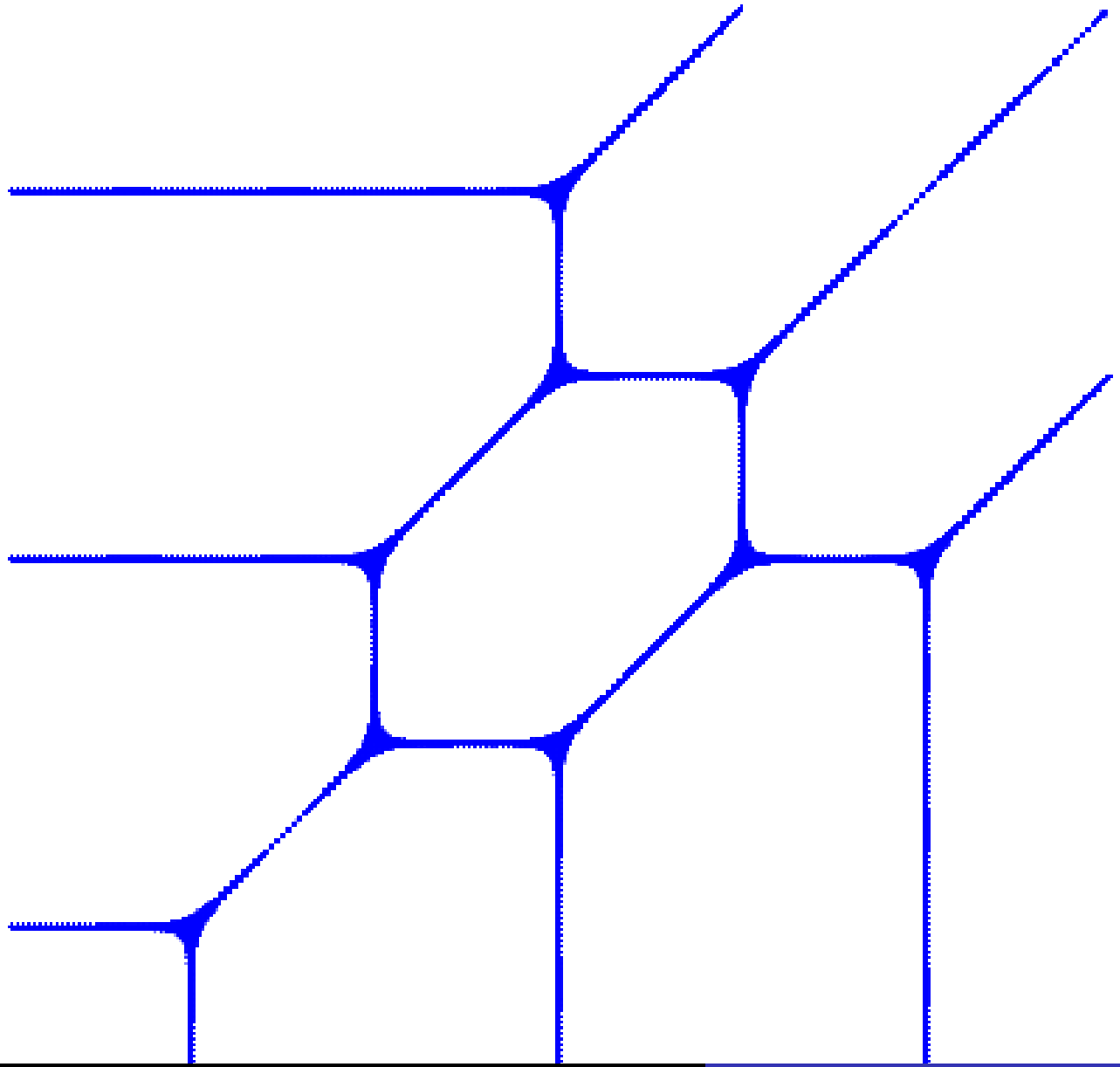
Towards the Tropical Limit

$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = 10^4$$



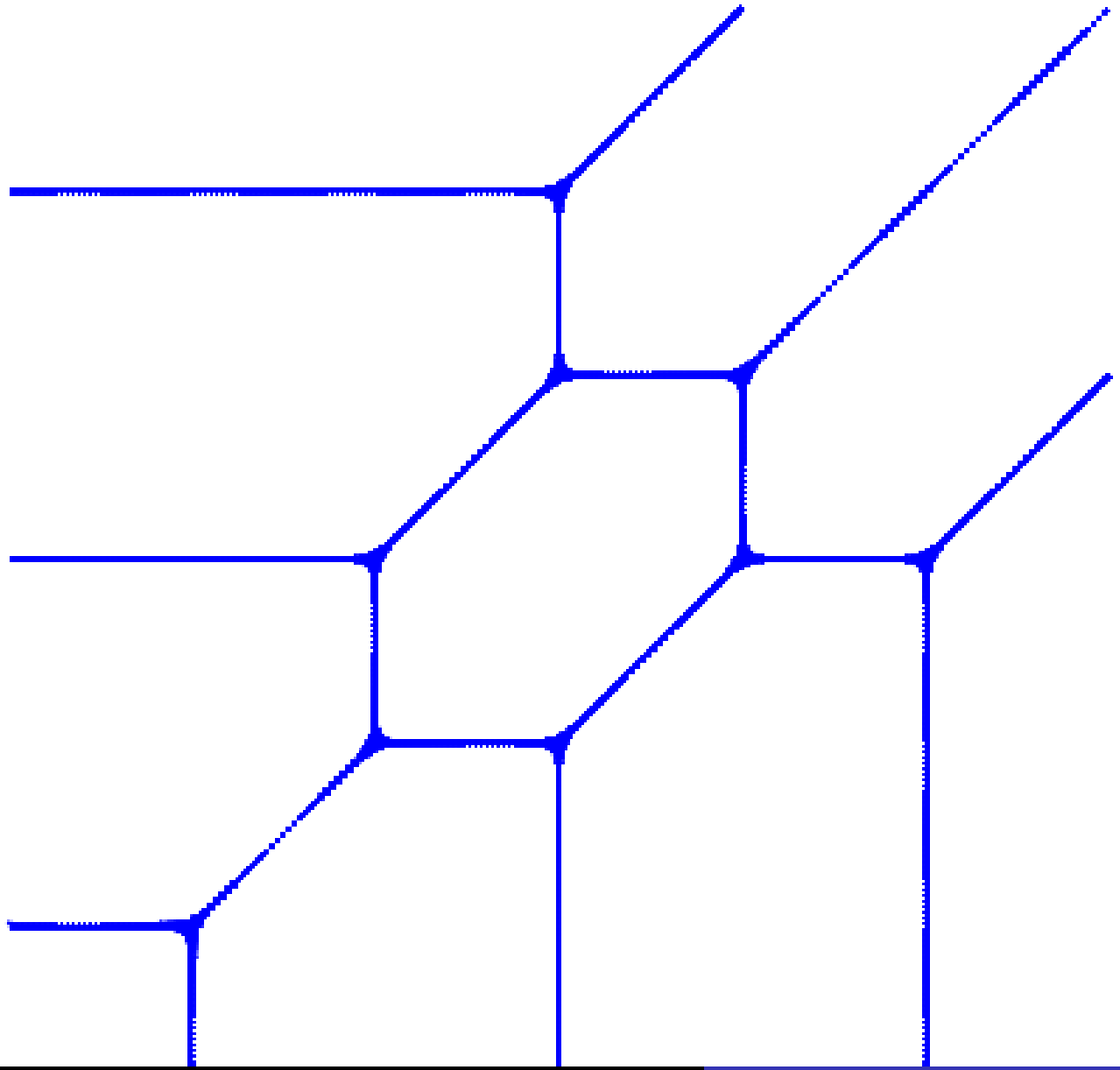
Towards the Tropical Limit

$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = 10^6$$

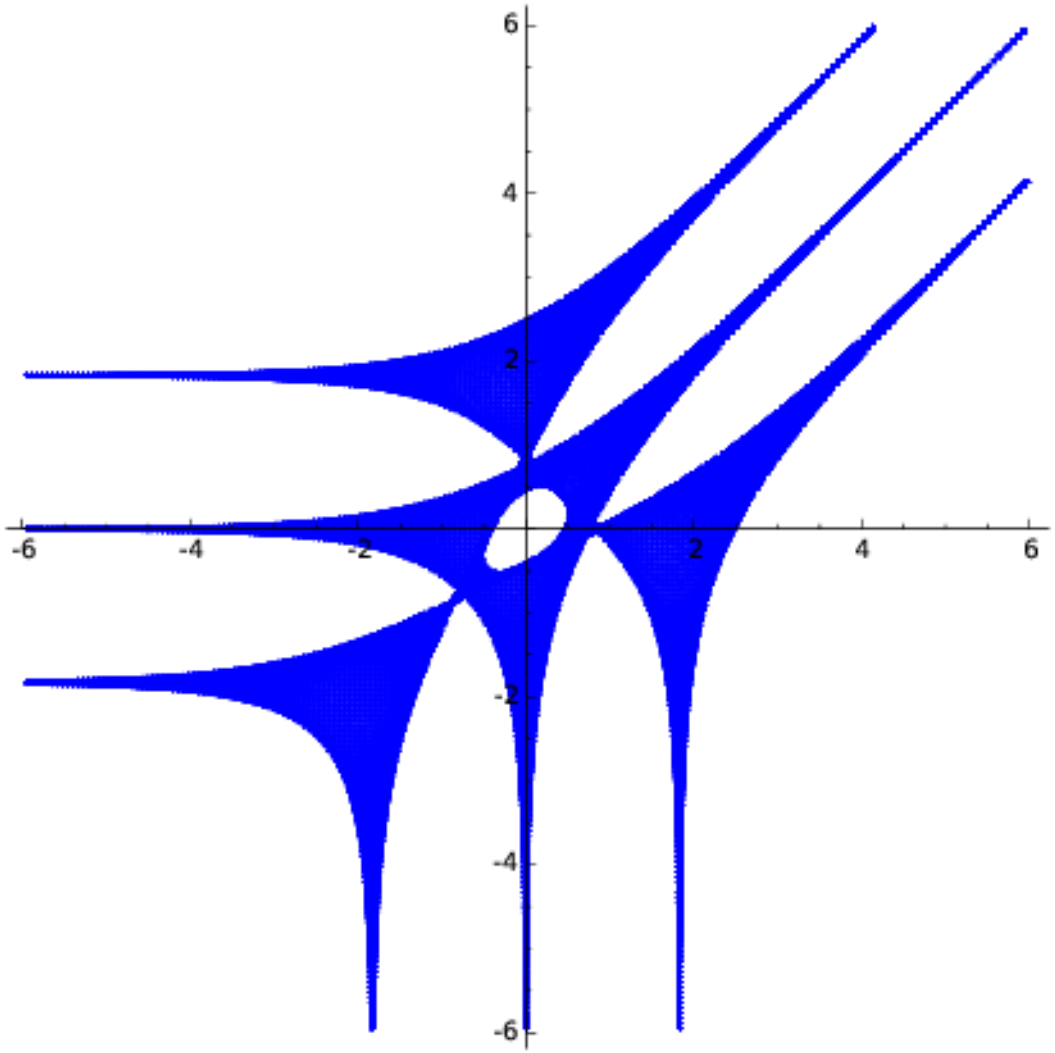


Towards the Tropical Limit

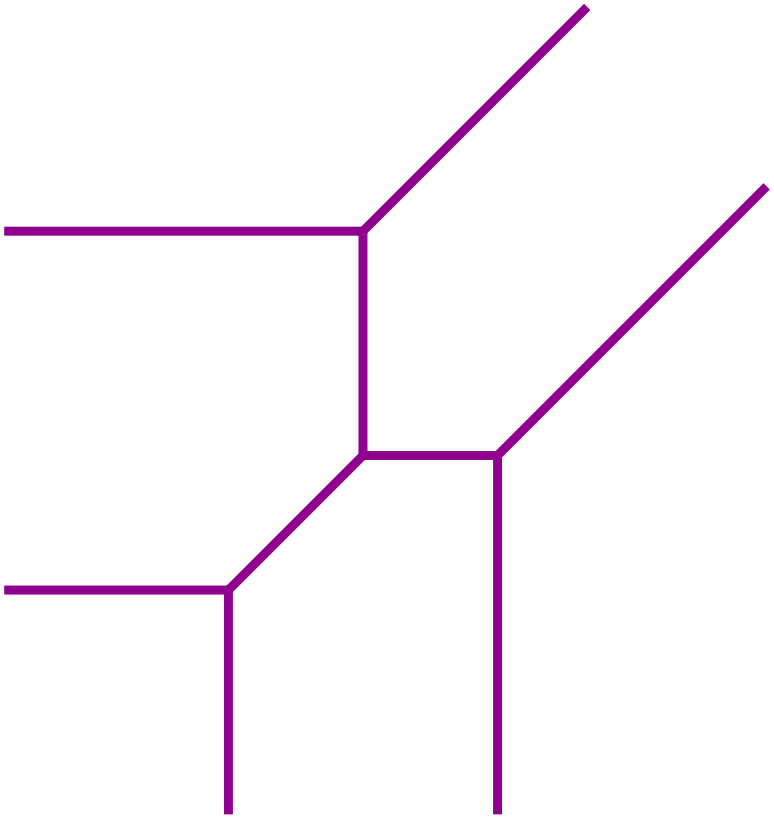
$$g = t^{-3}(1 + x^3 + y^3) + t^{-1}(x + y + x^2 + y^2 + x^2y + xy^2) - xy,\ t = 10^9$$



