Comparisons of split-linear fitting of wind curves

PHILIPPE C. BESSE*
Université Paul Sabatier / CNRS, Toulouse, France

NATHALIE RAIMBAULT
AEROSPATIALE MATRA AIRBUS, Toulouse, France

Abstract

The detection of slope change points in wind curves is associated with linear curve fitting. Classic algorithms based on smoothing and comparison of slopes are adapted and compared to a Bayesian method of curve fitting. After prior spline smoothing of the data, the algorithms are tested and the errors between the split-linear fitted wind and the real wind are estimated. In our case, adaptations of an edge-preserving smoothing algorithm allow the same good performance as automatic Bayesian curve fitting based on a Monte Carlo Markov chain algorithm.

Key words: slope change detection, split-linear fitting, bayesian curve fitting, MCMC, spline smoothing.

1 Introduction

This article comes within the large scope of a study dedicated to the improvement of the aircraft autopilot conception process. The autopilot allows landings in bad weather conditions and must guarantee passengers safety, touch-down confort and precision. We study in particular the influence of wind during automatic landing. Therefore, we focus on the effect of the linear wind components in the last 30 seconds since it is assumed to be a decisive factor in touch-down precision. It leads us to develop a method of split-linear fitting based on slope change detection adapted to our data.

**Address for correspondence:* Laboratoire de Statistique et Probabilités, UMR CNRS C5583, Université Paul Sabatier, 31062 Toulouse cedex 4, France. E-mail: besse@cict.fr.

This work is directed along the same lines as those proposed by Jones (1998) which aim at predicting the influence of discrete gusts on linear systems.

Slope change detection is associated with edge and peak detection. Two kinds of method can be adapted : classic ones, based on smoothing and a bayesian one based on Monte Carlo Markov Chains. Hall and Titterington (1992) proposed an edge-preserving smoothing algorithm. It is based on edge detection by comparisons of three smooths. The core of this method is to compute, for each given point, three smoothed estimates of the function, based on the data to the right, to the left and on both sides of the point. Each discontinuity is associated with a local maximum of the difference between the three fits. Wu and Chu (1993) took this algorithm and modified it using kernel smoothing instead of smoothing by linear combinations of observations. Loader (1996) showed convergence properties of this kind of change point estimate adapted to the search for one discontinuity in a function. Müller (1997) improved this kind of algorithm to increase the convergence rate of one discontinuity estimate. Otherwise, Green (1995) and Denison *et al.* (1998) applied reversible Monte Carlo Markov Chains and Bayesian model determination to curve fitting, including, in particular, piecewise linear polynomial fitting. The automatic Bayesian curve fitting algorithm proposed by Denison *et al.* (inspired by Green's (1995) algorithm) determines the number and the position of the slope change points so called knots by the estimation of their joint posterior distribution.

Our aim here is to derive, from these methods, an efficient split-linear fitting algorithm for our data. The specificity of our problem is that the quality of fit is not measured by the usual residual squared error. The criteria to be met are estimation of a parcimonious model by detecting only the significant slope changes in the curve and achievement of the same landing properties with the real wind and with the split-linear approximation. This is an important feature of this paper which especially involves adaptations of the algorithms which consist in prior smoothing of the data and algorithm refinements.

First of all, three classic split-linear fitting algorithms are computed and compared on samples of noise-free wind data measured during flight test. Then the best one is opposed to the automatic Bayesian curve fitting algorithm. The first algorithm is based on the comparison between consecutive slopes calculated in small intervals. The second one is inspired by Hall and Titterington (1992) as previously detailed. This method consists in computing and comparing three smooths. The local extrema of the difference between the three smooths are used as estimates of slope change points. Moreover kernel smoothing is here replaced by spline smoothing to improve the quality of the fitting as further detailled. Since our data consist in multiple slope changes, we take the time of calculation of each step of the algorithm into account. Therefore the improvement proposed by Müller (1997) is dropped here. The third method is a combination of the other two. Comparison between consecutive slopes is used to detect slope sign changes. Then the second method is needed to localize other slope changes between the previously detected ones.

Tests and comparisons show that the third method is the most appropriate one to use in our case among classic algorithms and that its performance is similar to that of the Bayesian linear curve fitting. Differences between touch-down with real wind and touch-down with split-linear fit are estimated with a complex simulation program which allows us to perfom free-choice but time-costly landing simulations. Implementation is carried out using SPLUS (1997).

The specific problem and the data transformations are explained in Section 2. In Section 3, we present the algorithms. The numerical results and the comparison between the methods are explored further in Section 4. We will conclude by a brief discussion in Section 5.

2 Aeronautical context and data transformations

As previously said, we aim at breaking up wind into linear components to study later their own influence on the behaviour of an aircraft autopilot. Actually, the stage of the flight we are interested in is the final approach, specifically the last 30 seconds before touch-down when wind perturbations are critical because of the imminence of the landing.

The dimension of the problem is restricted to the runway axis so that only longitudinal wind $W_x(t)$ is considered. Landing quality is estimated by the distance in meters between the runway threshold and the touch-down in the runway axis (X_{TP}). Our sample of data consists of 59 winds measured during flight tests. As we chose to focus on the latest 30 seconds before landing, each wind recording is composed of N=240 points $(t_i, w_i) = W_x(t_i)$.

Simulation software allows us to perform landing simulations choosing the flight parameters especially the wind. This tool is very useful as the criterion to be met is expressed in term of error between touch-down performance (X_{TP}) instead of the usual residual square error. This software is used to set the different values of the parameters and to evaluate the algorithms. The difficulty we have to cope with is that it cannot be included in an optimisation or a cross validation program as detailed further.

Now that the aeronautical context has been described, the data transformations have to be discussed here. As different ways were tested, successfully computations are made by using smoothed wind curves. Direct applications of the following algorithms on raw data are not satisfactory. Actually, we aim at determining only the significant slope change points avoiding finding too many knots. Experiments with raw data lead to non-significant point detection for classic algorithms and to difficulties in setting up the prior distribution of the number of knots in the Bayesian algorithm. Moreover, the estimated errors between touch-downs with the split-linear fits of raw data are too large.

So a first step of the method consists in clearing winds of non significant information. That is, components which do not change the touch-down. It may be measurement noise as well as some frequency components which do not disturb the aircraft during the flight. To be more specific, we know that high frequencies have no influence on our aircraft trajectory because of its inertia. Smoothing is applied to get rid of these high frequencies and allows the slope change detection to be efficient.

