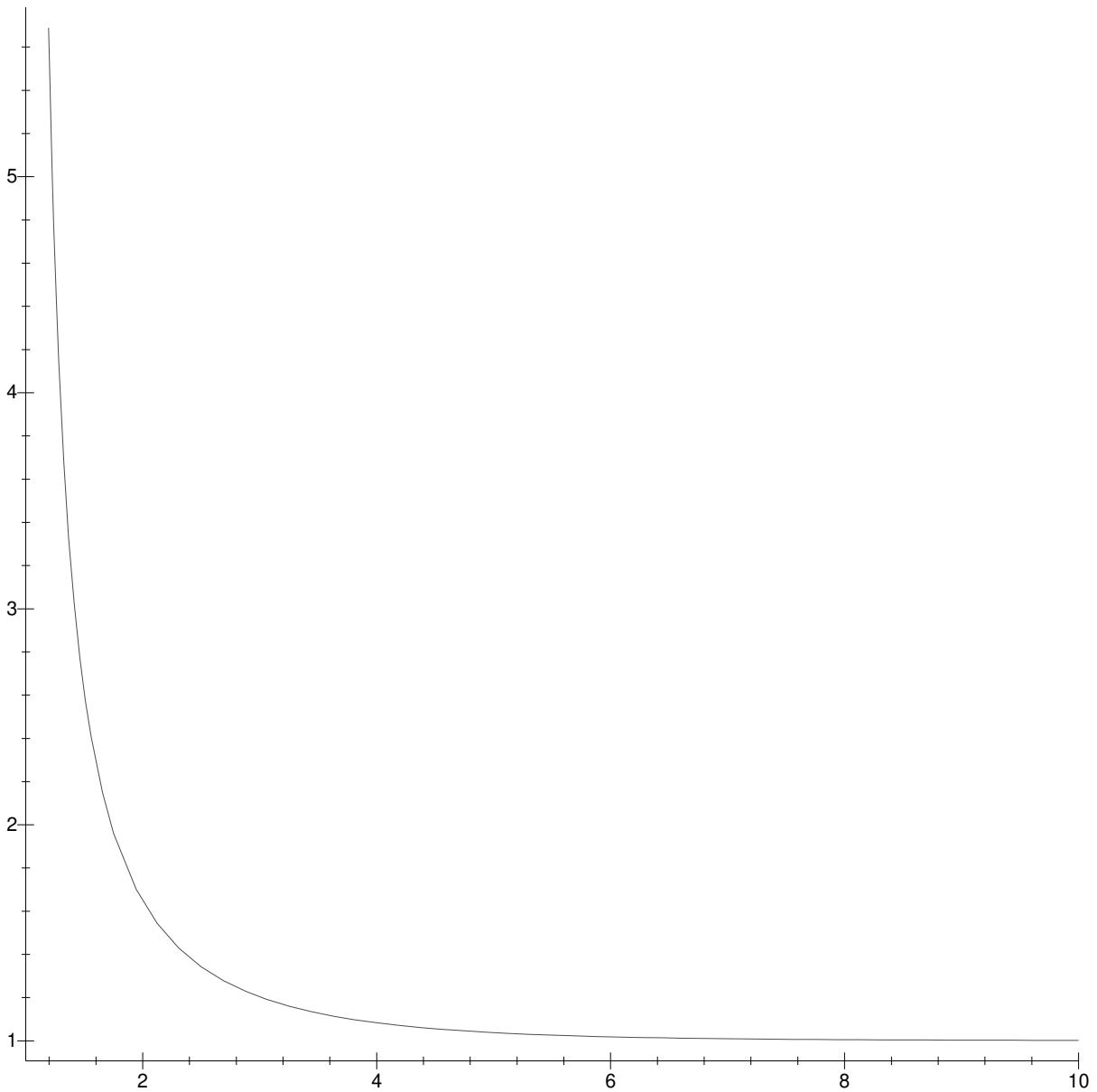
TD 5 : Series numeriques

```
> Sum(1/k^2, k=1..infinity