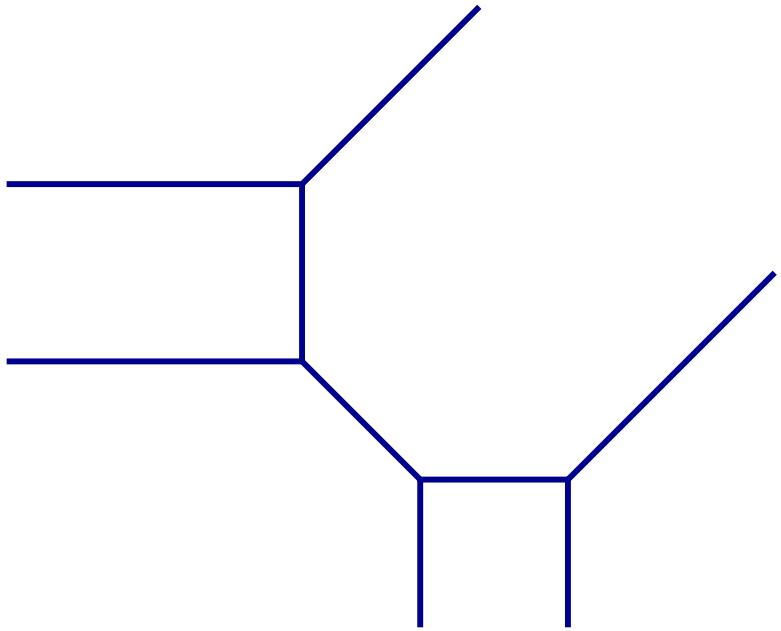
Amœbas



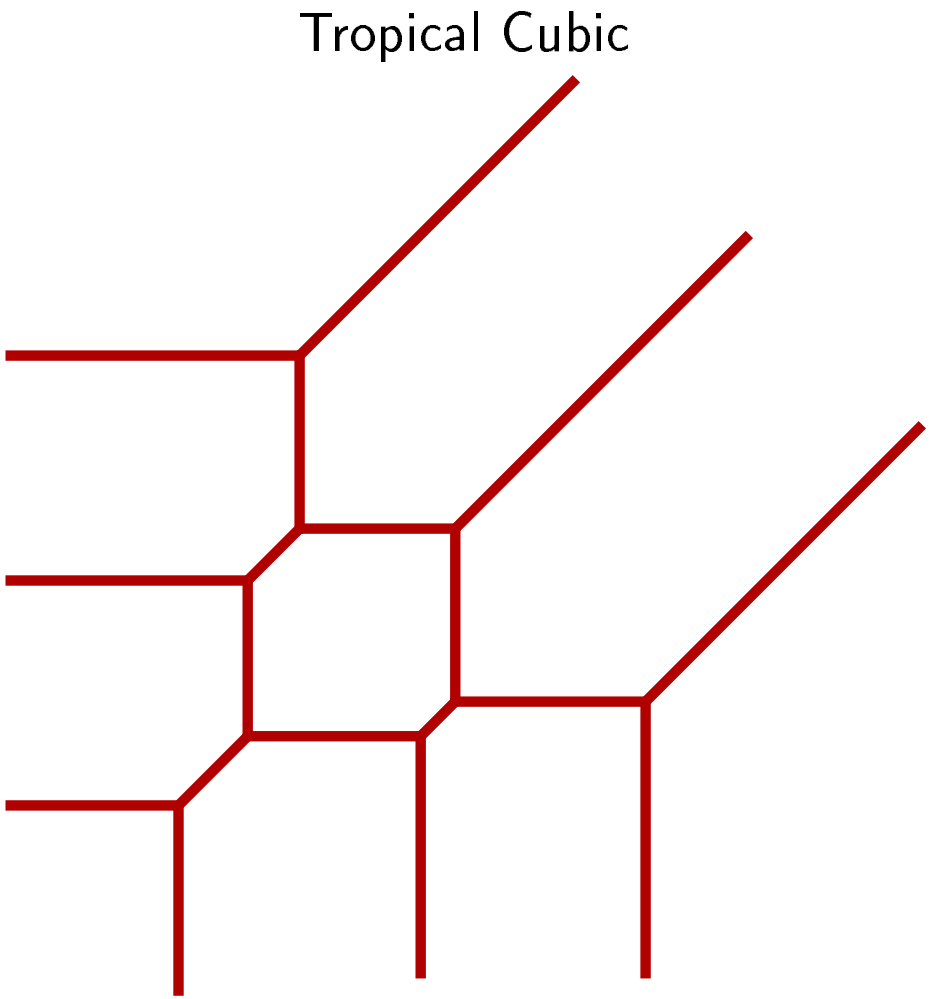
tropical conic



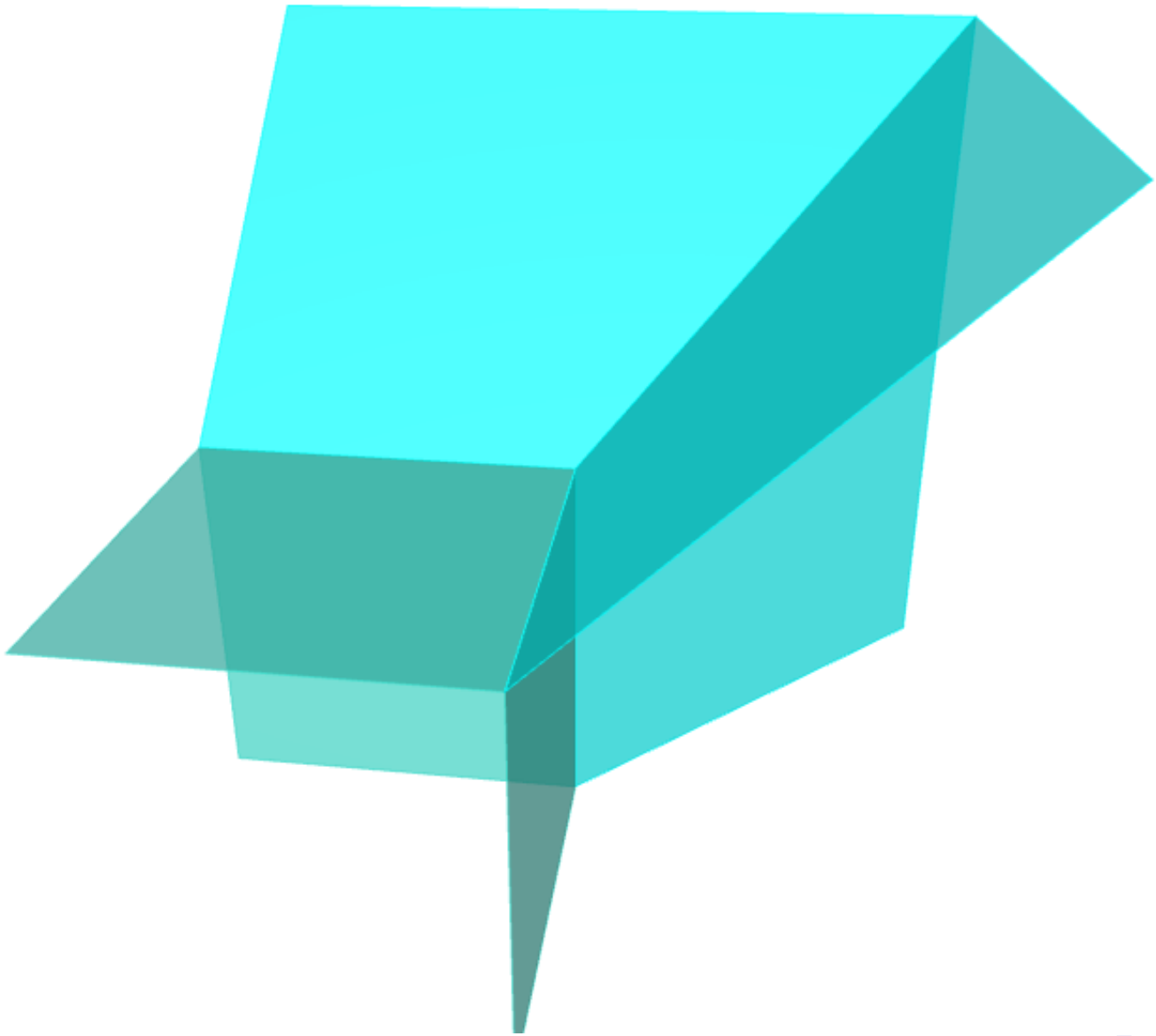
other tropical conic



