TD 3 : Dérivation, développements limités et intégration

Les exercices de familiarisation ne sont pas corrigés.

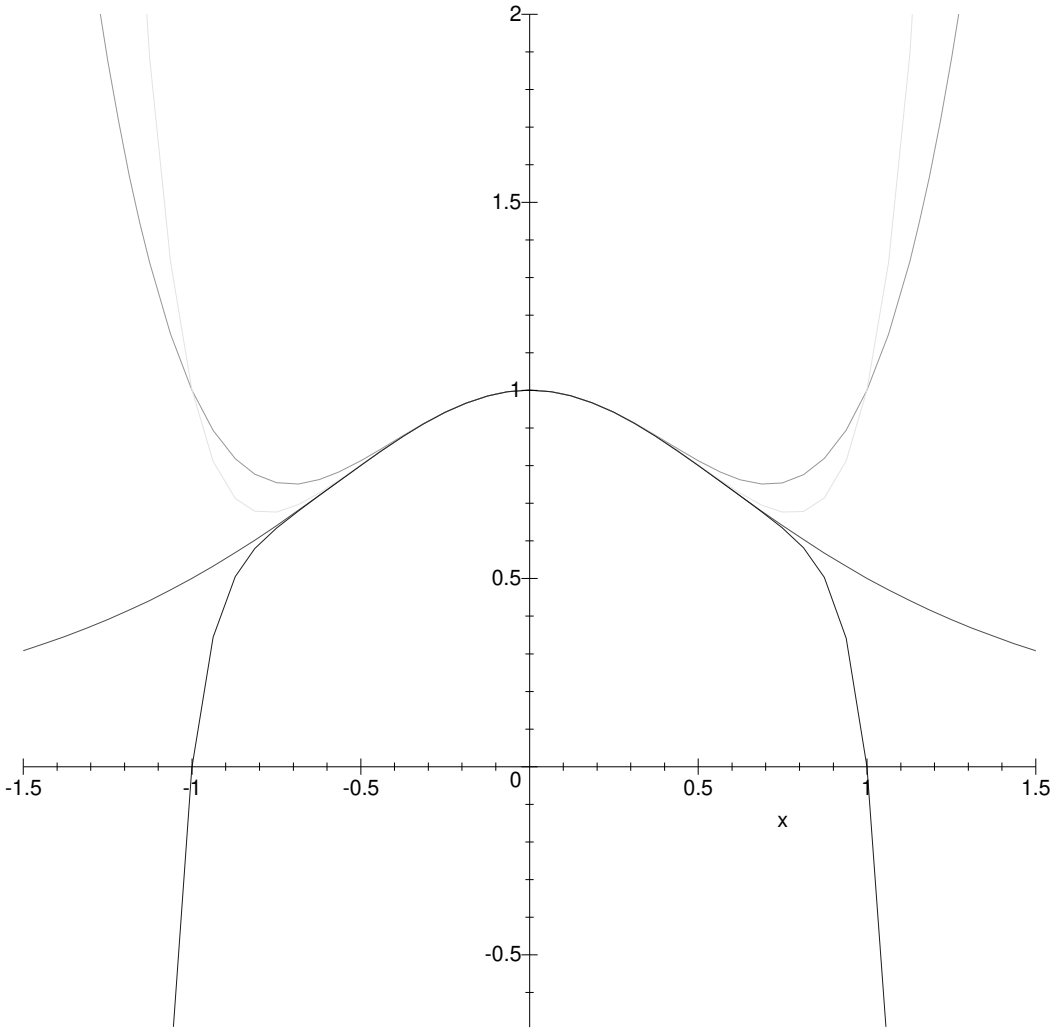
```
> f:=x->x^x;
  'f(0)'=limit(f(x), x=0);
  'df0'=limit((f(x)-1)/x, x=0);
  plot(f, 0..2):
```

$$f := x \to x^x$$

$$f(0) = 1$$

$$df0 = -\infty$$

```
> showtaylor:=proc(f, x0, xmin, xmax)
    local g1, g2, g3, ymin, ymax;
    g1:=convert(taylor(f(x), x=x0, 5), polynom);
    g2:=convert(taylor(f(x), x=x0, 10), polynom);
    g3:=convert(taylor(f(x), x=x0, 15), polynom);
    ymin:=minimize(f(x), x, evalf(xmin)..evalf(xmax));
    ymax:=maximize(f(x), x, evalf(xmin)..evalf(xmax));
    plot([f(x), g1, g2, g3] , x=xmin..xmax, ymin-1..ymax+1);
    end:
> showtaylor(x->x^11, 0.5, -1, 1):
> showtaylor(x->sin(x), 0, -2*Pi, 2*Pi):
> showtaylor(x->1/(1+x^2), 0, -1.5, 1.5);
```




```
> showtaylor(x->exp(-1/x^2), 0, -1.5, 1.5):
Error, (in series/exp) unable to compute series
> plot({exp(-1/x^2), 0}, x=-1..1):
> integrale1:=proc(f, a, b, N)
    local h, x, res;
    h:=(b-a)/N;
    res:=0;
    for x from a by h to b-h do
      res:=res+h*f(x);
    od;
    res;
  end:
> integrale2:=proc(f, a, b, N)
    local h, x, res;
    h:=(b-a)/N;
```

```
    res:=0;
    for x from a by h to b-h do
      res:=res+h*f(x+h/2);
    od;
    res;
  end:
> evalf(integrale1(x->exp(x), 0, 1, 10));
  evalf(integrale2(x->exp(x), 0, 1, 10));
  evalf(exp(1)-1);
```

