

# The Orr mechanism : Stability/Instability of the Couette flow

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April 30, 2018

The theory of hydrodynamic stability at high Reynolds number started already in the 19th century, with the likes of Stokes, Reynolds, Kelvin, Orr and others. Some of the first early theoretical works were done by Rayleigh, including for example, the inflection point theorem of the spectral instability on inviscid planar shear flows, and the exact solutions for Couette flow in the absence of boundaries constructed by Kelvin [Kel87] which showed linear stability independent of Reynolds number.

The Orr mechanism was introduced by Orr [Orr07] in 1907 to give an explanation to the so-called Sommerfeld paradox. The relevance of the Orr mechanism to hydrodynamic stability has been debated over the years. We will discuss the crucial importance of the Orr mechanism for nonlinear stability problems at high Reynolds numbers in 2D fluid mechanics, or at least for the Couette flow.

We will review the works [BM15, DM18] on the nonlinear asymptotic stability and instability of the Couette flow for the 2D incompressible Euler dynamic. On one hand, in [BM15], we prove that perturbations to the Couette flow which are small in Gevrey spaces  $G^s$  of class  $1/s$  with  $s > 1/2$  converge strongly in  $L^2$  to a shear flow which is close to the Couette flow. Moreover in a well chosen coordinate system, the solution converges in the same Gevrey space to some limit profile. On the other hand, we prove in [DM18] the existence of small perturbations in  $G^s$  with  $s < 1/2$  such that the solution becomes large in Sobolev regularity and hence yields instability.

We will also review few results in the viscous case [BGM17]. Indeed, when the viscosity is small, the notion of stability threshold relates the allowed size of the perturbation to the viscosity.

## References

- [BGM17] Jacob Bedrossian, Pierre Germain, and Nader Masmoudi. On the stability threshold for the 3D Couette flow in Sobolev regularity. *Ann. of Math. (2)*, 185(2):541–608, 2017.
- [BM15] Jacob Bedrossian and Nader Masmoudi. Inviscid damping and the asymptotic stability of planar shear flows in the 2D Euler equations. *Publ. Math. Inst. Hautes Études Sci.*, 122:195–300, 2015.
- [DM18] Yu Deng and Nader Masmoudi. Long time instability of the Couette flow in low Gevrey spaces. *preprint*, 2018.
- [Kel87] Lord Kelvin. Stability of fluid motion-rectilinear motion of viscous fluid between two parallel plates. *Phil. Mag.*, (24):188, 1887.
- [Orr07] W. Orr. The stability or instability of steady motions of a perfect liquid and of a viscous liquid, Part I: a perfect liquid. *Proc. Royal Irish Acad. Sec. A: Math. Phys. Sci.*, 27:9–68, 1907.