

Name:

Matricule number:

Mid-term test
Business Mathematics 2
Group 6
Winter 2014

<i>example</i>	<i>max.pts.</i>	<i>pts.</i>
1	3	...
2	3	...
3	3	...
4	3	...
<i>total :</i>	12	...

Instructions:

- No documents, no calculators
- Write your answers for an example in the corresponding indicated blank spaces
- All the answers must be justified
- The clarity and readability of the copy will be taken into account in the final mark

- 1) a) Find and sketch the definition domain of the following function f (two input variables and one output), defined by

$$f(x, y) = \frac{\sqrt{2x+2}}{\ln(1-y^2)}.$$

- b) Let $f(x, y) = 2x + y/2$. Find and sketch the contour lines $C_{f,c} = \{(x, y) \in \mathbb{R}^2; f(x, y) = c\}$, for $c = -1, 0, 1$.
- 2) a) Calculate the gradient vector of $f(x, y, z) = e^{x^2+z} - \frac{3}{y}$.
- b) Calculate the directional derivative of $f(x, y) = \cos(x^2y)$ at $P = (0, 1)$ in the direction $h = (2, -1)$.
- 3) a) Find the tangential plane of the function $f(x, y) = \sqrt{2x+y}$ at the point $P = (1, 1)$.

b) Let

$$h(x, y) = \begin{pmatrix} x^2 \\ 2y \end{pmatrix},$$
$$g(v, w) = \begin{pmatrix} e^v - w \\ v + w \end{pmatrix}$$

and let $f(x, y) = g(h(x, y))$. **Using the chain rule** calculate the Jacobian matrix of f . (No points will be given if you find the Jacobian matrix without using the chain rule).

- 4) a) Calculate the quadratic form

$$\begin{pmatrix} -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

and the matrix product

$$\begin{pmatrix} 1 & 0 & 1 \\ -2 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 4 \\ 1 & 1 \end{pmatrix}.$$

- b) Study the positiveness of the two following matrices

$$M_1 = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 4 & 3 \\ 2 & -1 & -1 \end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix} 5 & 2 \\ -2 & 6 \end{pmatrix}.$$

Answer to 1) a):

Answer to 1) b):

Answer to 2) a):

Answer to 2) b):

Answer to 3) a):

Answer to 3) b):

Answer to 4) a):

Answer to 4) b):