)=sum(1/k^2, k=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{6}\pi^2$$

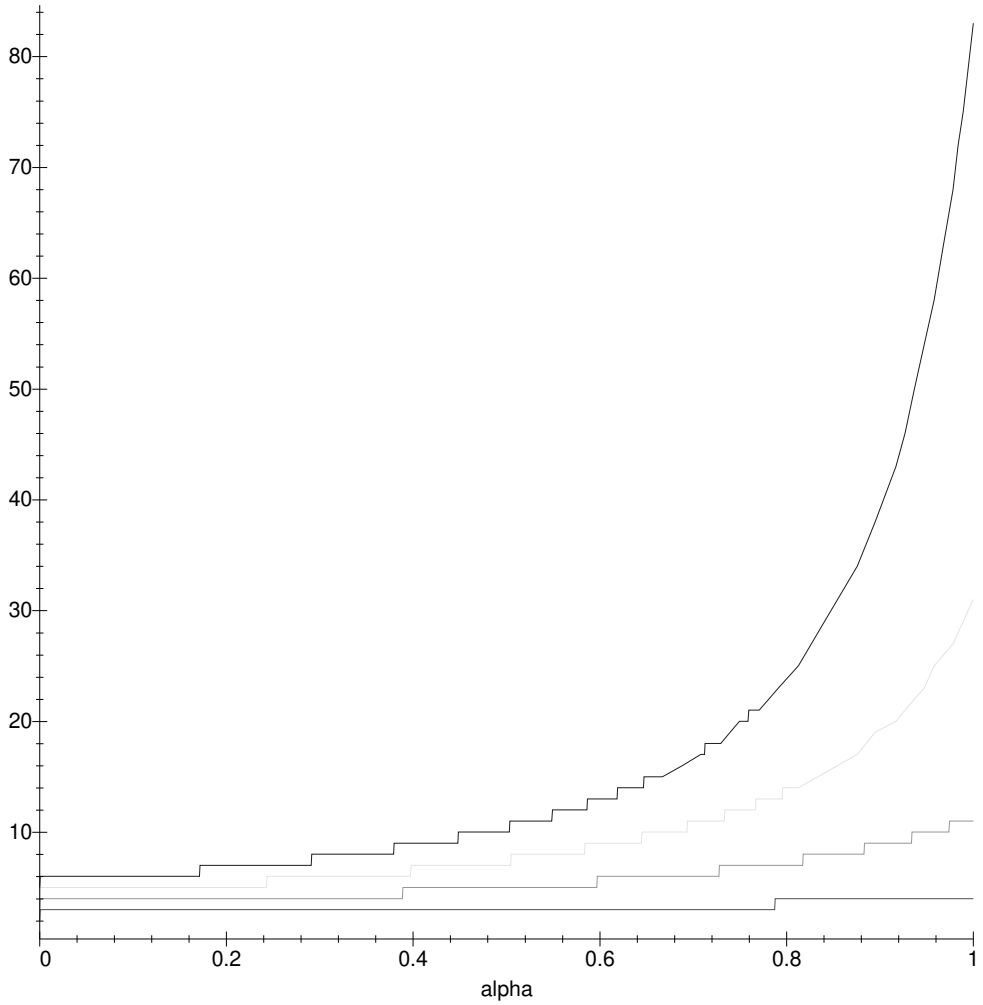
```
> plot(Zeta, 1..10);
```



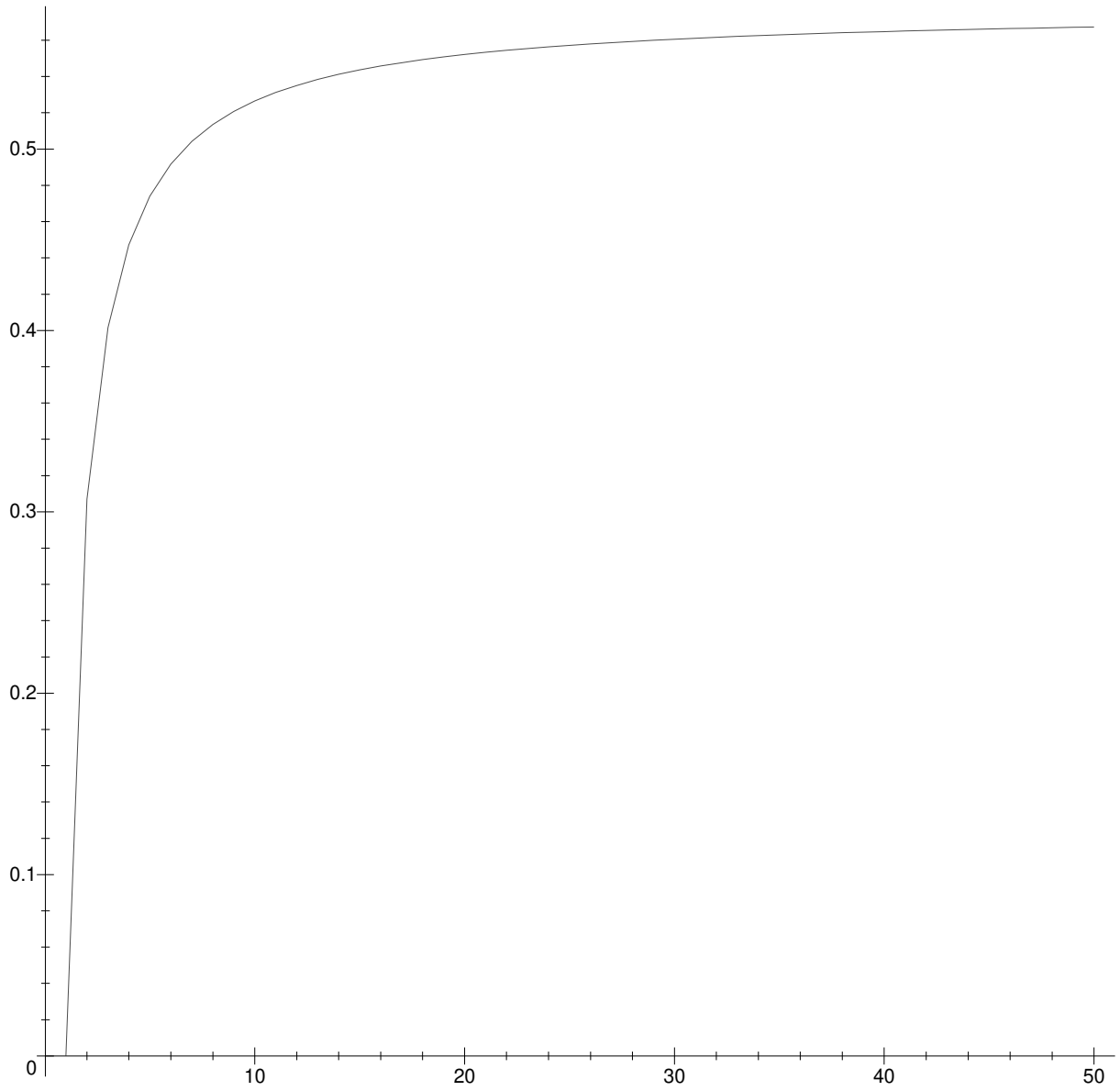

