REMARKS ON HERMAN'S MANUSCRIPT: "UNIFORMITY OF THE ŚWIĄTEK DISTORTION FOR COMPACT FAMILIES OF BLASHKE PRODUCTS..."

A. CHÉRITAT

In [H3], Herman proves that in a (pre)-compact family of Blaschke products *with bounded degree* and induicing homeomorphisms on the unit circle $\mathbb{S}^1$, the Świątek distortion stays bounded.

The notion of precompacity being here: $\exists \varepsilon > 0$ such that from all sequence one can extract a subsequence that converges uniformly on "$1 - \varepsilon < |z| < 1 + \varepsilon$".

The Świątek distortion is that of the lift $\widetilde{f} : \mathbb{T}^1 \to \mathbb{T}^1$ of $f : \mathbb{S}^1 \to \mathbb{S}^1$ by $x \mapsto \exp(2i\pi x)$.

In fact, the result remains true in any precompact family $\mathcal{F}$ of analytic homeomorphisms of $\mathbb{S}^1$ defined on a common ring "$1 - \varepsilon < |z| < 1 + \varepsilon$".

The proof of [H3] works almost without modification: let by contradiction $f_k \in \mathcal{F}$ be a sequence such that $SD(\widetilde{f}_k) \longrightarrow +\infty$. Up to a subsequence extraction, we may assume that f_k converges on the ring to some f, which is necessarily a analytic homeomorphism of $\mathbb{S}^1$. Note that the number of critical points, counted with multiplicity, of f_k in a given neighborhood of $\mathbb{S}^1$ is necessarily bounded. Let $p(k)$ be the number of intervals where the Schwarzian derivative Sf_k is non-negative. Then $p(k) \to +\infty$, otherwise according to [H3] the Świątek distortion $SD(\widetilde{f}_k)$ would be bounded. Therefore $Sf = 0$, otherwise the number of zeros of Sf_k in a neighborhood of $\mathbb{S}^1$ would be bounded. Hence f_k tends to a Möbius map and thus for k big enough $\operatorname{Var} \log D\widetilde{f}_k \leqslant M$, and thus $SD(\widetilde{f}_k) \leqslant e^{10M}$, which contradicts the hypothesis.

References

[H3] M.R. Herman, *Uniformité de la distortion de Świątek pour les familles compactes de produits de Blaschke*, manuscript.