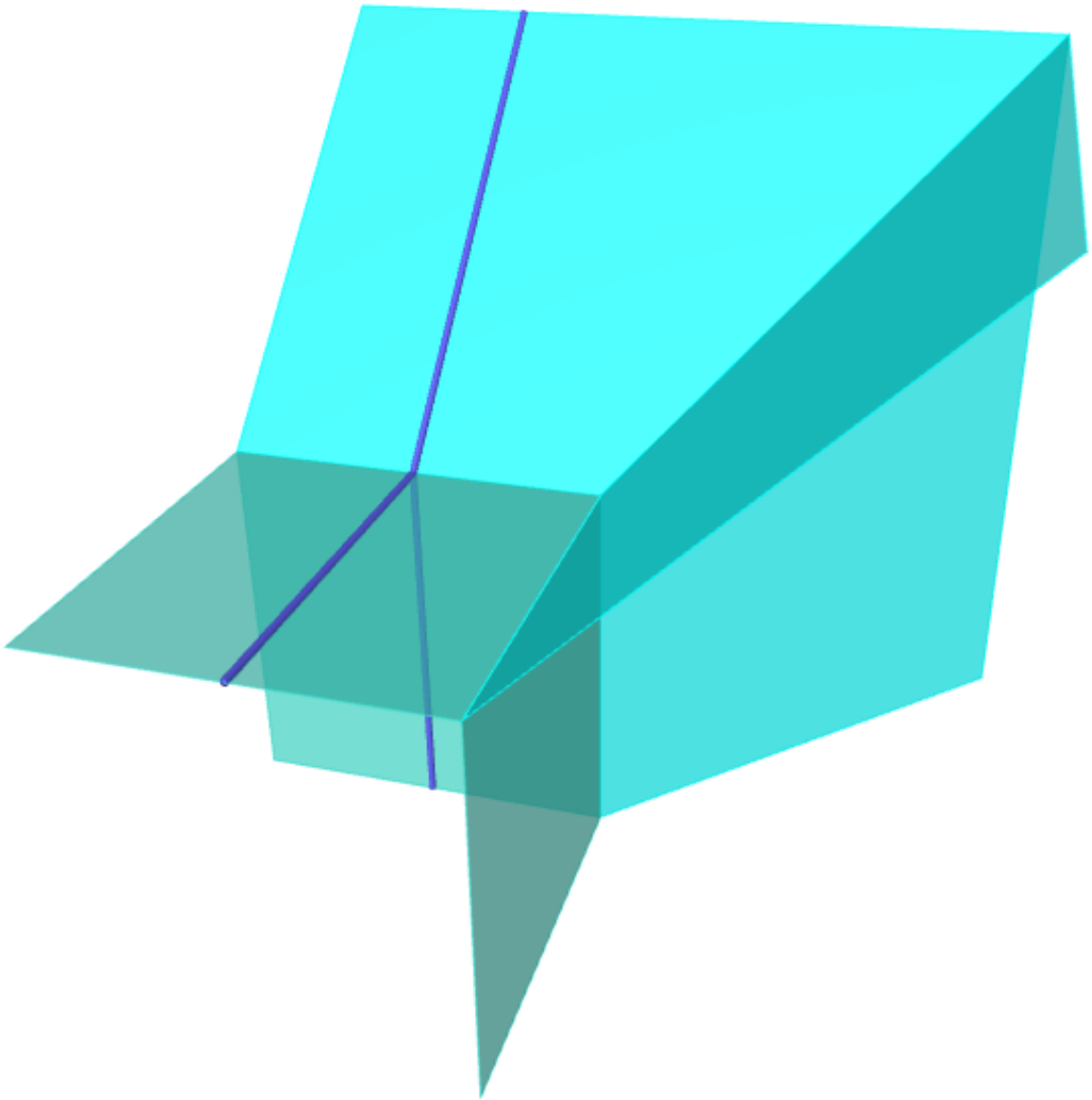
Example



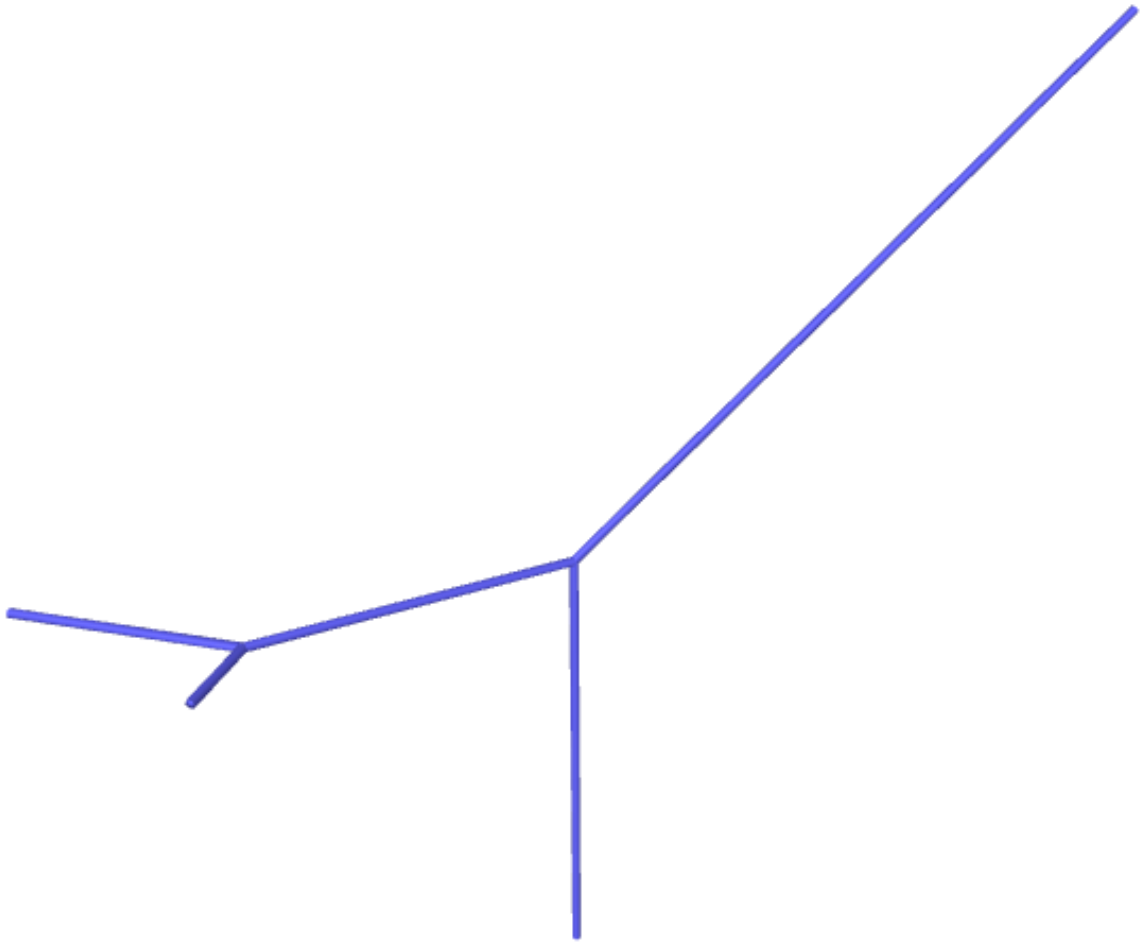
General Tropical Plane in $\mathbb{R}^3$



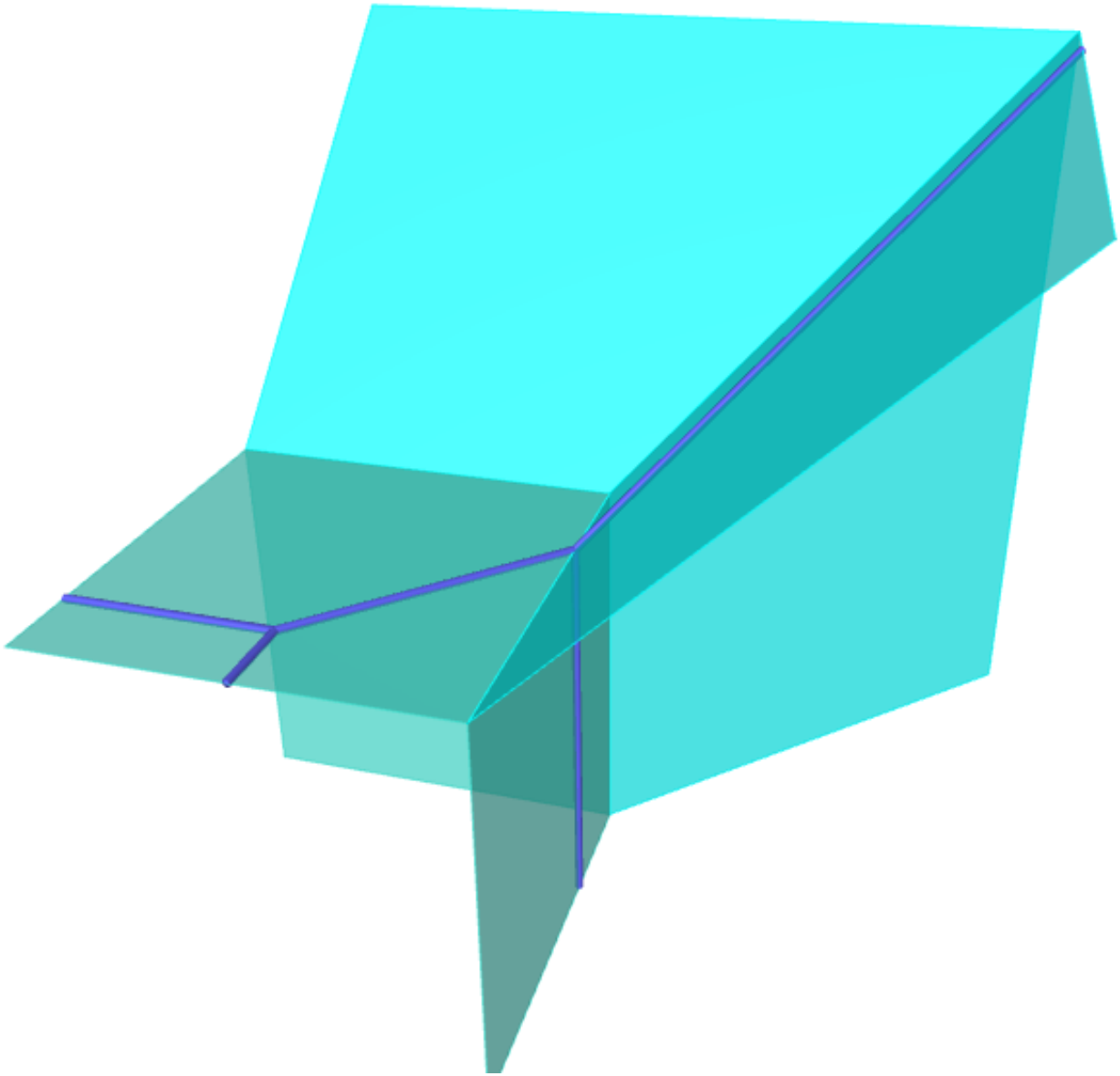