Spline smoothing is used in this case. Let $v(t_i)$ be the cubic spline fit of the (t_i, w_i). Then $v(t)$ is the cubic spline that minimizes the residual sum of squares plus a roughness penalty:

$$\sum_{i=1}^{N}(w_i - v(t_i))^2 + \lambda \int_a^b (v''(t))^2 dt \text{ where } w_i = W_x(t_i), \ \ \forall i = 1..N$$

The smoothing parameter λ usually governs the trade-off between smoothness and goodness of fit. It is usually set by cross validation. In our particular case, we aim at keeping only the significant components of the wind towards the landing. Therefore goodness of fit is estimated here by the error between the real X_{TP} touch-down with raw data and the one with smoothed data. This criterion replaces the usual residual error minimization.

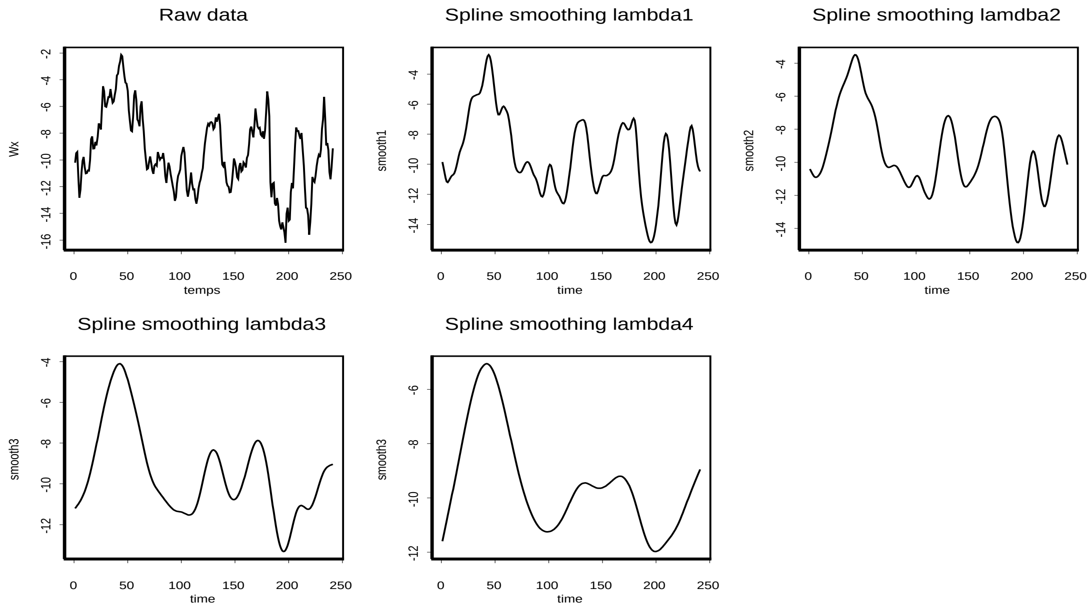

Figure 1: Influence of the smoothing parameter on wind speed versus time. Raw data are compared to smoothed data with different values of λ : $\lambda_1 = 10^{-6}$, $\lambda_2 = 10^{-5}$, $\lambda_3 = 10^{-4}$, $\lambda_4 = 10^{-3}$.

Simulating touch-down with modified winds involves a few laborious computation steps. So it is impossible to automate the search of the optimal value of the smoothing parameter in our case. We have to choose prior values and test them separately. Empirically, 4 values of the smoothing parameter λ are likely to be satisfactory : $\lambda_1 = 10^{-6}$, $\lambda_2 = 10^{-5}$, $\lambda_3 = 10^{-4}$, $\lambda_4 = 10^{-3}$. The data have to be smoothed enough without loosing too much information. Figure 1 depicts the influence of the 4 values of smoothing parameter on the raw data.

Then the 4 smoothing parameters are tested on 10 winds. Touch-downs are simulated so as to estimate the differences between the real flight X_{TP} with raw data and the one with smoothed data. The boxplots of results are represented on Figure 2. The second value of smoothing parameter is chosen since it is the stronger value which minimizes error mean and scatter

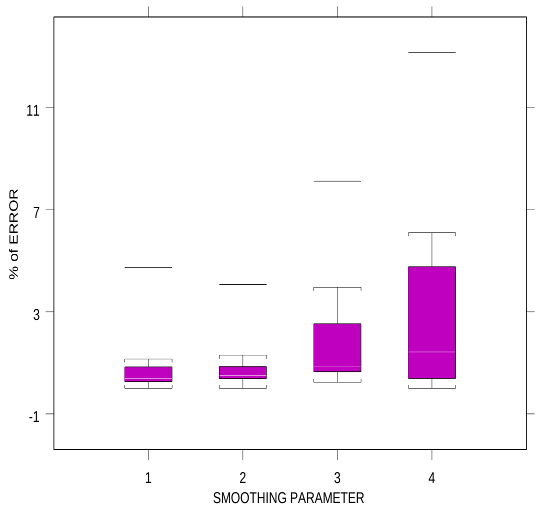

Figure 2: Choice of smoothing parameter. The boxplots represent the differences between the real flight X_{TP} with raw data and with smoothed data. The value λ_2 minimizes error mean and scatter.

(4% error maximum).

3 Split-linear fitting algorithms

The aim is to detect the slope change times t_j^* among the t_i, $i = 1..N$. The split-linear algorithms are detailed in this section. They are applied on smoothed wind data as previously explained in Section 2. Observations are still noted (t_i, w_i) and the values of the smoothed wind curves are called v_i.

Since one matching condition is to detect only significant slope changes, each method implies the search for the optimum values of the different parameters which govern the precision.

3.1 Algorithm based on slope comparisons

This first algorithm is based on five steps detailed hereafter. Three parameters have to be set to use this method : the number of intervals K, the threshold s and an optional minimum time interval T_{min} between two slope changes. They are specified in this section.

1. The smoothed wind curve is split into intervals I_k, $k = 1..K$.
2. In each interval, the slope a_k of the least-squares straight line is estimated.
3. The consecutive slopes a_k and a_{k+1} are compared.
   If $a_k.a_{k+1} < 0$, a point of slope sign change t_j^* is detected.
4. If the interval $[t_j^*, t_{j+1}^*]$ is larger than T_{min}, points of single sign changing t_j^{**} are sought by comparing the slopes estimated in the $I_k \subset [t_j^*, t_{j+1}^*]$ to a threshold value s.
5. Since every t_j^*, t_j^{**} has been detected, the split-linear fitting of the smoothed wind curves is carried out by linking the $v_j^* = v(t_j^*)$ or $v(t_j^{**})$.

