TD 1 : interpolation de Lagrange et phénomène de Runge

```
> produit:=proc(l)
  # calcule le produit des éléments de la liste l

    local i, total;
    total:=1;
    for i from 1 to nops(l)
    do
      total:=total*l[i];
    od;
    total;
    end;
```

produit := **proc**(*l*) **local** *i*, *total*; *total* := 1; **for** *i* **to** nops(*l*) **do** *total* := *total*∗*l*[*i*] **od**; *total* **end**

```
> interpole:=proc(lx, ly)
  # calcule le polynôme d'interpolation des points (lx(k), ly(k))

    local K, L, res, i, Q;
    res:=0; Q:=produit(map(x->X-x, lx));
    for i from 1 to nops(lx)
    do
      K:=Q/(X-lx[i]);
      L:=ly[i]*K/subs(X=lx[i], K);
      res:=res+L;
    od;
    expand(res);
  end;
```

interpole := **proc**(*lx*, *ly*)

local *K*, *L*, *res*, *i*, *Q*;

res := 0;

Q := produit(map($x \to X - x$, *lx*));

for *i* **to** nops(*lx*) **do** *K* := *Q* / (*X* − *lx*[*i*]); *L* := *ly*[*i*]∗*K* / subs(*X* = *lx*[*i*], *K*); *res* := *res* + *L*

od;

expand(*res*)

end

```
> x_uniforme:=proc(x_min, x_max, N)
  # retourne N+1 points uniformément répartis entre x_min et
  x_max

    local k, step;
    step:=(x_max-x_min)/(N);
    [seq(x_min+k*step,  k=0..N)];
  end;
```

x_uniforme := **proc**(*x_min*, *x_max*, *N*)

local *k*, *step*;

```
    step := (x_max - x_min) / N; [seq(x_min + k*step, k = 0 .. N)]
end
```

```
> x_chebishev:=proc(x_min, x_max, N)
  # retourne N+1 points entre x_min et x_max selon la distribution
  de Chebishev

    local k, step;
    [seq(evalf((x_min+x_max)/2+(x_max-x_min)/2*cos(k*Pi/N)),
  k=0..N)];
  end;
```

```
x_chebishev := proc(x_min, x_max, N)
local k, step;
    [
    seq(evalf(1 / 2*x_min + 1 / 2*x_max + 1 / 2*(x_max - x_min)*cos(k*π / N)), k = 0 .. N)
    ]
end
```

```
> runge:=x->1/(1+16*x^2);
```

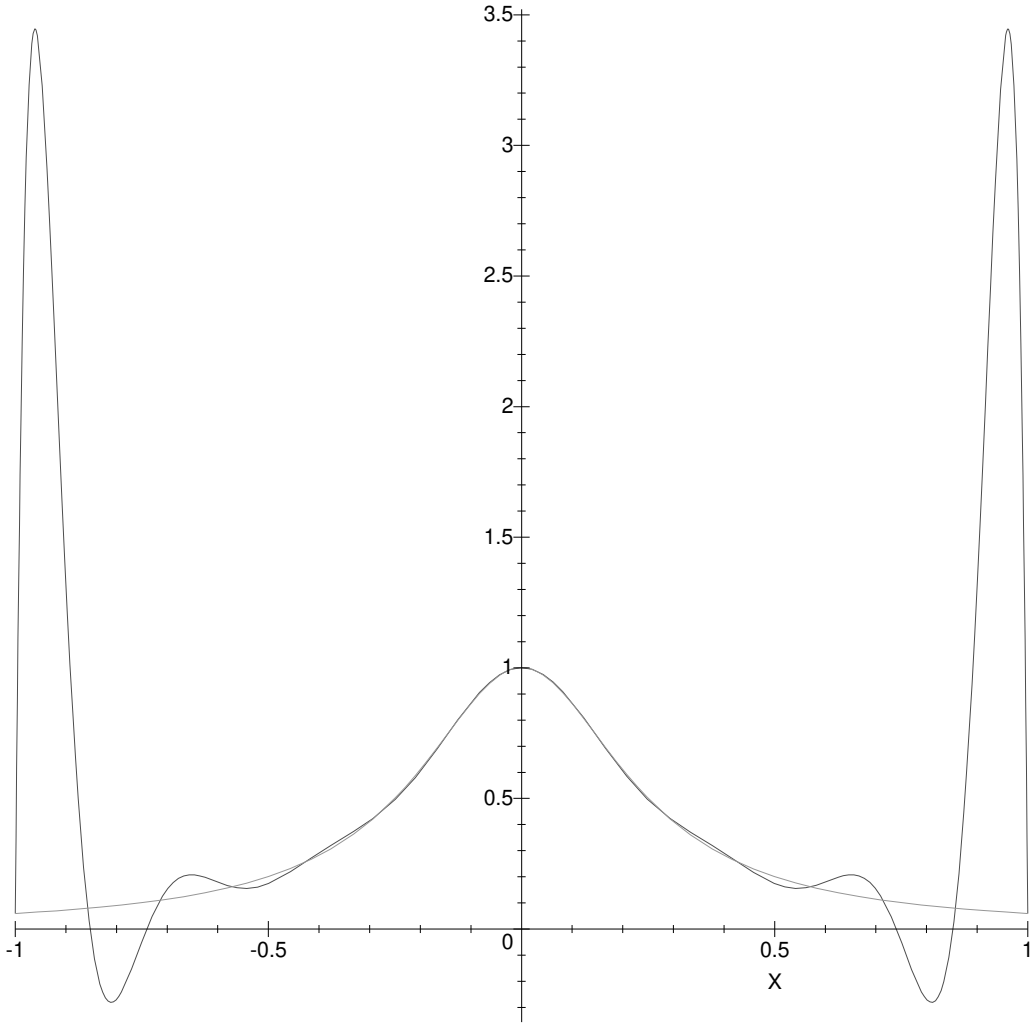
$$runge := x \to \frac{1}{1 + 16\,x^2}$$

```
> phenomene:=proc(N)
  # montre le phénomène de Runge avec N+1 points d'interpolation

    local lx, ly, P;
    lx:=x_uniforme(-1,1,N);
    ly:=map(runge, lx);
    P:=interpole(lx, ly);
    plot([P, runge(X)], X=-1 .. 1);
  end;
```

```
phenomene := proc(N)
local lx, ly, P;
    lx := x_uniforme(-1, 1, N);
    ly := map(runge, lx);
    P := interpole(lx, ly);
    plot([P, runge(X)], X = -1 .. 1)
end
```

```
> phenomene(14);
```



