ELECTROMAGNETISM

Faculty:	X. Antoine, A. Bendali, B. Bidégaray,
	C. Besse (50 %), P. Mazet (50 %), M. Masmoudi (50 %)
PhD :	Y. Boubendir (A. Bendali)
Ex. PhD :	N. Bartoli (A. Bendali, F. Collino, 00), B. Dah (P. Mazet, 01)
	J.-R. Poirier (A. Bendali, P. Borderies, 00)
	D. Reignier (B. Bidégaray, A. Bourgeade, P. Degond, 00)
	L. Ségui (P. Mazet & J.-P. Raymond, 00)

The research activities in the theme “Electromagnetism” can be gathered in three topics:

1. development of mathematical models and algorithms to solve complex problems in electromagnetism,

2. adaptation of the general algorithms of topological optimization and sensitivity analysis to the particular structure of the equations of electromagnetism,

3. development of efficient solvers when Maxwell’s equations are coupled with that related to the transport of particules and also extension of some techniques, specific to electromagnetism, to Schrödinger equation.

Since the earlier eighties, when they began to be extensively used in industrial problems, numerical methods, to solve the equations of electromagnetism, have been characterized by remarkable achievements. However, soon after, the industrial codes have been faced with the tremendous increase in the size of the problems due to the need to tackle cases involving higher frequencies or small details. The studies, conducted in the frame of the first topic, are all, more or less, related to problems which cannot be handled by a direct utilization of the existing general codes, even if they are tailored to run on high performance platforms involving a large amount of memory and processors. In this respect, several works have been undertaken and have resulted in new methods and also expertises. Limiting the description to the relevant domains, we can quote: boundary-element methods, finite volume methods, domain decomposition methods, perfectly matched layers, non-reflecting boundary conditions, rough surfaces, perfectly conducting metals coated by a thin dielectric layer, suppression of stair-case effects in grid approximation of the geometry, fast methods based on a matrix compression.

Concerning the second topic, the objective is to obtain efficient optimization methods based the use of the gradient for dealing with cases where the usual techniques based on genetic algoritms cannot apply. The topological optimization process has permitted to derive remarkable tools for the design of certain electrical devices. The application of these tools has permited an enhancement of the functionning of such devices which is unprecedented in some way.

The most important part of the activities developed in the frame of the third topic are strongly linked to the theme “transport”. One major problem is to construct robust numerical schemes coupling Maxwell’s system and a second set of equations describing physical phenomena other than electromagnetic waves. In this respect, splitting techniques have been developed to deal with the coupling of Bloch and Maxwell equations. Some conforming finite element and finite volume schemes have been worked out to handle the solution of Maxwell-Vlasov equations. In another direction, a deep and important result has been obtained for the derivation of non-reflecting boundary conditions for the Schrödinger equation.

The theme “Electromagnetism” has benefited from the contribution of young researchers recently recruited by the laboratory and of several PhD students. The number of external collaborations gives evidence for the importance of the undertaken studies. Future activities will be continued in the same axes directly inspired by or oriented to the physical and industrial applications.

Solution of Maxwell Equations

Impedance Boundary Condition Integral Equations Solution. *A. Bendali (in collaboration with M'B. Fares (CERFACS) and J. Gay (CEA-CESTA)), part of the PhD thesis of N. Bartoli (supervised by F. Collino (CERFACS)).*

