

Aspects symplectiques, réels et tropicaux de la géométrie énumérative

Rencontre du projet ANR ENUMGEOM

Abstracts

Omid Amini (Ecole Polytechnique)

Moduli of hybrid curves: higher rank inner products and metric degenerations of Jacobians.

I will discuss the asymptotic of complex geometry on Riemann surfaces close to the boundary of the Deligne-Mumford compactification. Part of this will be done by the introduction of inner product spaces of higher rank that allow to describe metric degenerations of polarized tori.

Based on joint works with Noema Nicolussi.

Benoît Bertrand (Institut de mathématiques de Toulouse)

Tropical cubics of any genus.

Joint work with E. Brugallé et L. López de Medrano.

I will explain how tropical cubics curves of any genus can be constructed in stark contrast with the complex algebraic world. Moreover, we also construct planar nonsingular tropical cubics of any genus. I'll try to shed light on this incongruity with some illustrating pictures.

Frédéric Bihan (Université Savoie Mont Blanc)

Optimal bounds for the number of connected components of fewnomial hypersurfaces.

During this talk I will present new bounds for the number of connected components of fewnomials hypersurfaces. When the codimension of the support is 2, the new bound is optimal. Key elements of the proof are A-discriminants, matroids and the Descartes bound for polynomial systems supported by a circuit.

This is a joint work with T. Humbert et S. Tavenas.

Baptiste Chantraine (Nantes Université)

Rigidité en géométrie localement conformément symplectique.

Dans cet exposé j'introduirais les notions de variétés localement conformément symplectique (lcs) et leur symétries. Bien étudiées du point de vue géométrique, peu de résultats rigides du point de vue topologique sont connus. Je montrerai dans cet exposé comment une construction de produit amalgamé (travail en collaboration avec Kevin Sackel), combiné à un résultat obtenu en collaboration avec Emmy Murphy, permet d'obtenir un lien entre orbites fermées de flot hamiltonien et le rang l'homologie de Novikov dans la classe de Lee d'une variété lcs. L'exposé sera introductif et je parlerai aussi des motivations à l'étude de ces structures venant de la géométrie complexe et de contact.

Alessandro Chiodo (IMJ-PRG)*Mumford's formula on the universal Picard stack.*

I will present work done in collaboration with David Holmes. We construct a derived pushforward of the r th root of the universal line bundle over the Picard stack of genus g prestable curves carrying a line bundle. We prove a number of basic properties, and give a formula in terms of standard tautological generators. After pullback, our formula recovers formulae of Mumford, of Pagani-Ricolfi-van Zelm and my Grothendieck-Riemann-Roch formula for r -spin curves. We apply these constructions to prove a conjecture expressing the double ramification cycle and several generalizations.

Sergey Finashin (Middle East Technical University, Ankara)*Topology of real lines on del Pezzo and elliptic surfaces.*

In our joint project with V.Kharlamov (being still in progress), we describe isotopy classes of real lines on real del Pezzo surfaces of degree 1 and use it to approach a similar task for real rational elliptic surface. In the latter case, studying of the action of real Mordell-Weil group is the key ingredient. In particular, I will discuss the difference between the algebraic and topological sections of real elliptic fibrations.

Thomas Guidoni (IMJ-PRG)*Intégrabilité des invariants de Gromov-Witten réels de \mathbb{P}^1 .*

Dans cet exposé, on présentera quelques résultats d'intégrabilité des invariants de Gromov-Witten réels de la sphère \mathbb{P}^1 .

Par exemple, la série génératrice des invariants stationnaires réels est une solution de la hiérarchie KdV. Ceci permet d'obtenir la formule explicite des invariants stationnaires réels de genre 0 de \mathbb{P}^1 .

On discutera aussi de l'analogie réel des contraintes de Virasoro. Ces contraintes de Virasoro réelles déterminent tous les invariants de Gromov-Witten réels de \mathbb{P}^1 à partir des invariants stationnaires réels.

Amanda Hirschi (University of Cambridge)*Global Kuranishi charts in symplectic GW theory.*

I will briefly describe the construction of a global Kuranishi chart for moduli space of pseudoholomorphic stable maps and how this can be generalised to the equivariant setting. I will state a virtual localisation formula in this setting, and if time permits, outline a comparison with the Ruan-Tian invariants if the manifold is semipositive. This is partially joint work with Mohan Swaminathan.

Viatcheslav Kharlamov (Université de Strasbourg)*On Smith deficiency of Hilbert squares.*

As it was observed in our recent joint work with Rares Rasdeaconu, it happens that starting from dimension 2 many of deformation classes of algebraic varieties do not contain any real variety whose Hilbert square is maximal. In this talk I intend to outline our latest resultss on this topic, especially in what concerns numerical Smith deficiency of the Hilbert square.