2d Line in a tropical plane in $\mathbb{R}^3$



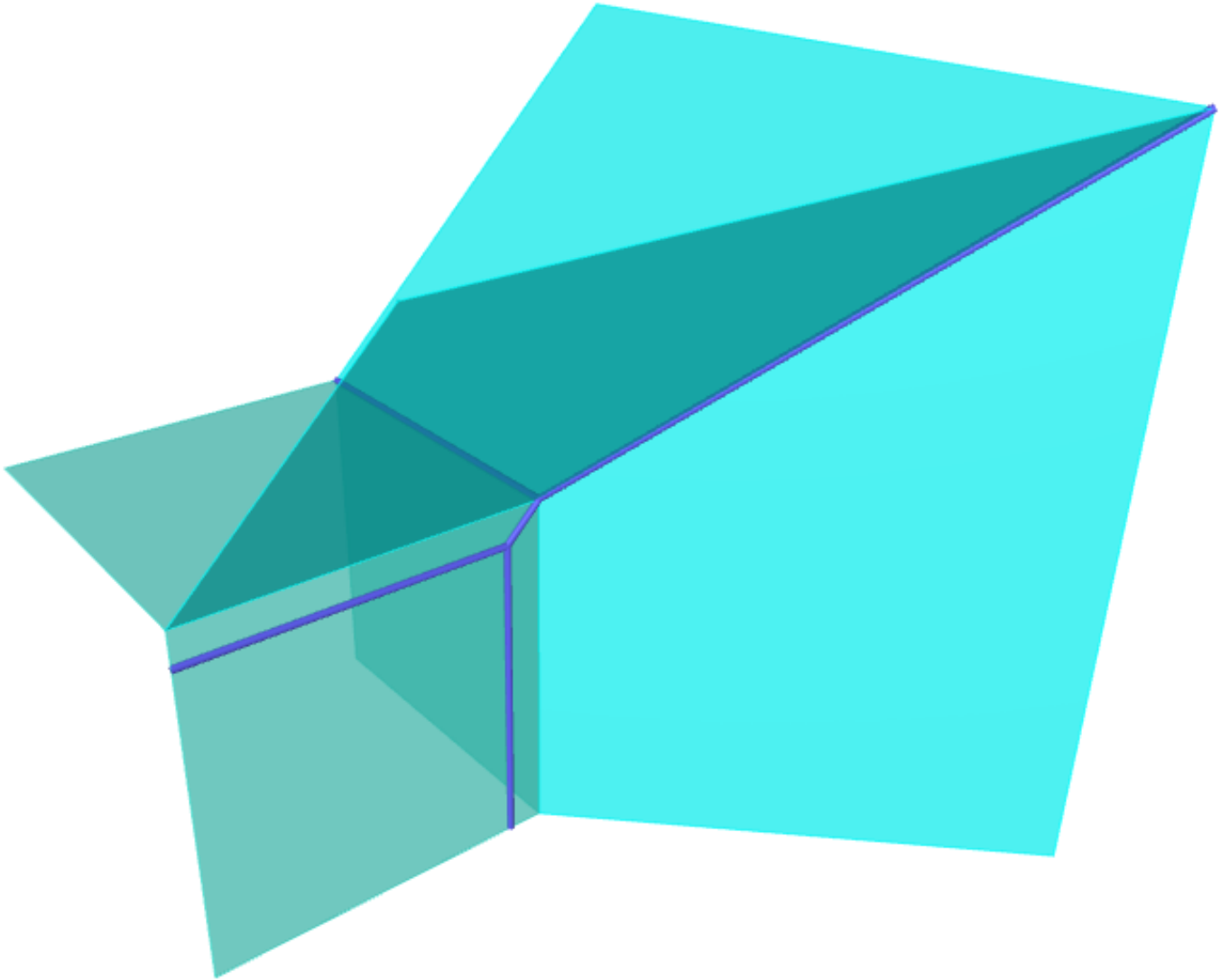
Two Vertices Line in a tropical plane in $\mathbb{R}^3$