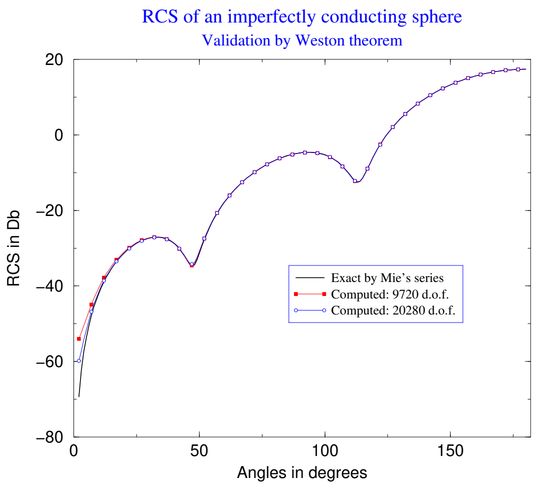

Figure 1: Boundary-Element solution for an imperfectly conducting sphere

Integral equations are perhaps the most efficient tool for solving time-harmonic problems in electromagnetism, mainly because they automatically incorporate the exact radiation condition at infinity in the formulation. In the other hand, several complicated instances of such problems as well as powerful techniques of solution are expressed in terms of an impedance boundary condition. Limiting the examples only to the researches undertaken in the laboratory, we can quote: modeling of rough surfaces and of a thin dielectric coating of a metal, imperfect conductors and domain decomposition methods. However, contrary to the use of a Finite Element Method (FEM), solving such a boundary-value problem by a boundary integral equation cannot be done through a straightforward procedure. As a result, a great deal of efforts has been devoted this subject. Figure 1 depicts the accuracy and the stability of the numerical procedure which can be reached. It permits to recover more than 80 Db for the extinction of the backscattering Radar Cross Section of an imperfectly conducting sphere. The computation has needed a parallel code (CERFACS) which has run on 36 processors of an SGI-origin 3000 of the CINES. The details can be found in [9] and [10].

Following a different path, Bartoli and Collino have adapted Després' integral equation method to carry all the treatment for the impedance condition over the right-hand side. A mathematical analysis as well as numerical tests depicting its effectiveness are reported in [8].

Fast solvers for integral equations. *A. Bendali (in collaboration with P. Borderies (ONERA-CERT)), PhD thesis of J.-R. Poirier (supervised by A. Bendali and P. Borderies).* The main limitation of the solution procedures based on an integral equation formulation is due to the large linear system with a dense matrix which has to be solved. Usual practical instances of these problems in industry involve several thousands of unknowns. Since the advent of the Fast Multipole Method (FMM), it has been recognized that not all of the information contained in the coefficients is required to compute at some accuracy a related matrix-vector product. The FMM exploits this property at a very high level of efficiency. However, it requires intricate analytical processes and a fine tuning of several parameters. It is also limited to some few Green kernels and some restricted situations. Less sharp, but more general and simple procedures, have been developed and implemented. They are based on an algebraic compression of the off-diagonal blocks of the dense matrices. Of course, due to the importance of interference phenomena in the propagation of waves, the compression process is not obtained by a simple threshold on the magnitude of the coefficients. It results from a QR decomposition of the off-diagonal blocks which permits to detect the few rows of Q and columns of R being significant. The resulting procedure has permitted to treat systems involving 30,000 unknowns on a sequential platform which otherwise can be used for systems involving 10,000 unknowns only. Its full description is reported in [17].

Finite volume methods. *B. Bidégaray (in collaboration with J.-M. Ghidaglia (ENS Cachan)).*

Multidimensional corrections to finite volume schemes for Maxwell equations have been studied by B. Bidégaray in collaboration with J.-M. Ghidaglia (ENS Cachan). The goal is to suppress mesh dependent structures which often occur in such simulations. This is performed taking into account the specific properties of the equations as well as exact solutions to the wave equation. This programme has been carried out for first and second order schemes paying particular attention to both conservativeness and an easy implementation. One article and a conference proceeding describe this work and a collaboration with Magdeburg University, Germany, is in progress.

Contribution to electromagnetic computation. *M. Masmoudi (in collaboration with X. Ferrières (CADOE)), PhD Thesis of J. Pommier (supervised by M. Masmoudi and X. Ferrières).*

The staircase approximation of the geometry by the FDTD method generates significant errors. By using a grid perturbation technique associated with the higher order derivatives method it is possible to improve the accuracy of the FDTD method and to reduce computing time. We refer to section ??? for more details.

Topological optimization. *M. Masmoudi (in collaboration with X. Ferrières (CADOE)), PhD Thesis of J. Pommier (supervised by M. Masmoudi and X. Ferrières).*

Most of the development in topological optimization concern structural mechanic with material properties optimization. For the topological optimization of electromagnetic structure, genetic algorithms are commonly used. But this method is very expensive and cannot be used for industrial applications.

The topological asymptotic has two interesting properties: its field of applications is very wide, it is possible to build fast optimization algorithms using a gradient type information (the topological sensitivity).