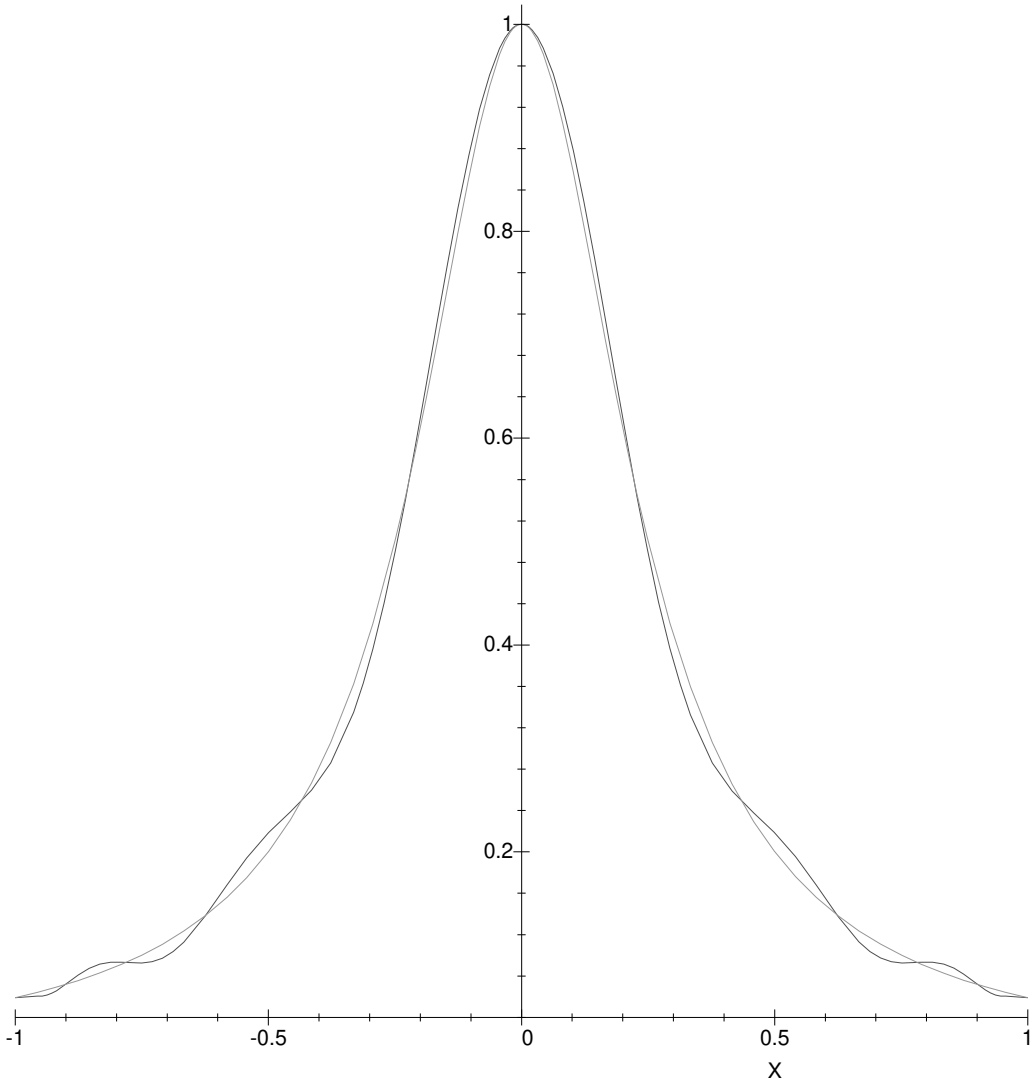

```
> solution:=proc(N)
  # montre que la distribution de Chebishev évite le phénomène de
  Runge

    local lx, ly, P;
    lx:=x_chebishev(-1,1,N);
    ly:=map(runge, lx);
    P:=interpole(lx, ly);
    plot([P, runge(X)], X=-1 .. 1);
  end;
```

solution := **proc**(*N*)

local *lx*, *ly*, *P*;

lx := x_chebishev(-1, 1, *N*);

ly := map(*runge*, *lx*);

```
    P := interpole(lx, ly);
    plot([P, runge(X)], X = -1 .. 1)
end
> solution(14);
```