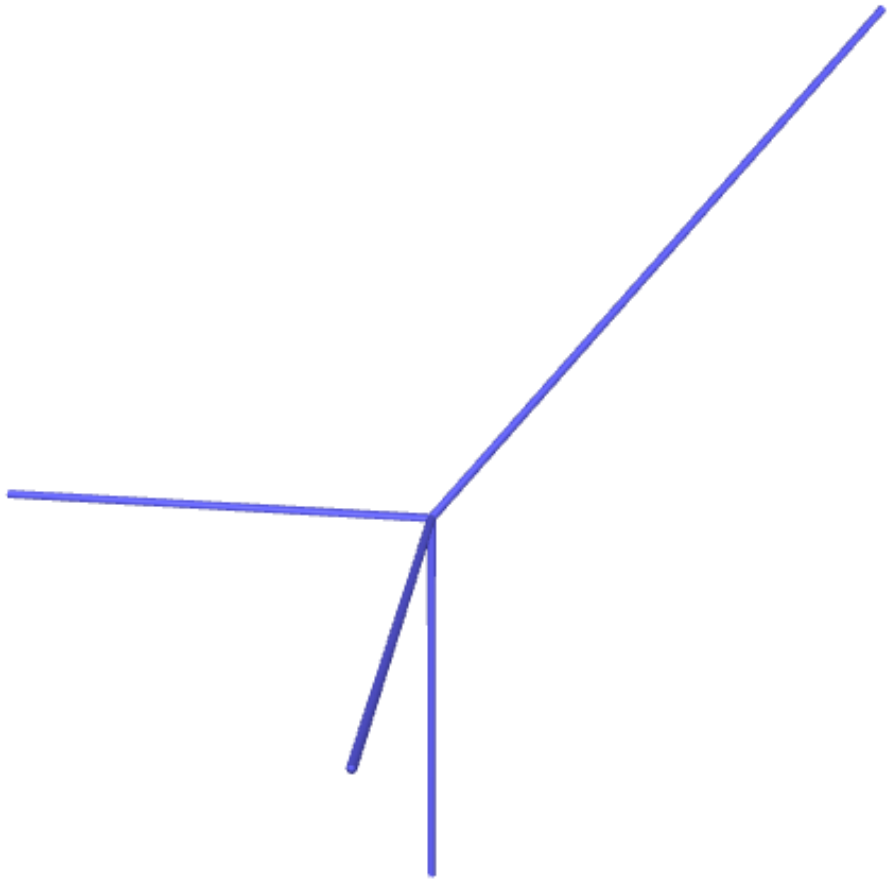
Two Vertices Line in a tropical plane in $\mathbb{R}^3$