The topological sensitivity gives the variation of the cost function with respect to the creation of a small hole at each point. We applied this method to design and to inverse problems. In most cases, the unknown shape is obtained after only one iteration. We refer to section for the presentation of some numerical results

Domain decomposition techniques. *A. Bendali (in collaboration with F. Collino, M'B. Fares (CERFACS)), PhD thesis of Y. Boubendir (supervised by A. Bendali).*

In general, Domain Decomposition Methods (DDM) are used in the frame of a "divide and conquer" strategy to iteratively solve a large size problem by reducing it to a system of coupled small problems. Indeed, it is a cleaver way to turn to one's advantage old ideas of substructuring. On the other hand, usual procedures coupling a FEM, to deal with nonhomogeneous materials, and integral equations, to cope with the radiation condition are not completely satisfactory due to the two kinds of matrices, sparse and dense, related to each method. Non-overlapping DDMs have been used, in this part of the activity, to design a powerful procedure to uncouple the FEM and the integral equation solutions. The DDM is based on a technique introduced by Després for FEM solutions. However, the original Després' algorithm only performs a damping of the propagative part of the error. It has been necessary to introduce some adaptation which also performs the damping of the evanescent part to obtain a procedure really efficient in the context of an integral equation solution. These studies are partly described in [12].

Physical and Numerical Modeling by Boundary Conditions

Perfectly Matched Layers for Maxwell equations. *P.-A. Mazet, L. Ségui, PhD thesis of B. Dah (supervised by P.-A. Mazet).*

Among many methods used to implement artificial boundary conditions, both in frequency and time domain cases, the so called "PML"method has a lot of very good numerical improvements. The PML medium described by the way of a suitable analytic extension of Green kernels in homogeneous mediums (vacuum but also dielectric or dispersive mediums) allows rigorous mathematical investigations, more of that, the properties of spatial Maxwell operator allows to describe the PML layers (the shape of layers may be plane, spherical or cylindrical) as an anisotropic dispersive medium so that numerical implementation is easy and existence and uniqueness proves can be derived (with bounded or unbounded domains in harmonic or time domain cases). What about more general shapes and PML theory in (for instance) stratified mediums?

On-Surface Radiation Conditions (OSRC). *X. Antoine and A. Bendali (in collaboration with H. Barucq (Pau Univ.))*

A possible alternative to integral equations is given by the *On-Surface Radiation Condition* (OSRC) method. It allows to fastly compute the unknown diffracted field with a reasonable error. In [4], an asymptotic analysis based on the use of pseudodifferential calculus has leading to both the improvement and generalization to arbitrarily-shaped surfaces of usual OSRCs involving in electromagnetic and acoustic studies. The approximation of the resulting OSRC equations has next been developed in a boundary element framework in two and three-dimensions [1], for different kind of effective boundary conditions and scattering obstacles. These new OSRCs has also proved to be efficient when they are used as non-reflecting boundary conditions for underwater acoustics simulations has recently shown by the numerical results obtained at the Center for Aerospace Structure à Boulder, Colorado (USA). In [3], these results have been extended to the case of strictly hyperbolic pseudodifferential systems and applied to the two-dimensional Maxwell system. The extension to the three-dimensional case is actually under progress.

Finally, these studies have shown that the OSRC method appears as a very promising tool for analyzing high-frequency scattering phenomena. Moreover, it can be very useful when used in conjunction with the integral equations [2] (construction of preconditioners for boundary element methods, initialization of an iterative solver used in integral equations, etc...).

Impedance Boundary Conditions (IBC) for scattering problems. *X. Antoine (in collaboration with H. Barucq and L. Vernhet (Pau Univ.)), A. Bendali, part of the PhD thesis of N. Bartoli (supervised by A. Bendali).*

The solution of the scattering problem of a plane wave by a metallic target coated by a thin dielectric substrate requires to solve a system of integral equations coupling the solution in the gain and the surrounding medium. Then, the resulting method is both time and memory consuming. Moreover, such a direct formulation suffers from numerical instabilities if the dielectric layer is too thin. An alternative to this approach consists in approximately simulating the interior propagation phenomenon by the way of a boundary condition, the so-called Impedance Boundary Condition (IBC), set on the surface of the obstacle and next to solve the associated scattering problem. In [8], a heuristic constructive process has been proposed to design high-order and quasi-local high-frequency IBCs for arbitrarily-shaped boundaries. Furthermore, the convergence and error estimates have been stated for the new schemes.

The computation of an electromagnetic or acoustic wave diffracted by a homogeneous dissipative body leads to solve a system of coupled integral equations in R^3. In [5], generalized IBCs have been developed by a high-frequency analysis. Their domain of validity has been numerically investigated to simplify the initial integral equation formulation. In [6], a detailed mathematical analysis of the resulting boundary value problem has been achieved and the extension of the method to arbitrarily-shaped surfaces has been given in the framework of integral equations. The treatment of the system of the full Maxwell equations is under progress.

