TD 11 : espaces vectoriels normés

Espaces vectoriels euclidiens

```
> restart; with(linalg):
  A:=evalm(matrix(4,4,1)-&*()):
Warning, new definition for norm
Warning, new definition for trace
```

```
> elpt:=eigenvects(A):
  vectp:=seq(op(x[3]), x=elpt):
  P:=concat(vectp):
  vp:=seq(x[1]$x[2], x=elpt):
  Di:=diag(vp):
  ok=iszero(A-evalm(P&*Di&*inverse(P)));
```

$$ok = true$$

```
> sqrtDi:=map(sqrt, Di):
  X:=evalm(P&*sqrtDi&*inverse(P));
  ok=iszero(A-map(simplify,evalm(X&*X)));
```

$$X := \begin{bmatrix} \%1 & \%2 & \%2 & \%2 \\ \%2 & \%1 & \%2 & \%2 \\ \%2 & \%2 & \%1 & \%2 \\ \%2 & \%2 & \%2 & \%1 \end{bmatrix}$$

$$\%1 := \frac{3}{4} I + \frac{1}{4}\sqrt{3}$$

$$\%2 := -\frac{1}{4} I + \frac{1}{4}\sqrt{3}$$

$$ok = true$$

Séries de Fourier

```
> restart;
```

```
> g:=x->2*x^2-x^5:
  alpha:=-1: beta:=1: T:=beta-alpha: omega:=2*Pi/T:
  f:=x-> if x<alpha then f(x+T); elif x<beta then g(x); else f(x-T); fi:
  plot(f, 2*alpha-beta..2*beta-alpha):
```

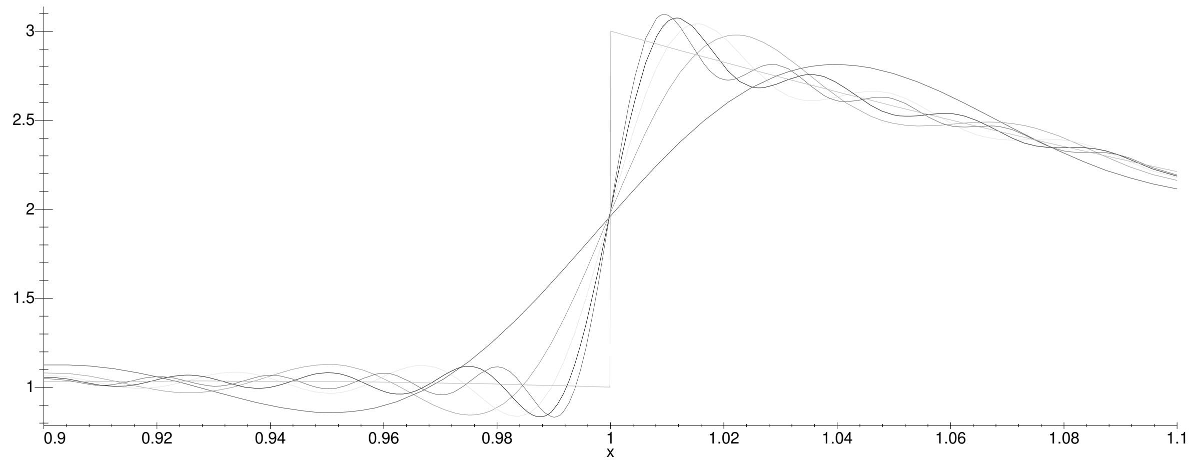
```
> assume(n, integer);
  a:=n->2/T*int(g(x)*cos(n*omega*x), x=alpha..beta):
  'a(n)'=simplify(a(n));
  b:=n->2/T*int(g(x)*sin(n*omega*x), x=alpha..beta):
  'b(n)'=simplify(b(n));
  S:=n->a(0)/2+sum('a(k)*cos(k*omega*x)+b(k)*sin(k*omega*x)', k=1..n):
```

$$\mathrm{a}(n) = 8\,\frac{(-1)^{n\sim}}{n\sim^2\,\pi^2}$$

$$\mathrm{b}(n) = 2\,\frac{(-1)^{n\sim}\,(n\sim^4\,\pi^4 - 20\,n\sim^2\,\pi^2 + 120)}{n\sim^5\,\pi^5}$$

```
> plot([S(10), 'f(x)'], x=2*alpha-beta..2*beta-alpha):
```

```
> exprs:=eval(seq('S'(20*k), k=1..5)):
  plot([exprs, 'f'(x)], x=beta-0.1..beta+0.1);
```



```
> g:=x->x:
  'a(n)'=a(n);
  'b(n)'=b(n);
  readlib(isolate):
  expand(simplify(Sum(b(n)^2, n=1..infinity)))=int(g(x)^2, x=alpha..beta):
  isolate(", Sum(1/n^2, n=1..infinity));
```

$$\mathrm{a}(n) = 0$$

$$\mathrm{b}(n) = -2\,\frac{(-1)^{n\sim}}{n\sim\,\pi}$$

$$\sum_{n\sim=1}^{\infty} \frac{1}{n\sim^2} = \frac{1}{6}\pi^2$$

Le joggueur et le chien

```
> restart; with(plots):
```

```
> v:=1; #vitesse du joggueur
  V:=1.5; #vitesse du chien
  x:=t->cos(v*t);
  y:=t->-sin(v*t);
```

$$v := 1$$

$$V := 1.5$$

$$x := t \to \cos(v\,t)$$

$$y := t \to -\sin(v\,t)$$

```
> eq1 := diff(X(t), t)=-V*(X(t)-x(t))/sqrt((X(t)-x(t))^2+(Y(t)-y(t))^2);
  eq2 := diff(Y(t), t)=-V*(Y(t)-y(t))/sqrt((X(t)-x(t))^2+(Y(t)-y(t))^2);
```

$$eq1 := \frac{\partial}{\partial t}\mathrm{X}(t) = -1.5\,\frac{\mathrm{X}(t)-\cos(t)}{\sqrt{(\mathrm{X}(t)-\cos(t))^2+(\mathrm{Y}(t)+\sin(t))^2}}$$

$$eq2 := \frac{\partial}{\partial t}\mathrm{Y}(t) = -1.5\,\frac{\mathrm{Y}(t)+\sin(t)}{\sqrt{(\mathrm{X}(t)-\cos(t))^2+(\mathrm{Y}(t)+\sin(t))^2}}$$

```
> sol:=dsolve({eq1, eq2, X(0)=-1, Y(0)=1}, {X(t), Y(t)}, numeric);
```

sol := **proc**(*rkf45_x*) ... **end**

```
> chien:=u->subs(sol(u), X(t)+I*Y(t));
  joggueur:=u->x(u)+I*y(u);
```

$$chien := u \to \mathrm{subs}(\mathrm{sol}(u), \mathrm{X}(t)+I\,\mathrm{Y}(t))$$

$$joggueur := u \to \mathrm{x}(u)+I\,\mathrm{y}(u)$$

```
> T:=2;
  p1:=complexplot('chien(t)' , t=0..T, color=black):
  p2:=complexplot('joggueur(t)', t=0..T, color=red):
  display([p1, p2], color=[black, green]);
```

$$T := 2$$