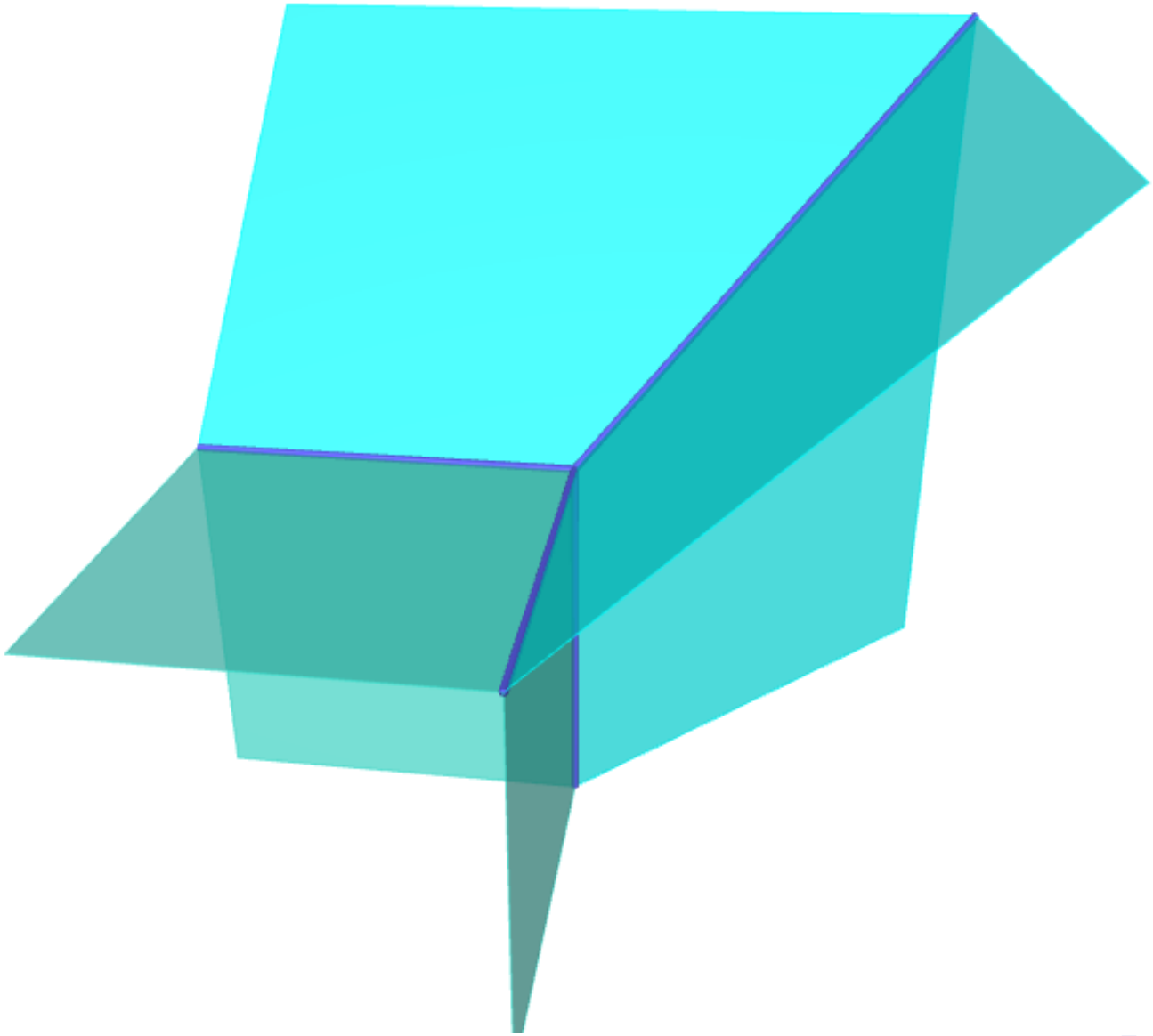
Another



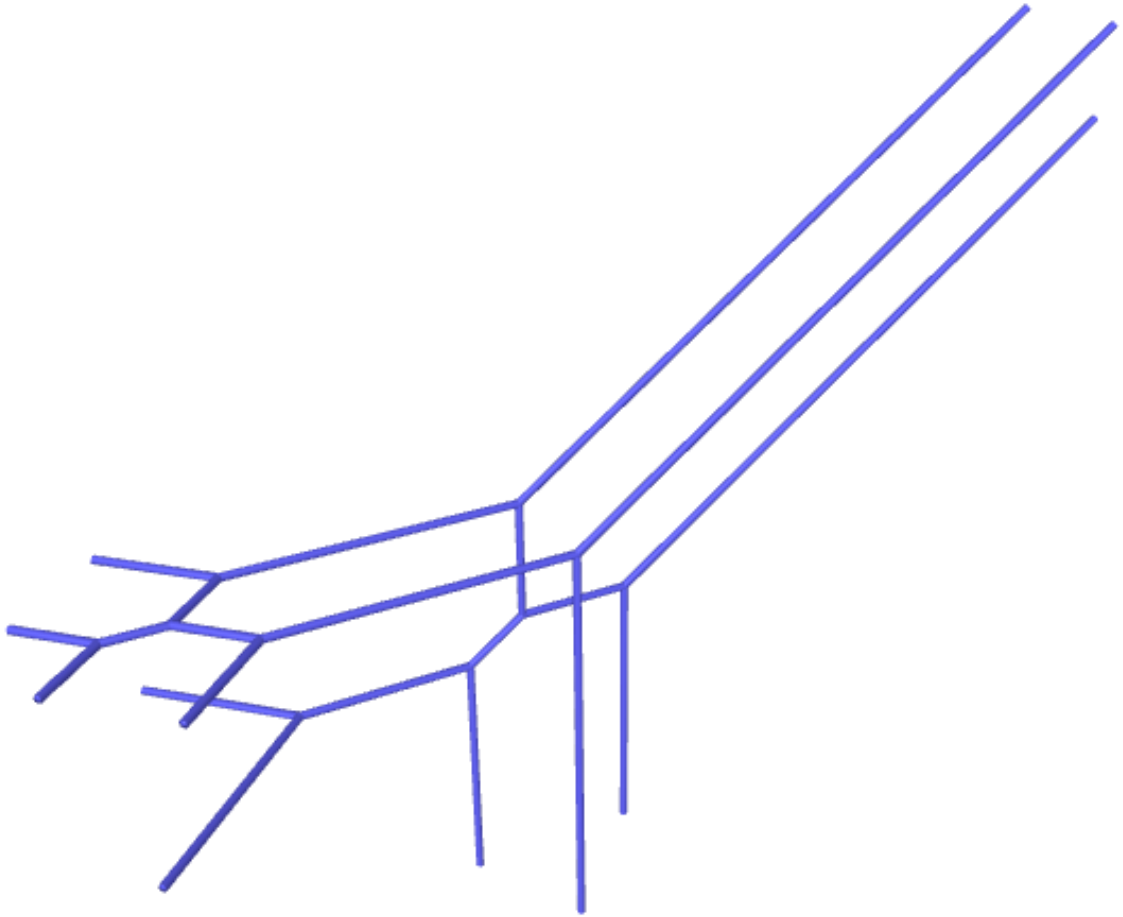
Four legs one vertex Line



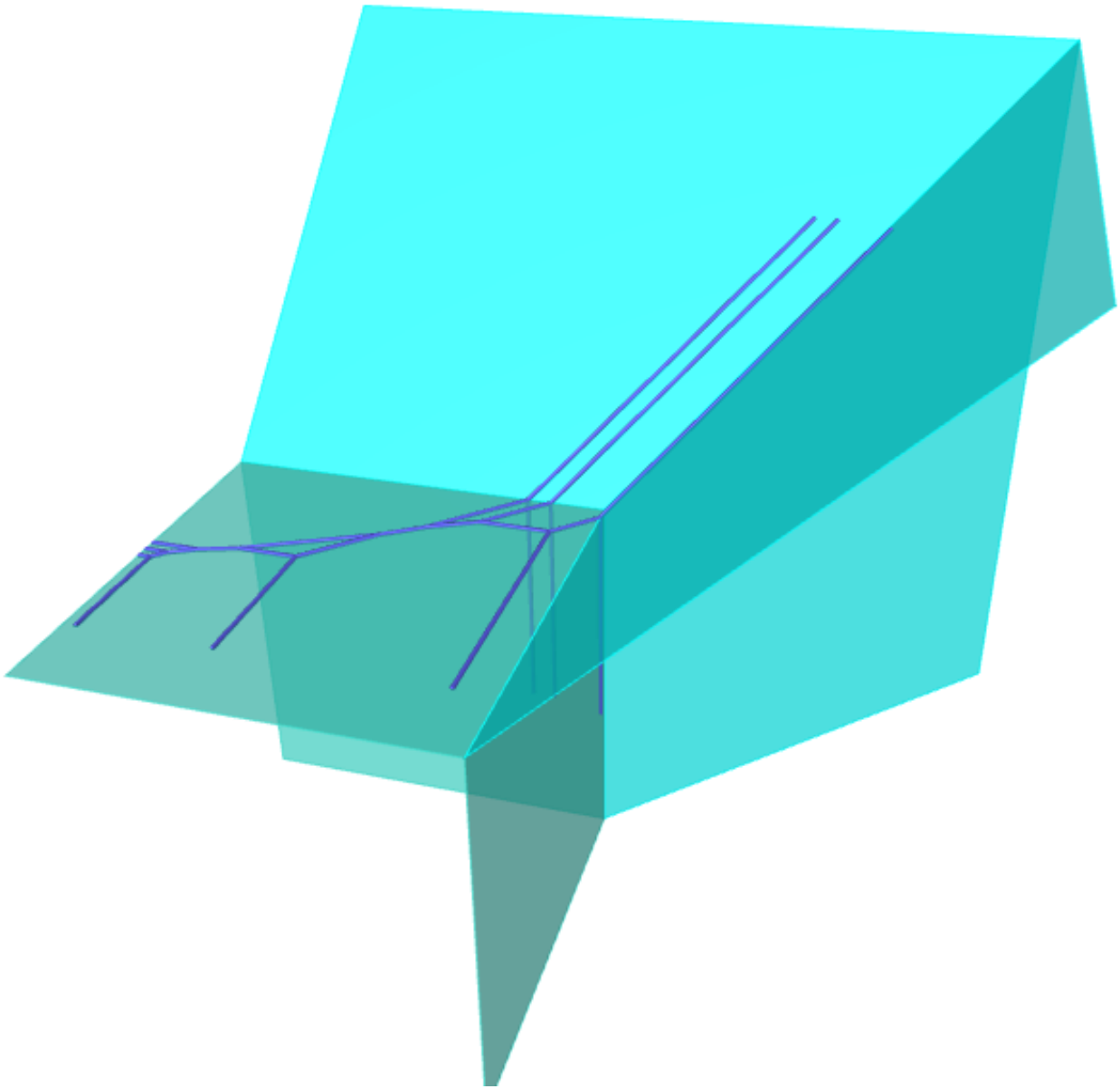
Four legs one vertex Line



