MML

16

Apparté

$(X_1, \dots, X_n) : \Omega \longrightarrow \mathbb{R}^n$

$\mathbb{P}_\theta$ proba on Ω

$\mathbb{E}_\theta \left[(\hat{\theta}(X_1, \dots, X_n) - \theta)^2 \right] = \int_\Omega (\hat{\theta}(X_1, \dots X_n) - \theta)^2 d\mathbb{P}_\theta$

$= \int_{\mathbb{R}^n} (\hat{\theta}(x_1, \dots x_n) - \theta)^2 d\mathcal{N}(\theta, \sigma^2)^{\otimes n}(x_1, \dots, x_n)$

$= \int_{\Omega'} (\hat{\theta}(X'_1, \dots, X'_n) - \theta)^2 d\mathbb{P}_{\theta'}$

where $\Omega' = \mathbb{R}^n$ ← output space $\quad X'_i(x_1, \dots x_n) = x_i$

$\mathbb{P}_{\theta'} = \mathcal{N}(\theta, \sigma^2)^{\otimes n}$ $\quad$ identity mapping

canonical construction

$= \mathbb{E}_{\mathbb{P}'_\theta} \left[(\hat{\theta}(X'_1, \dots, X'_n) - \theta)^2 \right]$

conclusion we can always assume that $\Omega = \mathbb{R}^n$

$\mathbb{P}_\theta = \mathcal{N}(\theta, \sigma^2)^{\otimes n}$, $X_i(w) = w_i$