```
> nombre:=proc(A: realcons, alpha: realcons)
    local n, s;
    if (evalf(alpha<0)) or (evalf(alpha)>1) then ERROR(`alpha doit être
  compris entre 0 et 1 !`); fi;
    n:=1; s:=0;
    while s<evalf(A) do
      s:=s+evalf(1/n^alpha);
      n:=n+1;
    od;
    n-1;
  end;
```

```
nombre := proc(A::realcons, α::realcons)
local n, s;
    if evalf(α < 0) or 1 < evalf(α) then ERROR( 'alpha doit être compris entre 0 et 1 !'  ) fi;
    n := 1;
    s := 0;
    while s < evalf(A) do s := s + evalf(1 / n^α); n := n + 1 od;
    n − 1
end
```

```
> plot([seq('nombre'(A, alpha),A=2..5)], alpha=0..1);
```




```
> plot([[n, sum(1/k, k=1..n-1)-ln(n)] $n=1..50]);
  asympt(sum(1/k, k=1..n-1)-ln(n), n, 3);
```



$$\gamma - \frac{1}{2}\frac{1}{n} - \frac{1}{12}\frac{1}{n^2} + \mathrm{O}\left(\frac{1}{n^4}\right)$$

```
> S:=sum(ln(k+n), k=1..n);
  'S'=expand(asympt(S, n, 3));
```

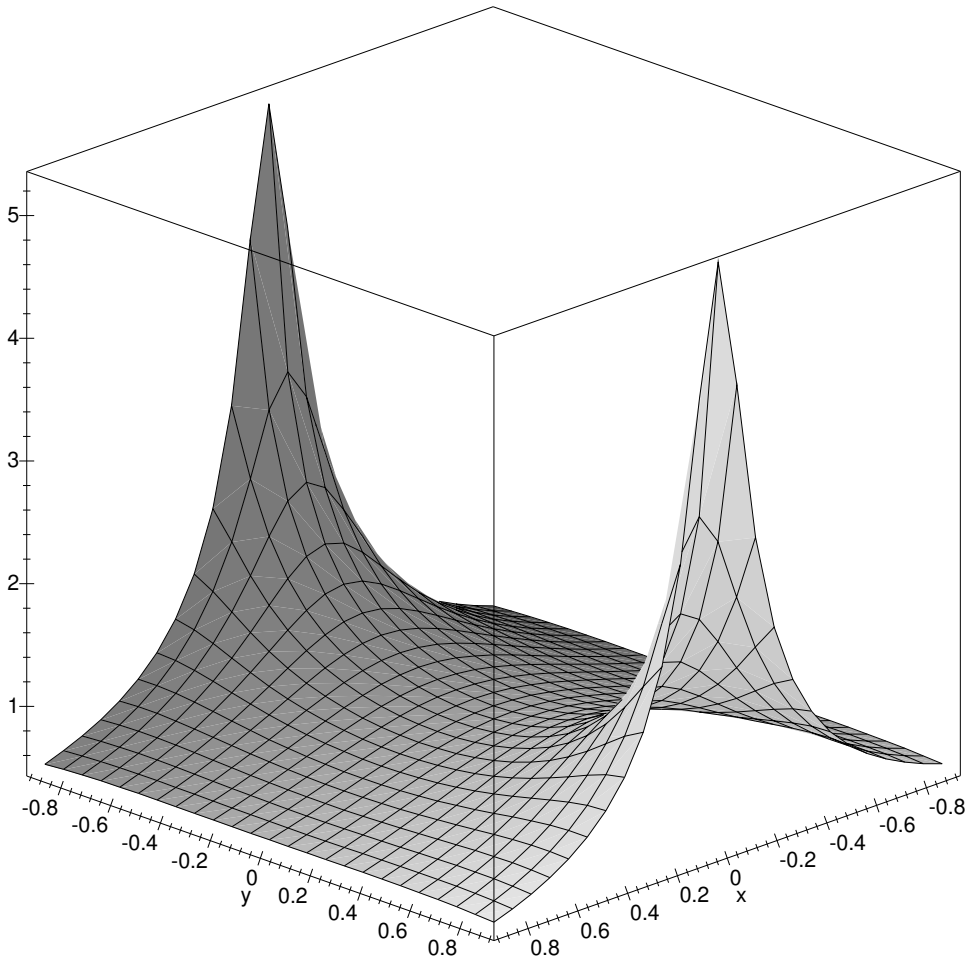
$$S := \ln(\Gamma(2n+1)) - \ln(\Gamma(n+1))$$

$$S = n\ln(n) - n + n\ln(4) + \frac{1}{2}\ln(2) + \mathrm{O}\left(\frac{1}{n}\right)$$

```
> S1:=sum(z^k, k=0..infinity);
  S2:=sum((-1)^k*z^(2*k), k=0..infinity);
  plot3d((abs(1/(1+(x+I*y)^2))), x=-0.9..0.9, y=-0.9..0.9);
```

$$S1 := -\frac{1}{z-1}$$

$$S2 := \frac{1}{z^2+1}$$




```
> with(linalg);
  P:=matrix(3, 3, [1, 1, 0, 1, -1, 0, 0, 0, 1]);
  Di:=matrix(3, 3, [1, 0, 0, 0, 2, 0, 0, 0, 3]);
  A:=evalm(P &* Di &* inverse(P));
  Sum((A &^ k)/k!, k=0..infinity)=exponential(A);
  'exponential(A)'='P' &* 'exponential(Di)' &* 'inverse(P)';
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stack*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*]

$$P := \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Di := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A := \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{-1}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\sum_{k=0}^{\infty} \frac{A \,\&\hat{}\, k}{k!} = \begin{bmatrix} \frac{1}{2}\mathbf{e} + \frac{1}{2}\mathbf{e}^2 & -\frac{1}{2}\mathbf{e}^2 + \frac{1}{2}\mathbf{e} & 0 \\ -\frac{1}{2}\mathbf{e}^2 + \frac{1}{2}\mathbf{e} & \frac{1}{2}\mathbf{e} + \frac{1}{2}\mathbf{e}^2 & 0 \\ 0 & 0 & \mathbf{e}^3 \end{bmatrix}$$

