TD 7 : Powseries, Von Koch

```
> with(powseries):
> f:=evalpow(sin(x)/(1+tan(x))): tpsform(f,x,5);
  g:=compose(f, f): tpsform(g, x, 5);
```

$$x - x^2 + \frac{5}{6}x^3 - \frac{7}{6}x^4 + O(x^5)$$

$$x - 2x^2 + \frac{11}{3}x^3 - \frac{15}{2}x^4 + O(x^5)$$

```
> a:=powsolve((1+x^2)*diff(y(x),x)=x*y(x),y(0)=1):
  tpsform(a, x, 10);
  s:=rhs(dsolve({(1+x^2)*diff(y(x),x)=x*y(x),y(0)=1}, y(x)));
  series(s,x=0,9);
```

$$1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 - \frac{5}{128}x^8 + O(x^{10})$$

$$s := \sqrt{1+x^2}$$

$$1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 - \frac{5}{128}x^8 + O(x^{10})$$

```
> c:='c':
  powcreate(c(n)=-2*c(n-1)/n, c(0)=1);
  tpsform(c, x, 10);
  evalpow(exp(-2*x)): 'exp(-2*x)'=tpsform(", x, 10);
```

$$1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \frac{4}{45}x^6 - \frac{8}{315}x^7 + \frac{2}{315}x^8 - \frac{4}{2835}x^9 + O(x^{10})$$

$$\mathbf{e}^{(-2x)} = 1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \frac{4}{45}x^6 - \frac{8}{315}x^7 + \frac{2}{315}x^8 - \frac{4}{2835}x^9 + O(x^{10})$$

```
> c:='c':
  powcreate(c(n)=-2*c(n-1)*(n-1)/n, c(0)=0, c(1)=2);
  tpsform(c, x, 10);
  evalpow(ln(1+2*x)): 'ln(1+2*x)'=tpsform(", x, 10);
```

$$2x - 2x^2 + \frac{8}{3}x^3 - 4x^4 + \frac{32}{5}x^5 - \frac{32}{3}x^6 + \frac{128}{7}x^7 - 32x^8 + \frac{512}{9}x^9 + O(x^{10})$$

$$\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 - 4x^4 + \frac{32}{5}x^5 - \frac{32}{3}x^6 + \frac{128}{7}x^7 - 32x^8 + \frac{512}{9}x^9 + O(x^{10})$$

```
> with(plots): J:=exp(2*I*Pi/3):
> transfo_segment:=proc(p1,p2) (2*p1+p2)/3, (p1+p2)/2+I*sqrt(3)/6*(p2-p1),
  (p1+2*p2)/3, p2;end;
```

transfo_segment :=

proc(*p1*, *p2*) 2 / 3**p1* + 1 / 3**p2*, 1 / 2**p1* + 1 / 2**p2* + 1 / 6**I**sqrt(3)*(*p2* − *p1*), 1 / 3**p1* + 2 / 3**p2*, *p2* **end**

```
> transfo_courbe:=proc(c)
    local i, res;
    res:=c[1]:
    for i from 1 to nops(c)-1 do
      res:=res, transfo_segment(c[i], c[i+1]):
    od:
  [res]:
  end;
```

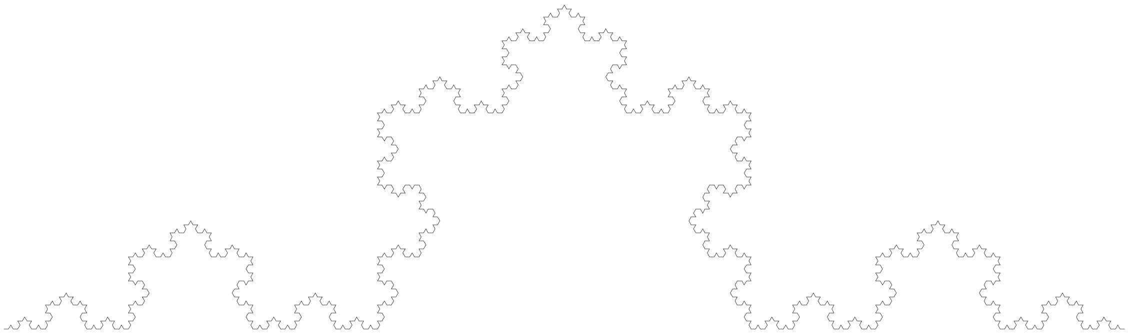
transfo_courbe := **proc**(*c*)

local *i*, *res*;

$res := c[1]$; **for** i **to** nops$(c) - 1$ **do** $res := res$, transfo_segment$(c[i], c[i+1])$ **od**; $[res]$

end

```
> courbe1:=[0.0, 1.0]:
  for i from 1 to 5 do courbe1:=transfo_courbe(courbe1): od:
  complexplot(courbe1, axes=NONE, scaling=CONSTRAINED);
```



```
> courbe2:=[1, J^2, J, 1]:
  for i from 1 to 4 do courbe2:=transfo_courbe(courbe2): od:
  complexplot(courbe2, axes=NONE, scaling=CONSTRAINED);
```