Linear least-squares model

The size of the intervals governs the trade-off between accuracy of the method and calculation time. In our case, tests lead us to select 4-points intervals. Since each wind curve is composed of N=240 measurments, the

30s time interval is split into K=60 intervals.

In each interval, the linear least-squares model is fitted on the data :

$$v_i = a_k t_i + b_k, \ \forall i \in [t_{4(k-1)+1}, t_{4k}] \ \forall k = 1..K.$$

Therefore, K slopes a_k, $k = 1..K$ have to be compared.

Slope sign changing

Assume that a_k (resp. a_{k+1}) is estimated on $I_k = [t_{4(k-1)+1}, t_{4k}]$ (resp.$I_{k+1} = [t_{4k+1}, t_{4k+4}]$).

If $a_k.a_{k+1} < 0$, the point of slope sign change $t_j^* = t_{4k+1}$ is detected.

Comparisons of slopes

Assume that two points of slope sign change t_j^* and t_{j+1}^* are detected. If $t_{j+1}^* - t_j^* \geq T_{min}$, then points of single slope change are sought. We choose to set this optional parameter representing a minimum time interval since studies have shown the poor influence of high frequencies on a long range aircraft. We thus decide not to search for too close wind gradients. In our case, $T_{min} = 5sec$ is a reasonable value.

For each interval $I_k \subset [t_j^*, t_{j+1}^*]$, the slopes a_k compared as follows :

If $\lvert a_{k+1} - a_k \rvert > s$	then $t_j^{**} = t_{4k+1}$
	and a_{k+1} is compared similarly to the next slope a_{k+2}.
If $\lvert a_{k+1} - a_k \rvert \leq s$	then $a_{k,k+1}$ is estimated on $I_k \cup I_{k+1}$
	and $a_{k,k+1}$ is compared to the next slope a_{k+2}.

The gathering of intervals improves the significance of the t_j^{**} detected compared to a step-by-step comparison.

The threshold value s has to be set arbitrarly at first and then adapted to the data. A low threshold lead to the detection of non-significant difference of slope whereas a high threshold make the method inaccurate. Cross validation can be used to choose the optimum s value in a general case considering the usual residual square error between the wind and its split-linear fitting. We recall here that our specific problem leads us to focus on touch-down performance. Cross validation cannot be actually applied in this case. Therefore the best threshold value is empirically chosen on some data and checked on the others. According to the different tests, the value $s = 0.1$ is suitable. We will discuss further on the dependence of this value on the data.

Construction of the split-linear estimation

The points (t_j^*, v_j^*) $j = 1..J$ are joined up to create the estimation. The tests made with the raw data $(t_j^*, w_j^*) = W_x(t_j^*))$ $j = 1..J$ give bad results according to landing simulations.

3.2 Algorithm based on differences between smooths

The basic features of the second algorithm inspired by Hall and Titterington (1992) are as follows:

1. At each point t_i, left, right and centre smooths are computed on the smoothed wind curves as explained further.
2. Slope changes t_j^* are detected by comparisons between the three smooths.
3. Every v_j^* corresponding to a t_j^* are linked so as to produce the split-linear fit of the smoothed wind curves.

Two parameters have to be set here : the smoothing parameter λ and the bandwith h corresponding to the number of points involved in the left, right and centre smooths.

Left, right and centre smoothing

First of all, the bandwith value h has to be chosen. This parameter h determines how large is the neighborhood of each point used to calculate the three smooths.

For each point in $[1+h, \mathrm{N}-h]$, left, right and centre smoothing are computed. Hall and Titterington propose smoothers such as particular linear combinations of the observations around each point. This smoothing has been tested and gave worse results on our data than spline smoothing (see

Section 4.1). So spline smoothing has been chosen for left and right smoothing whereas centre smoothing is simply a mean calculation.

Let $f_r(i)$ be the value of the right smooth at the point i computed on $(t_k, v_k),\ k \in [i, i+h]$. Let $f_l(i)$ be the value of the left smooth at the point i computed on $(t_k, v_k),\ k \in [i-h, i]$. Let $f_m(i)$ be the mean of the $\{v_k,\ k \in [i-h, i+h]\}$.

For the two spline smooths, the smoothing parameter λ has to be chosen. Moreover, the bandwith h has to be large enough to avoid side-effects of the smoothing. But since $2h$ points are lost for the slope change detection, a trade-off has to be sought.

These two parameters λ and h can be optimized by cross validation. In our case, we have to choose prior satisfactory empirical values and perform the choice futher as regards the numerical results in Section 4. Tests show that the bandwith can be 10, 15 or 20 points large and that three values of λ are possible : $\lambda_1 = 10^{-6}$, $\lambda_2 = 10^{-5}$, $\lambda_3 = 10^{-4}$.

Detection of slope change

The diagnosis is made by estimating for each point $i \in [1+h, \mathrm{N}-h]$, the difference d_1 defined by $d_1(i) = |f_r(i) + f_l(i) - 2f_m(i)|$. If the point i is a slope change point then this difference will be locale maximum.

Let us remark that, since our data are smoothed before the algorithm is applied, $f_r(i) \simeq f_l(i),\ \forall i \in [1+h, \mathrm{N}-h]$. So $d_1(i)$ can be replaced arbitrarly by $d_2(i) = |f_r(i) - f_m(i)|$. The detection of the local maxima is made by the comparison of each point with its two neighbours. A point i is a local maximum if :

$$|f_r(i) - f_m(i)| > |f_r(j) - f_m(j)|\ for\ j = i-1\ and\ i+1$$

Each local maximum of d_2 noted i is associated with a slope change point $t_j^* = t(i)$. Since every (t_j^*, v_j^*) has been detected, the interpolation is computed as in the first algorithm.

3.3 Combination of the first two algorithms

This algorithm is based on five steps combinating the other two methods:

1. The smoothed wind curve is split into intervals $I_k,\ k = 1..K$.

2. In each interval, the slope a_k of the least-squares straight line is estimated.

3. The consecutives slopes a_k and a_{k+1} are compared.
If $a_k.a_{k+1} < 0$, a point of slope sign change t_j^* is detected.