Modeling of rough surfaces. *A. Bendali (in collaboration with P. Borderies (CERT-ONERA)), PhD thesis of J-R. Poirier,(supervised by A. Bendali and P. Borderies).*

When a boundary element method is used to compute the electromagnetic field diffracted by a rapidly oscillating rough surface, two severe drawbacks can be observed: the linear system to be solved has a size which can dramatically increases and it becomes ill-conditioned. Consequently, this prevents the use of an iterative solver making the determination of the solution unreachable. Based on techniques issued from homogeneization theory, a model which allows to replace the two-scale periodic homogeneities by an IBC has been derived in [20]. A detailed numerical study has also shown that this model is very useful to analyze scattering problems involving small periodic rugosities or rough surfaces defined through some statistical properties. Figure 2 compares the magnitude of the scattered electromagnetic field directly computed using a huge mesh involving 150 points per wavelength and that obtained with the IBC using an usual mesh of 10 points per wavelength. Surprisingly enough, these models are able also to take into account small details on the surface. Finally, the model for the three-dimensional Maxwell system has been analytically derived.

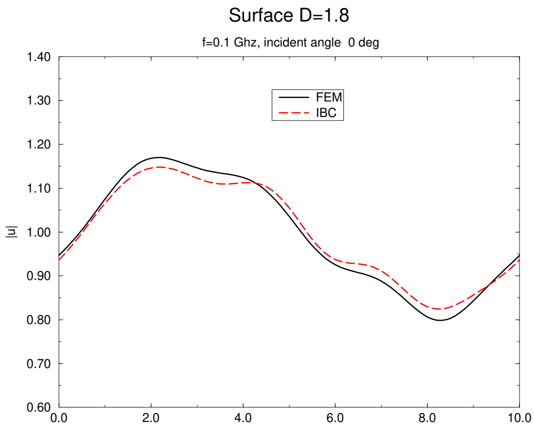

Figure 2: Magnitude of the field obtained through a brute force computation and an effective IBC

Non-reflecting Boundary Conditions for Waveguides, *A. Bendali and Ph. Guillaume.*

Non-reflecting Boundary Conditions for Waveguides, which can be used as simple and powerfull terminating boundary condition beyond FEM meshes, have been constructed. Following the words of the referee who has accepted the article [11]: “it is for the first time that provable error estimates have been established for a truncating approximate boundary condition”.

Nonlinear wave interaction

Quantum Optics, *B. Bidégaray, PhD theses of D. Reignier and O. Saut (supervised by C. Besse, B. Bidégaray, P. Degond and A. Bourgeade (CEA-CESTA)).*

The most precise model for the semi-classical description of wave-matter interaction is Maxwell-Bloch model. Numerous numerical problems have been pointed out when trying to use classical methods used in previous codes to model more complex physics. We therefore addressed positiveness problems both on the continuous model (which leads to conditions on the relaxation terms) and on the numerical schemes. Physical and numerical models are now more robust in the frame-work of centrosymmetric media such as gases. The study of more complex materials is in progress.

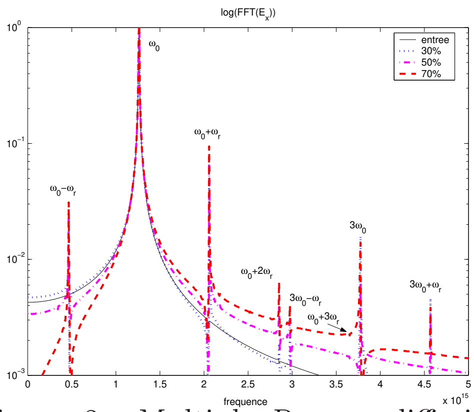

Figure 3: Multiple Raman diffusion and higher harmonics generation.

Crank-Nicolson scheme is usually used to discretize Bloch equations. It proves to be inadequate since it does not preserve positiveness properties of the continuous model. Therefore a splitting scheme has been developed which handles these positiveness problems and besides allow significant computational time savings. The efficiency of the coupling with Maxwell equations has also been improved.

The progresses made for the introduction of relaxation terms in Bloch equations have made possible the simulation of more realistic physical test cases (such as coherence transfer, Raman diffusion, laser cavities, ...). The computational improvements allow simulations over relatively larger times. We may therefore consider the comparison with asymptotic models in order to validate them and to make clear the modelling limits.