$$1.633799400$$

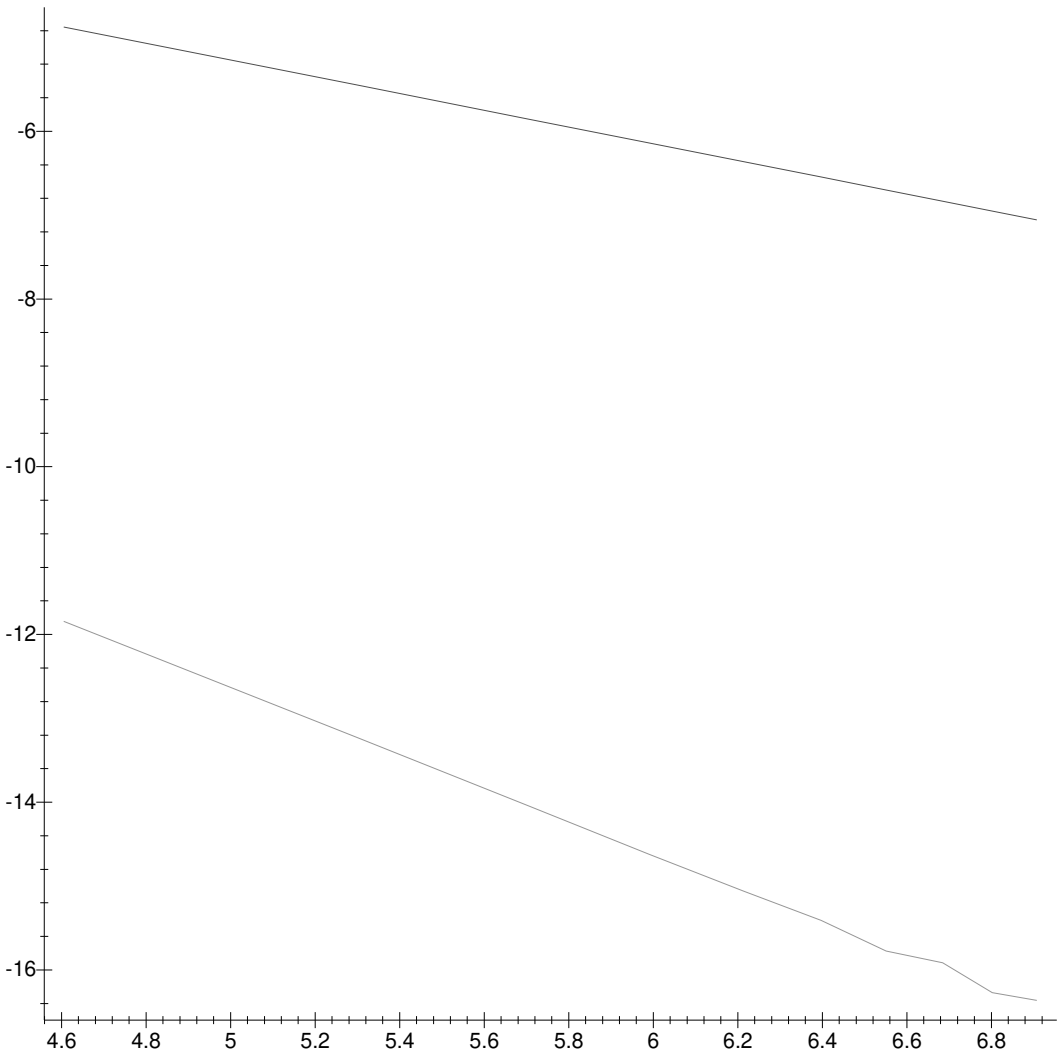
$$1.717566087$$

$$1.718281828$$

```
> delta:=100; nb:=10;
  p1:=[seq([log(delta*n), log(abs(evalf(integrale1(exp, 0, 1,
  delta*n)-(exp(1)-1))))], n=1..nb)]:
  p2:=[seq([log(delta*n), log(abs(evalf(integrale2(exp, 0, 1,
  delta*n)-(exp(1)-1))))], n=1..nb)]:
  plot({p1, p2});
```

$$\delta := 100$$

$$nb := 10$$


-6
-8
-10
-12
-14
-16
4.6
4.8
5
5.2
5.4
5.6
5.8
6
6.2
6.4
6.6
6.8