4. If the interval $[t_j^*, t_{j+1}^*]$ is larger than T_{min}, points of single sign change t_j^{**} are sought by the comparison of the three smooths detailed in method 2.

5. Since every t_j^*, t_j^{**} has been detected, the split-linear fitting of the smoothed wind curves is carried out by linking the $v_j^* = v(t_j^*)$ or $v(t_j^{**})$.

This combination allows us to avoid the difficult choice of the threshold of the first method and to make up for the weaknesses of the second method detailed further in Section 4.

The parameters to set are : the number of intervals K, the optional T_{min}, the smoothing parameter λ and the bandwith h. The difficulties of choice are the same as above.

3.4 Bayesian Automatic Curve Fitting

Recall here that we aim at identifying the slope change times t_j^* among the t_i. Therefore, the algorithm of Denison *et al.* is applied on our data so as to obtain a split-linear fit and only modified to obtain the final knots. The Bayesian basis of the algorithm is briefly described here according to the article of Denison *et al.* (1998).

The observations are still noted $v = (v_1, ...v_N)$ and denote the smoothed value of the wind curves. The general model of piecewise polynomial fit is as follows:

$$\begin{aligned} v_i &= f_{K,l}(t_i) + \epsilon_i && (1) \\ &= \sum_{n=0}^{l} \beta_{n,0}(t_i - t_0^*)_+^n + \sum_{j=l}^{K} \sum_{n=l_0}^{l} \beta_{n,j}(t_i - t_j^*)_+^n + \epsilon_i,\ \ i = 1, ...N && (2) \end{aligned}$$

In our case, $l = l_0 = 1$.

Basis of the algorithm

The feature is here that the dimension of the true model is unknown. So that the final model comes from the class of models $M_0, M_1, ...$ where M_K represents a model with K interior knots. Then the parameters of a model M_K are $(K, \theta^{(K)})$ where K represents the number of knots, $\theta^{(K)} = (t_1^*, ..., t_K^*, \sigma^2)$ with $t_0^* = t_1$, $t_{K+1}^* = t_N$ and σ^2 corresponds to the variance of the model error $\epsilon_i = v_i - v_i^*$, $i = 1, ..N$.

Inference about K and $\theta^{(K)}$ is based on the joint distribution of $(K, \theta^{(K)}, v)$. The best $(K, \theta^{(K)})$ are assumed to maximize the posterior distribution $p(K, \theta^{(K)}/v)$. The coefficients $\beta_{n,j}$ in equation (2) are then estimated by least-squares minimization.

This posterior distribution is the normalised product of the prior probability for each model, the parameter prior distribution and the likehood.

$$p(K, \theta^{(K)}/v) = \frac{1}{Z} p(K) p(\theta^{(K)}/K) p(v/K, \theta^{(K)})$$

The prior distribution for the error variance is taken to be $\pi(\sigma^{-2}) = gamma(10^{-3}, 10^{-3})$. The prior probability of the dimension K of the model is a Poisson distribution with parameter λ_p:

$$p(K) = \frac{\lambda_p^K exp(-\lambda_p)}{K!}, \quad K = 0, 1, 2, ...$$

The choice of λ_p is discussed further.

Since working with an analytic or numerical expression of this distribution is impossible, we have to simulate samples of the joint posterior distribution.

Denison *et al.* propose the design of three types of move between the subspace Θ_K for θ_K which allows us to explore all the subspaces, searching for the (K, θ_K) which minimizes :

$$MSE = \frac{1}{n} \sum_{i=1}^{N} v_i - f_{K,1}(t_i)^2$$

The three possible transitions of the Markov Chain are:

(a) birth : addition of knot

(b) death : deletion of knot

(c) movement of knot

Each move is chosen with probablity b_K, d_K and η_K which depends on the prior distribution of K.

The algorithm of Denison *et al.* can be summarized in three steps as follows:

1. Initialization : choose λ_p knots uniformly among the t_i and set k equal to the number of interior knots.

2. Choice of the type of move : generate u uniformly and compare it to b_k and d_k to choose a birth, death or move transition.

3. Iteration : repeat step 2 until the MSE no longer decreases.

Let us remark here that the quality of fit is measured by the MSE since, as already explained, it is impossible to replace the criterion by the one we focus on, that is difference between touch-down with the fitted wind curves and the real ones. Anyway, verifications are carried out (see estimation of error in Section 4.) to ensure that the fits were satisfactory according to our criterion.

Choice of the prior distribution of K and spline smoothing

One important step is the choice of the prior distribution of K according to the final number of knots aimed at. Recall that K is assumed to follow a Poisson distribution with parameter λ_p:

$$p(K) = \frac{\lambda_p^K exp(-\lambda_p)}{K!}, \quad K = 0, 1, 2, ...$$

Two examples are taken here to illustrate the sensiblity of λ_p. Few values of λ_p are tested on the two raw and on the two smoothed winds. The final number of knots after 10000 iterations is plotted against the different values of λ_p. The two top panel of Figure 12 corresponds to the raw data and the other two to the smoothed data.

These plots confirm that the Denison *et al.* algorithm gives better performance on "smooth" curves and show that the value of λ_p is quite impossible

