TP 3 : Intégration

```
> Int(exp(-x^2), x=-infinity..infinity)=int(exp(-x^2),
  x=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} \mathbf{e}^{(-x^2)}\, dx = \sqrt{\pi}$$

```
> Int(sin(x)/x, x=-infinity..infinity)=int(sin(x)/x, x=-infinity..infinity);
```

$$\int_{-\infty}^{\infty} \frac{\sin(x)}{x}\, dx = \pi$$

```
> Gamma:=x->int(t^(x-1)*exp(-t), t=0..infinity);
  'Gamma'(x)=series(Gamma(x), x=0, 1);
  'Gamma'(x)=series(Gamma(x), x=infinity, 1);
```

$$\Gamma(x) = x^{-1} - \gamma + \mathrm{O}(x)$$

$$\Gamma(x) = \frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{\frac{1}{x}} + \mathrm{O}\left(\left(\frac{1}{x}\right)^{(3/2)}\right)}{\left(\frac{1}{x}\right)^{x} \mathbf{e}^{x}}$$

```
> f:=x->(a*x^8+b*x^7+c)/((x-1)^3*(x+2)^5);
```

```
> decomp:=convert(f(x), parfrac, x):
  equas:=coeff(decomp, 1/(x-1))=0, coeff(decomp, 1/(x+2))=0;
  sol:=solve({equas}, {a,b,c});
  primitive:=normal(int(subs(sol,f(x)),x));
```

$$equas := \frac{49}{729}\,a + \frac{11}{243}\,b + \frac{5}{729}\,c = 0, -\frac{5152}{729}\,a + \frac{232}{243}\,b - \frac{5}{729}\,c = 0$$

$$sol := \{ c = -56\,a, a = a, b = 7\,a \}$$

$$primitive := \frac{a\,(54 + 28\,x - 74\,x^2 + 19\,x^4 + x^7 + 6\,x^6 - 48\,x^3 + 22\,x^5)}{(x-1)^2\,(x+2)^4}$$

```
> quad1:=(f, a, b, N)->evalf((b-a)/N*sum(f(a+k*(b-a)/N), k=0..N-1));
  quad2:=(f, a, b, N)->evalf((b-a)/N*sum(f(a+(k+1/2)*(b-a)/N), k=0..N-1));
  quad1(exp, 0, 1, 50),quad2(exp, 0, 1, 50),exp(1.)-1;
```

$$1.718281828, 1.701156286, 1.718253193$$

```
> with(plots):
  p1:=loglogplot([[N^2, abs('quad1'(exp, 0, 1, N^2)+1-exp(1))] $ N=1..50]):
  p2:=loglogplot([[N^2, abs('quad2'(exp, 0, 1, N^2)+1-exp(1))] $ N=1..50]):
  display([p1, p2]);
```

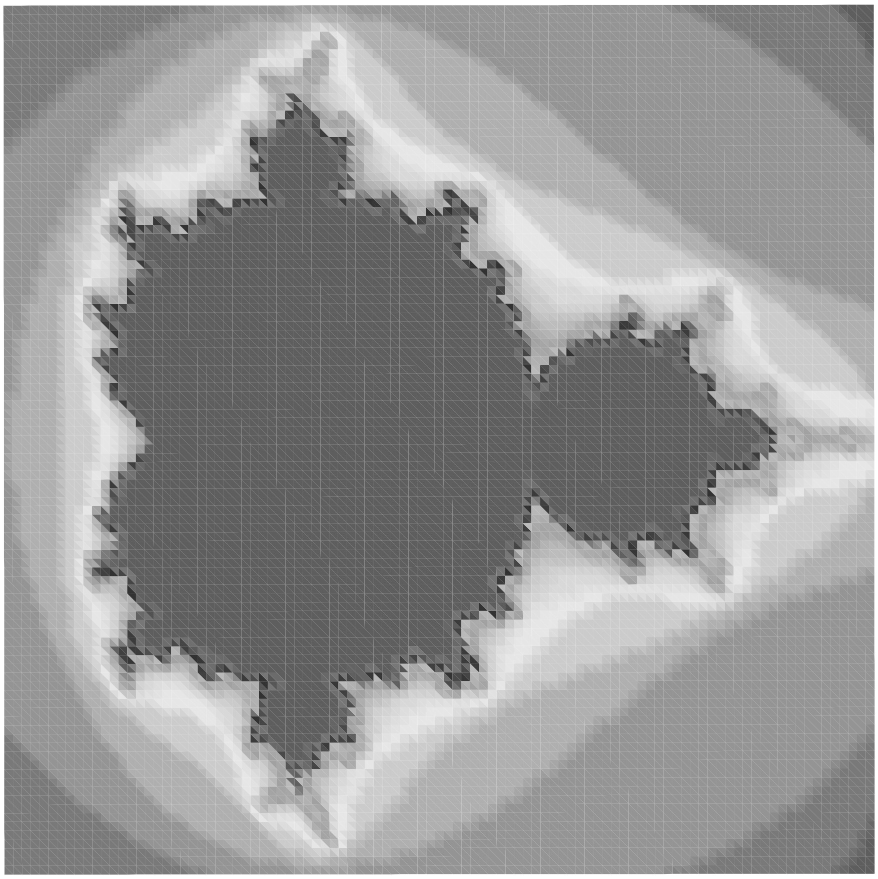
```
> mandel:=proc(x, y)
    local k, u, z;
    z:=evalf(x+I*y);
    u:=z;
    for k from 1 to 30 while abs(u)<2 do
```

```
      u:=u^2-z;
    od;
    k;
  end;
```

mandel :=

proc(x, y) **local** k, u, z; z := evalf($x + I*y$); u := z; **for** k **to** 30 **while** abs(u) < 2 **do** u := u^2 − z **od**; k **end**

```
> plot3d(0, -0.7 .. 1.7, -1.2 .. 1.2, orientation=[-90,0],
  grid=[100, 100], style=patchnogrid, color=mandel);
```



```
>
```