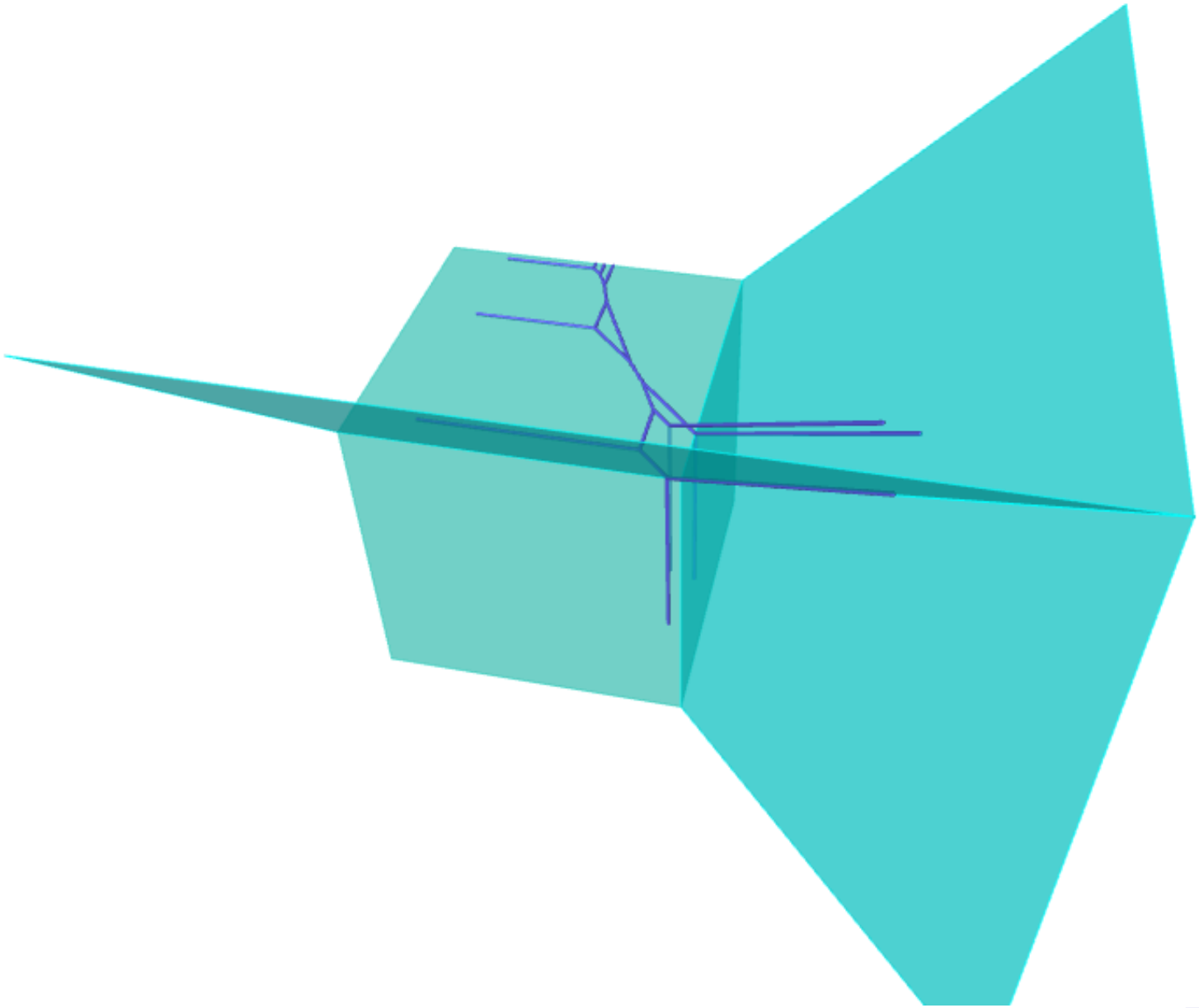
Rational Cubic



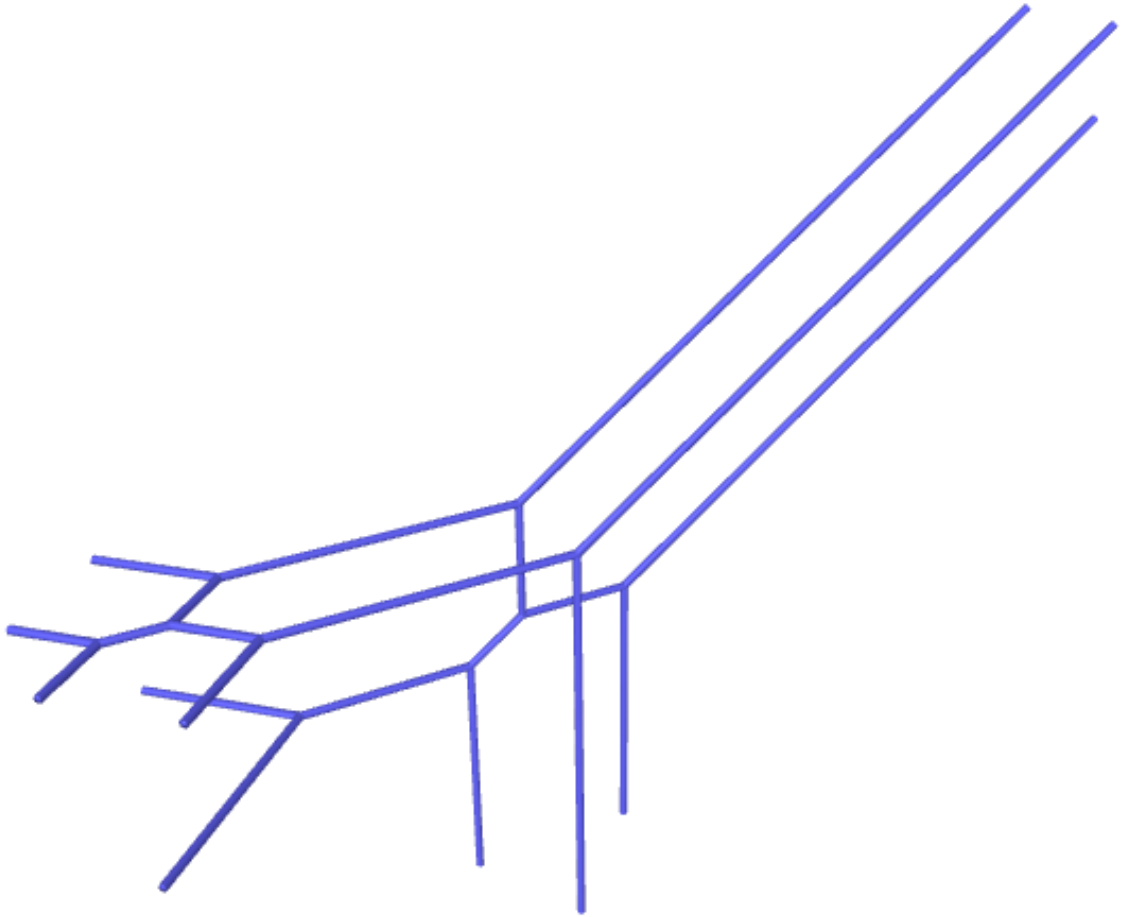
Genus 1 planar Cubic in $\mathbb{R}^3$



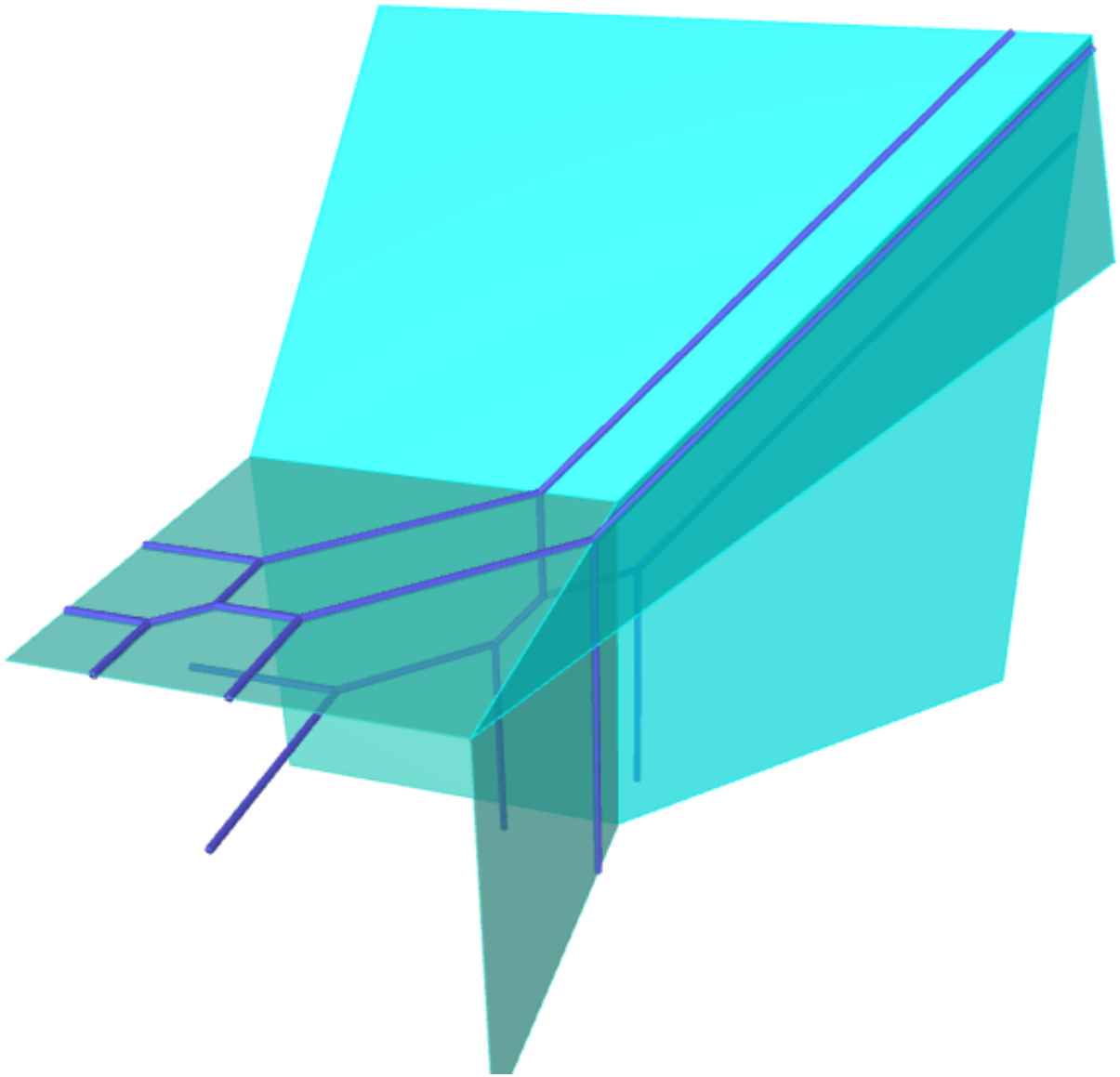
Genus 1 non-planar Cubic in $\mathbb{R}^3$