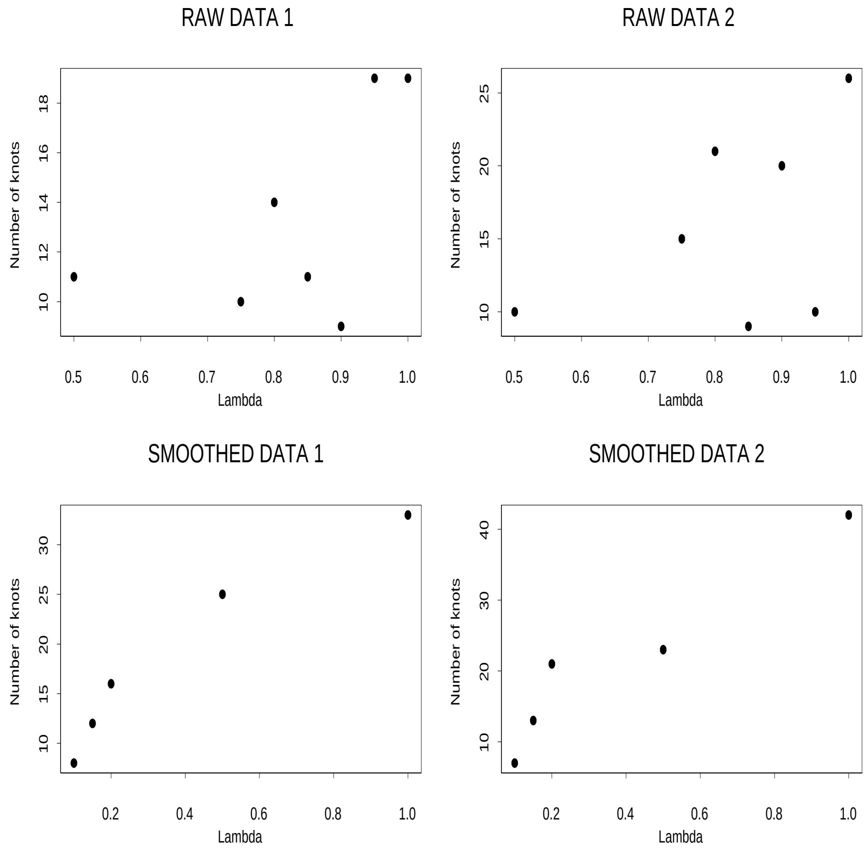

Figure 3: This Figure depicts the influence of λ_p on the final number of knots K obtained for two raw and smoothed winds. The difficulty to set λ_p for raw data leads us to use prior spline smoothing on our data.

to set for raw data. So we use the same prior spline smoothing on our data as for the classic algorithms.

So the parameters to be set for this algorithm are the smoothing parameter λ and the paramater λ_p of the prior distribution of K. Further details will be found in the next section for the choice of λ_p which is made according to the number of knots wanted as the influence of this parameter on the number of knots is quite 'smooth' on the smoothed data.

4 Practical comparisons

In this section, we apply our algorithms on the wind data described in Section 2. The test sample is composed of all 59 available winds. Each algorithm is tested separately at first and then a numerical comparison is carried out. The three classic algorithms are tested at first and the best one is compared to the Bayesian approach.

Graphic comparisons are made between raw data and split-linear fitting to check the quality of the methods. Then touch-down are simulated with the two kinds of wind and errors are estimated and compared so as to choose the best algorithm.

For the three classic algorithms, the tests are carried out with empirically best values of the different parameters to be set. Actually, each test involves a few laborious steps so a global optimisation algorithm is impossible to compute. To get around this difficulty, the parameters have to be set empirically at first and then the best values are chosen. These tests lead us to choose the third method among the classic ones which gives the best results in terms of graphic comparisons and error minimization. Therefore a special effort is made to improve the choice of the needed values of parameters.

Then this third algorithm which corresponds to the adaption of the Hall and Titterington algorithm is compared to the Denison *et al.* automatic Bayesian linear curve fitting algorithm. To carry out a fair comparison, we have to focus on the final number of knots K (slope change point) as we are searching for a parcimonious linear-fits. The comparison protocol is as follows:

1. Choose the maximum number of knots allowed K_{max} (in our case, $K_{max} = 20$ since too close slope changes do not have any physical meaning).
2. For each wind, check that the number $K_{class} < K_{max}$ (K_{class} is the number of knots obtained by the classic algorithm).
3. For each wind, set λ_p (parameter of the Poisson distribution of the number of knots) such as $|K_{class} - K_{Bayesian}| \leq 3$ with $K_{Bayesian}$ number of knots given by the Bayesian linear curve fitting algorithm.
4. Carry out the graphical comparisons for each wind.
5. Carry out the numerical comparisons for each wind by simulating the X_{TP} impact obtained with the two split-linear fits.

4.1 Graphic comparisons

Test of the algorithm based on on slope comparisons

The algorithm is tested on 59 winds with the threshold value s set to 0.1. As explained in Section 3, this value is the best empirical one for a few winds and is checked on the other data. In a general case, it could have been set by a rigourous cross-validation protocol.

Eighty percent of the graphic comparisons are satisfactory. Figure 4 depicts satisfactory fit. Every significant slope change are detected. Little errors on the two bounds of the time interval are due to the method which excludes the first and the last interval. The influence of wind far from touch-down is negligible and wind in the last seconds (1 or 2) has no influence on the aircraft. Therefore, this kind of error does not have any practical consequence.

However, in some cases, fitting can be very bad, as in Figure 5. A significant slope change at time 215 is completely forgotten whereas non-significant slope changes are detected around time 50. The threshold value may not fit this case but as we are searching for a threshold value suitable for each wind, this drawback is serious.

Test of the algorithm based on differences between smooths

The algorithm is tested on 59 winds with the parameter values set to $\lambda = 10^{-5}$ and $h = 15$ points. These values are the best empirical ones. Further

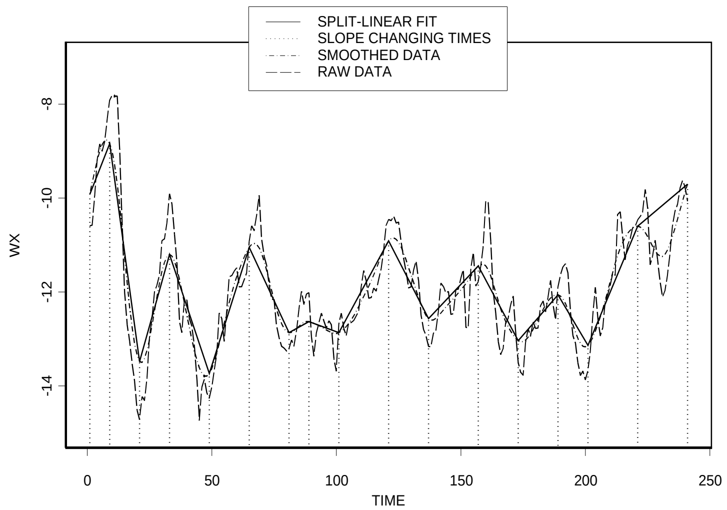

Figure 4: Example of a satisfactory graphic comparison between the raw data and the split-linear fitting obtained with the algorithm based on slope comparisons. Every significant slope change points are detected.