Nonlinear Optics, *C. Besse and B. Bidégaray (in collaboration with S. Descombes (ENS Lyon)).*

Schrödinger-Debye system models the propagation of an electromagnetic wave in the paraxial approximation in a Kerr medium with finite response time. From the mathematical point of view it is one of the simplest perturbation of the cubic nonlinear Schrödinger equation we may think of, but it proves to be quite complex.

Local in time existence results and uniqueness of both strong and weak solutions to the Cauchy problem have been proved and recently improved. There are also rigorous results for the limit as the response time tends to zero. But the global in time Cauchy problem seems to resist to classical analyses up to now. Therefore different numerical methods have been studied and compared to approximate this system. They seem to display some finite time blow-up. This study also led to new results for the order of splitting methods for the nonlinear Schrödinger equation.

Non-Reflecting boundary conditions and associated discretization schemes for the simulation of Schrödinger-type equations *X. Antoine and C. Besse.*

Linear or nonlinear Schrödinger-like equations fundamentally appear in various physical areas. They especially involve in the paraxial approximation of Helmholtz equation in a priviligeated direction as for instance for computing the far field pattern of a strongly directed antenna interacting with a complex environment or for simulating the propagation of an underwater acoustic wave. They also play a central role in the comprehension of the propagation of a nonlinear optical wave but also in the study of quantum phenomena. This last aspect has naturally given rise to a collaboration between the transport and the electromagnetic teams to develop some efficient numerical methods for simulating Schrödinger-type equation.

High order non-reflecting boundary conditions (local in space and nonlocal in time) have been constructed for general boundaries for the two dimensional linear Schrödinger equation. Moreover, this study has been completed by stating several precise characterizations of the Steklov-Poincaré operator by using the microlocal analysis. Some unconditionally stable numerical schemes are being developed and implemented for computing the propagation of a guided optical wave by the Fresnel equation, a one dimensional Schrödinger equation.

Future directions of research will be the extension of the numerical schemes to the two dimensional case and their applications to the techniques of parabolic equations in electromagnetism scattering. Moreover, collaborations will be strongly intensified for various concrete applications in quantum mechanics.

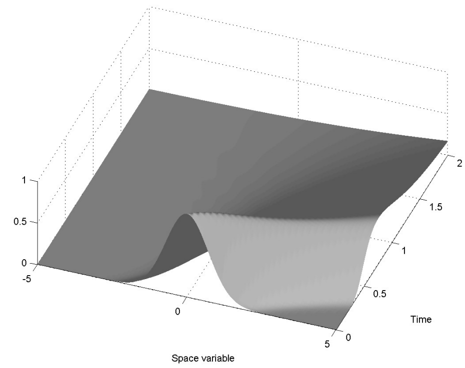

Figure 4: Simulation of the solution of the 1D Schrödinger equation using the non-reflecting boundary condition.

Isotopic separation device by ionic cyclotron resonance *PhD thesis of P. Omnès (supervised by P. Degond, in collaboration with P. Louvet (CEA-Saclay)).*

This work deals with the study of a magnetically confined plasma submitted to a sinusoidal surfacic electric current. The behaviour of the ions is modeled by a nonrelativistic Vlasov equation whereas the electrons are governed by a conductivity tensor of a cold specie. The Maxwell-Vlasov equations are solved in periodic regime and bounded domain by a fixed point method by only retaining the first harmonic. The electromagnetic field is computed by conform finite elements and the ionic current by smoothed particle method well adapted to the periodic regime. This process, applied to the simulation of the isotopic separation device by ionic cyclotron resonance, converges in a few iterations. Furthermore, a finite volume method for solving the time-dependent Maxwell equations has been developed. This approach has proved to be suitable to take into account the Gauss law as well as preserving the hyperbolicity of the system to solve.

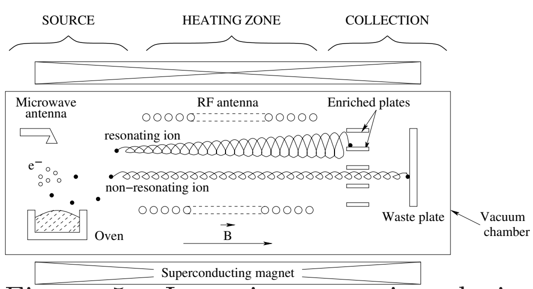

Figure 5: Isotopic separation device scheme

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