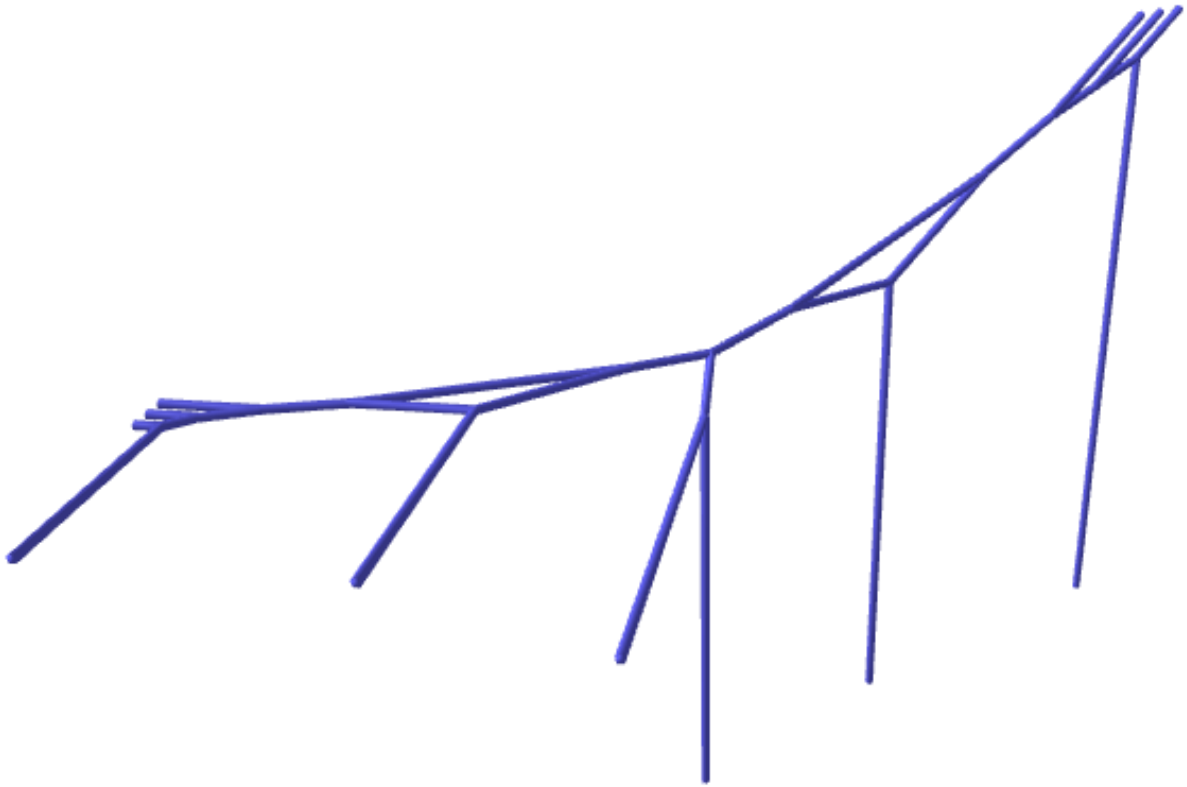
Rational Cubic



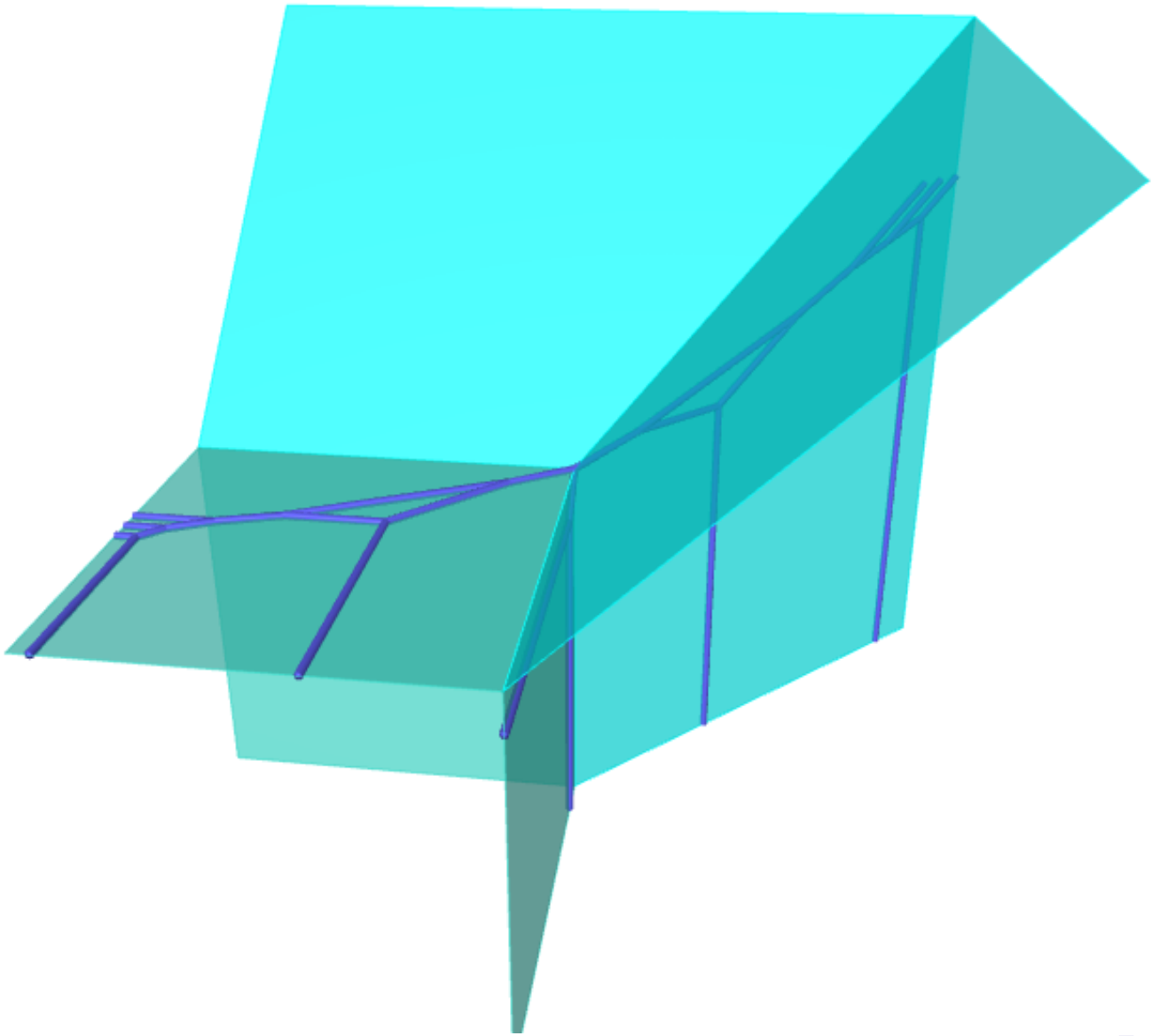
Rational Cubic in $\mathbb{R}^3$



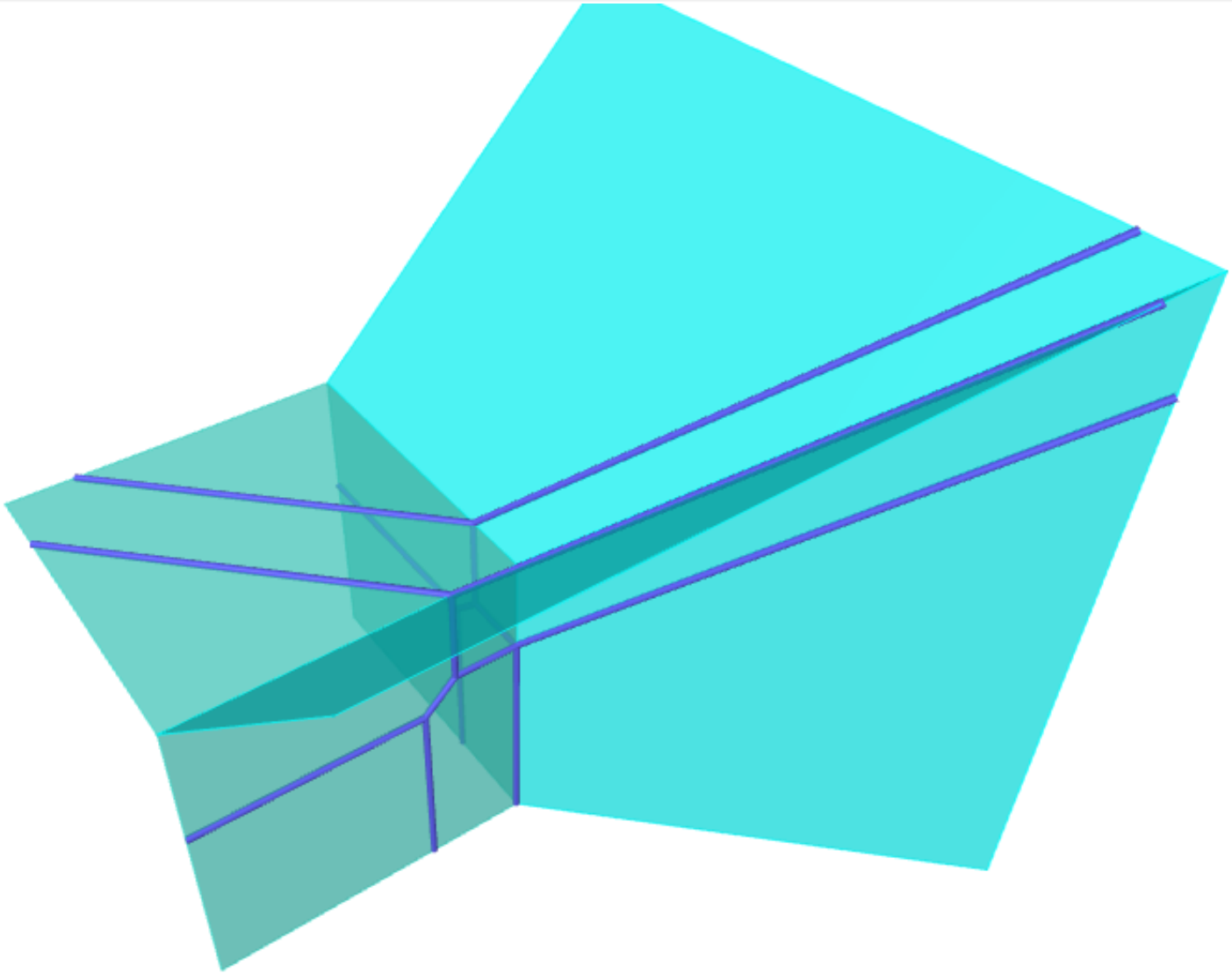
Genus 2



Genus 2 Planar CUBIC!



Genus "non-singular" O Planar Cubic



Genus −2 "non-singular" Planar Cubic