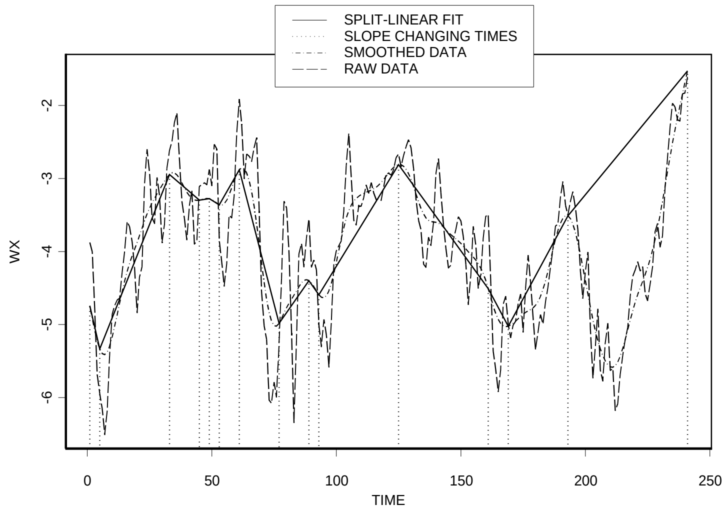

Figure 5: Example of an unsatisfactory graphic comparison between the raw data and the split-linear fitting obtained with the algorithm based on slope comparisons. A significant slope change at time 215 is completely forgotten whereas non-significant slope changes are detected around time 50.

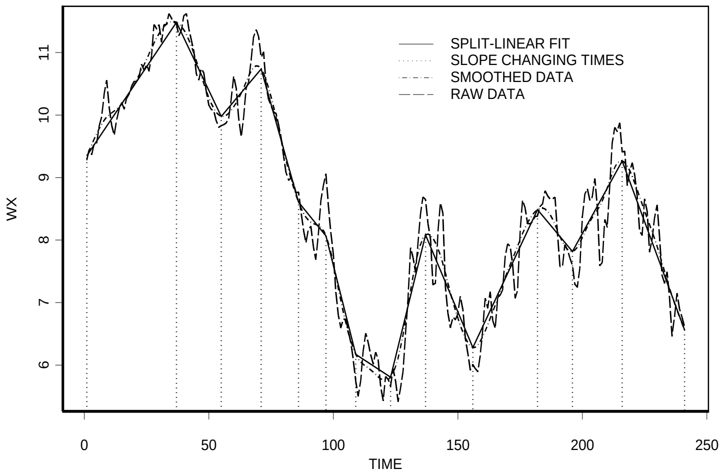

Figure 6: Example of a satisfactory graphic comparison between the raw data and the split-linear fitting obtained with the algorithm based on differences between smooths. Every significant slope change points are detected.

results in Section 4.2 will show that they also are the best as regards error minimization.

Eighty percent of the graphic comparisons are satisfactory. An example is plotted in Figure 6 where all the slope change points are detected. Let us remark that this split-linear is carried out with spline smoothing instead of the linear combination smoothing proposed by Hall and Titterington. This plot can be compared to Figure 7 where the linear combination smoothing is applied on the smoothed data with the same bandwidth and confirms the interest of using spline smoothing.

However, a few wind are not correctly fitted with this algorithm. An unsatisfactory case is presented in Figure 8. At least five slope changes are forgotten at times 15, 190 and between times 210 and 250. The direct application of the algorithm inspired by Hall and Titterington does not allow the full detection of sign slope change. Therefore the two algorithms have been combined to get around this difficulty.

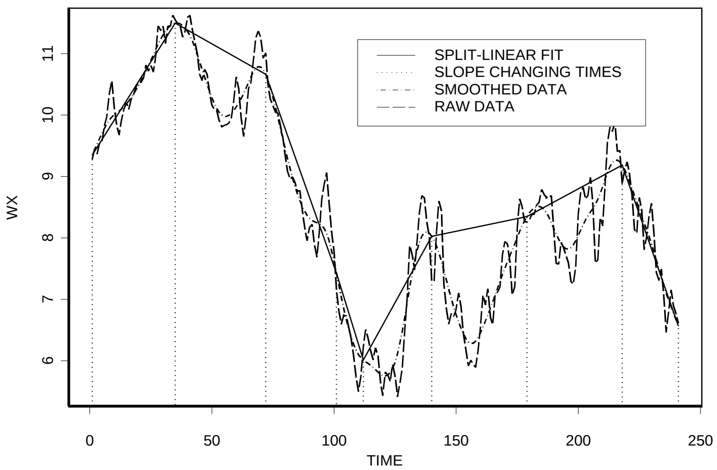

Figure 7: Comparison with direct application of the Hall and Titterington method with linear combination smoothing. The result is not satisfactory.

Test of the combination of the two algorithms

The combination of the two algorithms allows us to get around their respective drawbacks and to correct the unsatisfactory split linear fits presented above.

The parameters to be set are the same as for the second algorithm : $\lambda = 10^{-5}$ and $h = 15$ points. These values are the best empirical ones. Further results in Section 4.2 will show that they also are the best as regards error minimization. There is no need to set a threshold value as in the first method since only slope sign changes are detected with slope comparisons.

Figure 9 corresponds to Figure 5 and Figure 10 is associated with Figure 8. The slope changes forgotten by the first method in these two cases are detected by the third algorithm. Every graphic comparisons are satisfactory. Further results will confirm this tendancy.

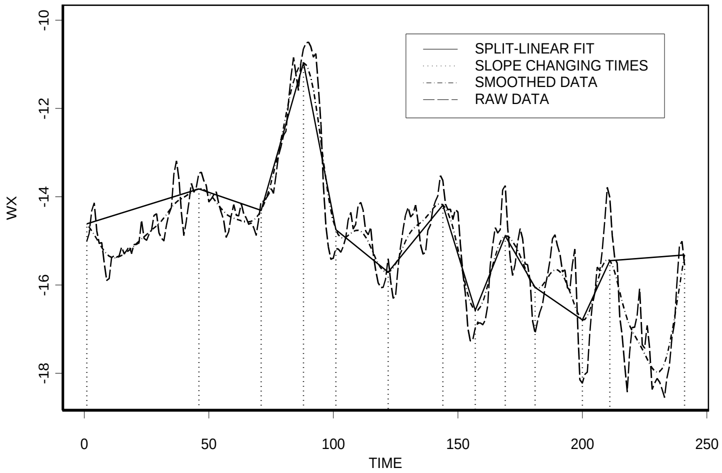

Figure 8: Example of an unsatisfactory graphic comparison between the raw data and the split-linear fitting obtained with the algorithm based on differences between smooths. At least five slope changes are forgotten at times 15, 190 and between times 210 and 250.