Frédéric Mangolte (Aix-Marseille Université)*Comessatti's Theorem on Rational Surfaces and Real Fano threefolds.*

From the classification of real rational surfaces worked out by Comessatti at the beginning of the 20th century we get the following striking characterization of real rational surfaces: a geometrically rational real surface is rational if and only if its real locus is non-empty and connected. In a work in progress with Andrea Fanelli, we explore real loci of geometrically rational Fano threefolds and study the rationality of these.

Grigory Mikhalkin (Université de Genève)*Tropical wave fronts and caustics.*

Based on the intrinsic geometric structure of the tropical plane, we define the evolution of planar convex domains and their boundaries (wave fronts). This evolution is subject to Huygens' principle. Interestingly enough, even if the boundary is smooth, the result of its evolution after an arbitrary small time is polygonal. Furthermore, it turns out that the caustic of this wave front propagation is a tropical curve itself. Joint work with Mikhail Shkolnikov.

Jean-Philippe Monnier (Université d'Angers)*The Hilbert 17th problem for a ring.*

Real algebra has been developed to answer Hilbert 17th problem: polynomials non negative on \mathbb{R}^n are exactly the sum of squares of rational functions. For polynomials in the coordinate ring of an irreducible affine algebraic variety V a difference appears that a sum of squares in the fraction field may be nonnegative only on the central locus of V (which consists in the euclidean closure of the non-singular points).

We deal with central algebra of a general ring A , studying two particular spaces of orderings : the central and the precentral loci living both in $\text{Spec}_r A$, the real spectrum of A . Central orderings are of topological nature whereas precentral orderings are defined by a natural algebraic condition. We study the differences between these spaces of orderings and prove that they give the smallest and largest subsets of the real spectrum that answer to an abstract Hilbert 17th problem.

This is a joint work with Goulwen Fichou and Ronan Quarez.

Alexandru Oancea (Université de Strasbourg)*Morse and Floer theory with DG coefficients.*

Building on a 2004 construction of Barraud-Cornea, I will explain how Morse and Floer theory can be formulated in order to accommodate differential graded local coefficients, also known under the name of infinity local systems. This construction allows, for example, to speak of and compute the homology with coefficients in chains on the fibers of a fibration over the base space. This is joint work with Jean-Francois Barraud, Mihai Damian and Vincent Humilire, partly accomplished and partly in progress. I will also discuss potential developments.

Nermin Salepci (Université Claude Bernard Lyon 1)*Morse shellings on finite simplicial complexes.*

(Joint work with Jean-Yves Welschinger)

Morse shellings generalize the classical notion, shellings, on simplicial complexes

and they are strongly related to discrete Morse functions introduced by R. Forman. In this talk, I will present some fundamental properties of Morse shellings and discuss their existence on triangulated closed manifolds.

Antoine Toussaint (IMJ-PRG)

Real structures of phase tropical surfaces.

Phase tropical surfaces can appear as a limit of a 1-parameter family of smooth complex algebraic surfaces. A phase tropical surface admits a stratified fibration over a smooth tropical surface. We study the real structures compatible with this fibration and give a description in terms of tropical cohomology. As an application, we deduce combinatorial criteria for the type of a real structure of a phase tropical surface.

Xiaohan Yan (IMJ-PRG)

Divisor equations in quantum K-theory.

Quantum K-theory studies a K-theoretic analogue of the Gromov-Witten invariants, defined as holomorphic Euler characteristics over the moduli spaces of stable maps. The K-theoretic GW invariants satisfy similar axioms as the cohomological ones do, except the divisor equation. In genus-zero case, substitutes of the divisor equation in quantum K-theory have been proposed through the idea of toric fixed point localization and that of the so-called adelic characterization, both leading to interesting applications. For instance, explicit formulas of the K-theoretic J-functions of flag varieties may thus be justified, and the phenomenon named the quantum Serre duality may be demonstrated. Higher-genus substitutes of the divisor equation remain however largely unknown.

Dimitri Zvonkine (Laboratoire de Mathématiques de Versailles)

Quantum Hall effect via the Grothendieck-Riemann-Roch formula.

We will explain how the fractional quantum Hall effect on a Riemann surface C of genus g can be studied using algebraic geometry. Mathematically, the goal is to compute the characteristic classes of a vector bundle over the Jacobian $\text{Jac}(C)$ of the surface, obtained as a pushforward of a line bundle on a large symmetric power of C . The computation is performed via the Grothendieck-Riemann-Roch formula. Physically, the vector bundle in question, called the Laughlin vector bundles is the vector bundle of ground states of a quantum system.

This is joint work with Semyon Klevtsov.