Name:

Matricule number:

End-term test
Business Mathematics 2
Group 6
Winter 2014

example	*max.pts.*	*pts.*
1	3	...
2	3	...
3	3	...
4	3	...
total :	12	...

Instructions:

- No documents, no calculators
- Write your answers for an example in the corresponding indicated blank spaces
- All the answers must be justified
- The clarity and readability of the copy will be taken into account in the final mark

1) a) Let $f(x,y) = x^3 - y^3 + xy$. Find two critical points of f. (You do not have to determine their nature).

b) Let $f(x,y) = e^{x^2+1} - e^{y^2+1}$. Then $(0,0)$ is a critical point of f (you do not have to show it). What is the nature of the critical point $(0,0)$?

2) a) **Using the graphical method**, find the global maximizer for the problem

$$\begin{array}{ll} \max & y \\ s.t. & x^2 + y^2 = 1. \end{array}$$

(No points will be given if you do not use the graphical method).

b) Using the method of your choice, find a local maximizer for the problem

$$\begin{array}{ll} \max & xy \\ s.t. & x + y = 1. \end{array}$$

3) a) Show that the function $f(x,y,z) = e^x + z^2 + (1+y)^2$ is convex on $\mathbb{R}^3$.

b) Show that the set $\{(x,y) \in \mathbb{R}^2; x^2 \leq 1, x \leq 2 - y^2\}$ is convex.

4) a) Consider the problem

$$\begin{array}{ll} \min & x \\ s.t. & x \geq 0 \\ & x^2 + y^2 \leq 1. \end{array}$$

Can $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ be a local minimizer for this problem?

b) Consider the problem

$$\begin{array}{ll} \min & (x+1)^2 + y^2 \\ s.t. & x \leq 0 \\ & y \leq 0. \end{array}$$

Find a point (x^*, y^*) that satisfies the KKT conditions and show that no other points can satisfy the KKT conditions. Which constraints are active at the point (x^*, y^*).

Answer to 1) a):

Answer to 1) b):

Answer to 2) a):

Answer to 2) b):

Answer to 3) a):

Answer to 3) b):

Answer to 4) a):

Answer to 4) b):