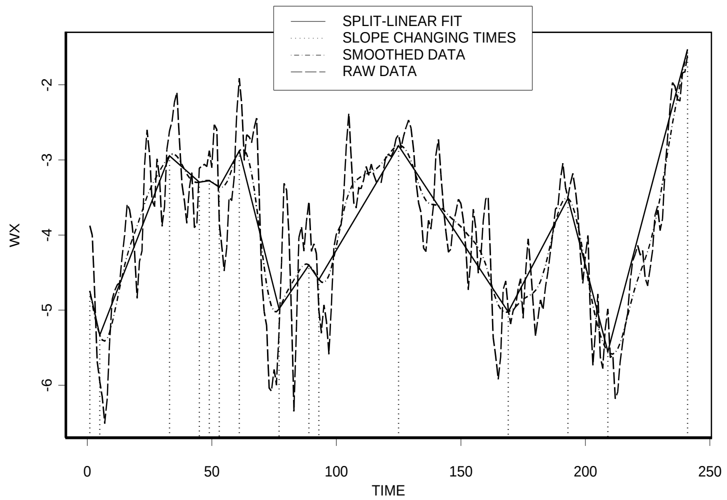

Figure 9: Example of a satisfactory graphic comparison between the raw data and the split-linear fitting obtained with the combined algorithms. This corresponds to Figure 5. The quality of fitting is significally improved.

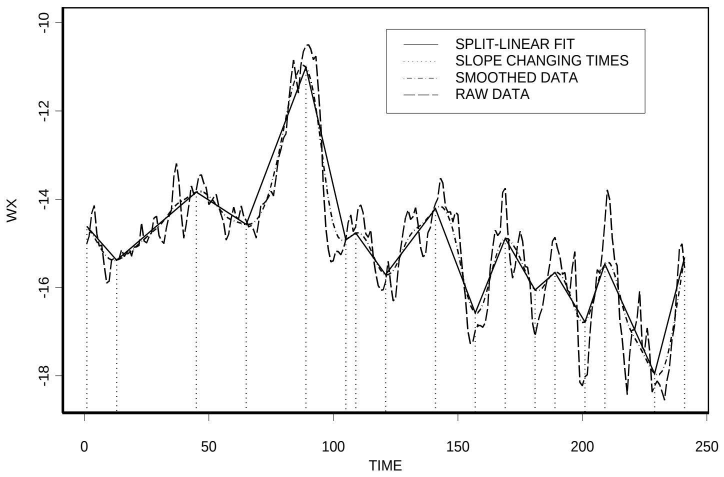

Figure 10: Example of a satisfactory graphic comparison between the raw data and the split-linear fitting obtained with the combined algorithms. This corresponds to Figure 8. Every slope sign change is detected.

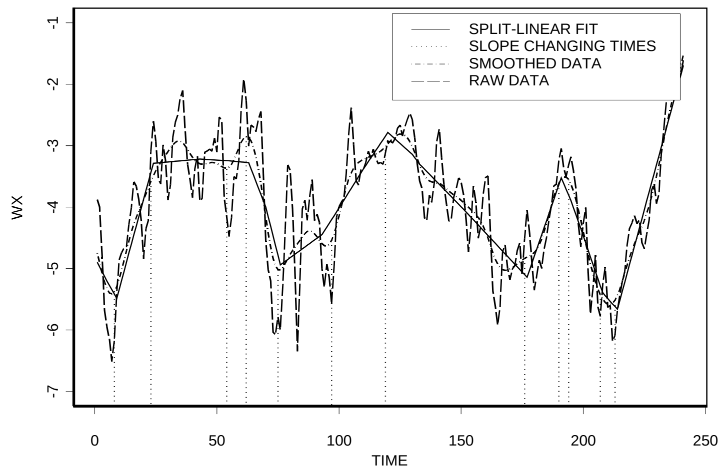

Figure 11: The raw data and the split-linear fit obtained on smoothed data with the Bayesian algorithm are plotted. This Figure is associated with Figure 9.

Test of the Bayesian automatic curve fitting algorithm

The two graphic examples shown in Section 4.1 (Figure 9 and 10) are then compared to the results given by the Denison *et al.* algorithm on the same two winds. The comparison protocol is followed. For each wind, K_{class} is estimated then λ_p is chosen such that $|K_{class} - K_{Bayesian}| \leq 3$.
For the first wind (Figure 13), $K_{class} = 13$, $K_{Bayesian} = 12$ for $\lambda_p = 0.15$.
For the second wind (Figure 14), $K_{class} = 15$, $K_{Bayesian} = 13$ for $\lambda_p = 0.15$.

The graphical results are less satisfactory than the one given by the classic algorithm. But as our criterion to be met is mainly to obtain the same X_{TP} touch-down with the split linear fit and the raw wind as explained in Section 4.2, we carry out numerical comparisons.

4.2 Estimation of error

Graphic comparisons bring out the fact that the combination of the first two algorithms give the best results as regards the full detection of the slope

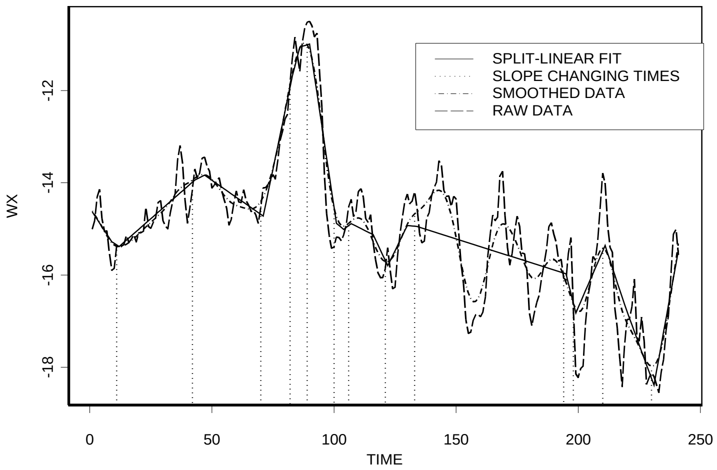

Figure 12: The raw data and the split-linear fit obtained on smoothed data with the Bayesian algorithm are plotted. This Figure is associated with Figure 10.

change points among classic methods. Moreover the Bayesian algorithm is also satisfactory. As previously said, the main criterion of satisfaction consists in the estimation of error. We replace here the usual residual error minimization by the minimization of the difference between touch-down with raw data and the split-linear fit.