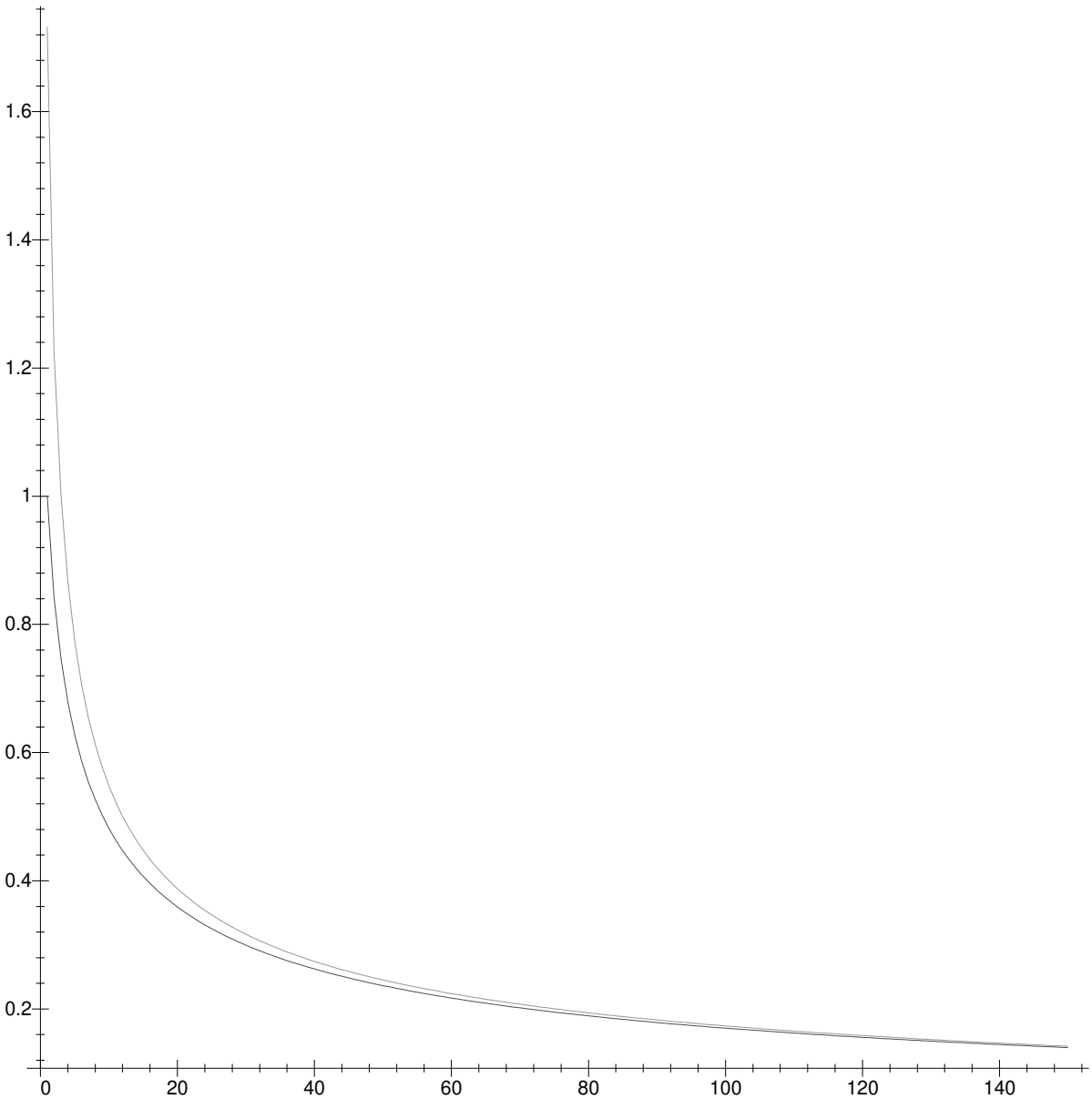
$$\text{exponential}(A) = P \,\&*\, \text{exponential}(Di) \,\&*\, \text{inverse}(P)$$

```
> u0:=Pi/2;
  u:=proc(n: integer)
  option remember;
  if(n=0) then u0; else sin(u(n-1)); fi;
  end;
```

$$u0 := \frac{1}{2}\pi$$

$$u := \mathbf{proc}(n::integer)\ \mathbf{option}\ remember;\ \mathbf{if}\ n = 0\ \mathbf{then}\ u0\ \mathbf{else}\ \sin(\mathrm{u}(n-1))\ \mathbf{fi}\ \mathbf{end}$$

```
> plot([[n, 'u(n)']$ n=1..150], [[n, sqrt(3/n)] $ n=1..150]]);
```


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
20
40
60
80
100
120
140