TD 9 : Algèbre linéaire générale

```
> restart:with(linalg):alias(Id=&*()):
Warning, new definition for norm
Warning, new definition for trace
```

```
> A:=matrix(3, 3, [1, 0, 0, 0, -2, -9, 0, 1, 4]):
  '(A-Id)^2'=evalm((A-Id)^2);
  'A^n'=evalm(Id+n*(A-Id));
```

$$(A-Id)^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3\,n+1 & -9\,n \\ 0 & n & 3\,n+1 \end{bmatrix}$$

```
> f:=(i, j) -> if i>j then 0 else j-i+1 fi:
  A:=matrix(5, 5, f);
  'A^(-1)'=evalm(A^(-1));
```

$$A := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{A} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> v:=[a, b, c, d];M:=vandermonde(v):
  for i from 1 to 4 do
    M[i, 4]:=M[i, 4]*v[i]:
  od:
  M=evalm(M);
  'det(M)'=factor(det(M));
```

$$v := [a, b, c, d]$$

$$M = \begin{bmatrix} 1 & a & a^2 & a^4 \\ 1 & b & b^2 & b^4 \\ 1 & c & c^2 & c^4 \\ 1 & d & d^2 & d^4 \end{bmatrix}$$

$$\det(M) = (-a+d)\,(-a+b)\,(b-d)\,(-a+c)\,(c-d)\,(c-b)\,(c+d+a+b)$$

```
> A:=randmatrix(5, 5);
  'theoreme'=iszero(transpose(evalm(adjoint(A))&*A-det(A)*Id));
```

$$A := \begin{bmatrix} -85 & -55 & -37 & -35 & 97 \\ 50 & 79 & 56 & 49 & 63 \\ 57 & -59 & 45 & -8 & -93 \\ 92 & 43 & -62 & 77 & 66 \\ 54 & -5 & 99 & -61 & -50 \end{bmatrix}$$

$$theoreme = true$$

```
> f:=(i, j) -> if i=j then 1+v[i]^2 else v[i]*v[j] fi:
  A:=matrix(3, 3, f);
```

```
Vp:=eigenvects(A);
```

$$A := \begin{bmatrix} 1+a^2 & a\,b & a\,c \\ a\,b & 1+b^2 & b\,c \\ a\,c & b\,c & 1+c^2 \end{bmatrix}$$

$$Vp := \left[1, 2, \left\{\left[-\frac{c}{a}, 0, 1\right], \left[-\frac{b}{a}, 1, 0\right]\right\}\right], \left[c^2+b^2+1+a^2, 1, \left\{\left[\frac{a}{b}, 1, \frac{c}{b}\right]\right\}\right]$$

```
> A:=matrix(3, 3, [1, alpha, a, 1, beta, b, 1, delta, c]):
  P:=matrix(3, 3, [1, 1, 1, 1, 0, -1, 1, -1, 0]):
  Di:=evalm(P^(-1)&*A&*P): eqns:=Di[1, 2]=0:
  for i from 1 to 3 do
    for j from 1 to 3 do
      if i<>j then eqns:=eqns, Di[i, j]=0: fi:
    od:
  od:
  solve({eqns});
  Vp:=eigenvects(subs(", evalm(A)));
```

$$\{\delta = 1, a = 1, \alpha = 1, \beta = 1, b = 1, c = 1\}$$

$$Vp := [0, 2, \{[-1, 1, 0], [-1, 0, 1]\}], [3, 1, \{[1, 1, 1]\}]$$

```
> x:=[1.1, 1.9, 3.2, 4.0, 5.3]:
  y:=[1.6, 2.9, 4.9, 6.1, 8.1]:
  A:=matrix(5, 2, (i, j)-> if j=1 then x[i] else 1 fi);
  sol:=linsolve(evalm(transpose(A)&*A), evalm(transpose(A)&*y)):
  alpha:=sol[1]; beta:=sol[2];
```

$$A := \begin{bmatrix} 1.1 & 1 \\ 1.9 & 1 \\ 3.2 & 1 \\ 4.0 & 1 \\ 5.3 & 1 \end{bmatrix}$$

$$\alpha := 1.542342339$$

$$\beta := -.06126124991$$

```
> p1:=plot([seq([x[i], y[i]],i=1..5)], 0..6, 0..10, style=point,
  color=black, symbol=box):
  p2:=plot(x->alpha*x+beta, 0..6, 0..10):
  display[plot](p1, p2);
```