A subset of 10 winds is extracted. A split-linear fit is estimated with each algorithm using the same values of parameters as in Section 4.1. Then touch-down simulations are computed comparing the value of X_{TP} obtained with each raw wind and its three fits. The errors are estimated in percent of the X_{TP} calculated with the raw wind.

The numerical comparison is carried out in the same way as the graphic comparison. The three classic algorithms are compared at first. Since the combination of the first two algorithms gives the best results, special attention is paid to the setting of the parameters and this optimized method is then compared to the Bayesian approach.

Comparison of the three classic algorithms

Figure 13 depicts the three boxplots of errors of the three algorithms. The third algorithm allows us to summarize the influence of a wind by a split-linear fit while losing less than 5% precision on the X_{TP} estimation. This method has been fully tested for the parameter values set to $\lambda = 10^{-5}$ and $h = 15$ points which correspond to the best empirical values. Since this algorithm gives the best results, we have sought further the optimum choice of the values of λ and h.

Optimum setting of the parameters of the third algorithm

The difficulties of the optimization of these values consists in the steps necessary to test the influence of the fits on touch-down. So, a few values have to been chosen then tested instead of a fully optimization algorithm. The tested values for λ and h are as follows : $\lambda = 10^{-4}$, 10^{-5}, 10^{-5} and $h = 10$, 15, 20. Tests mixing these values are carried out to confirm the best empirical choice and explore the interaction between λ and h. The error mean (%) is plotted for each level of our two factors in the left panel of the Figure 14. In the right panel, the influence of the bandwidth h on the error mean

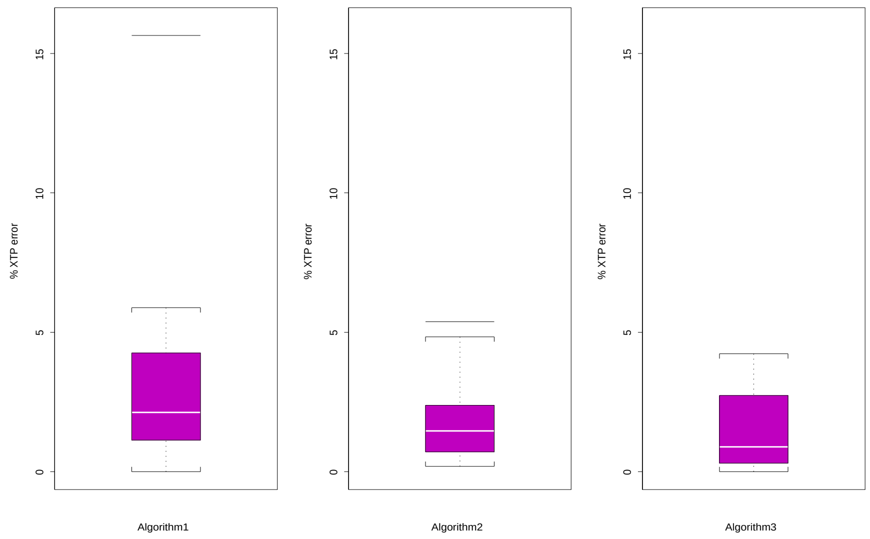

Figure 13: Boxplots of errors of the three algorithms estimated in percents of the X_{TP} calculated with the raw wind. The third algorithm gives the best results with 4.5% maximum error.

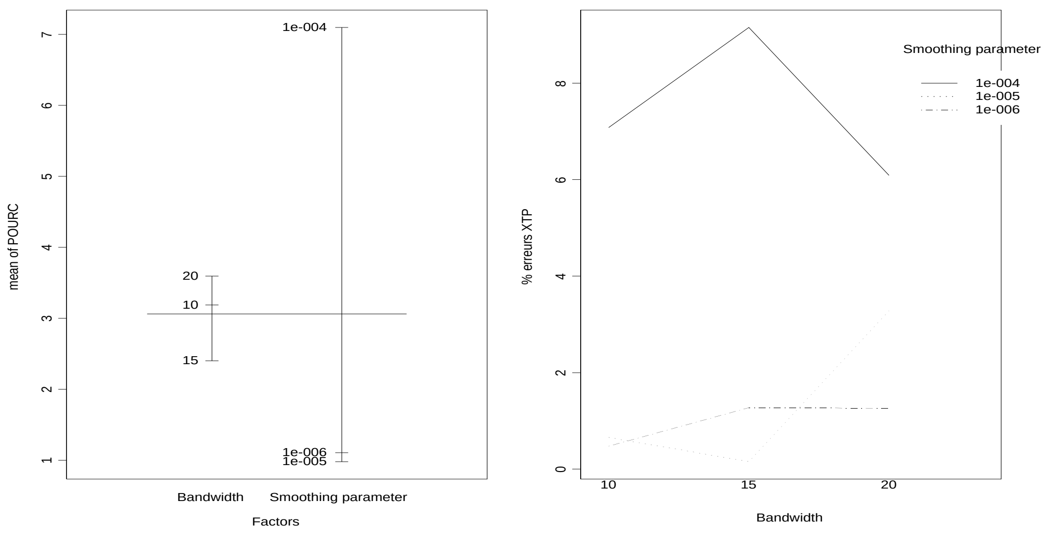

Figure 14: In the left panel, the mean of the error (%) is plot for each level of the factors. In the rigth panel, the influence of the bandwidth h on the error mean is plotted for each value of the smoothing parameter λ. The values of these two parameters have to be chosen simultaneously and the best choice is $\lambda = 10^{-5}$ and $h = 15$.

is plotted for each value of the smoothing parameter λ. It shows a little interaction between h and λ which significates that the two values may not be chosen separatly. The best combination is $\lambda = 10^{-5}$ and $h = 15$.

Let us remark that the results of the combination $\lambda = 10^{-6}$ and $h = 15$ are also satisfactory. But our goal is to maximize the simplication of information. Since $\lambda = 10^{-5}$ is a stronger smoothing parameter, we prefer to set this parameter to this value.

The adaptation of the Hall and Titterington algorithm leads us to adapt and compute an efficient split-linear fit for our data which allows us to resume the influence of wind on the behaviour of automatic landing pilot. The winds are split up into linear components corresponding to gradients. The loss of information is less than 4.5% on the touch-down simulation and allows to work with less simplified data than raw winds for further studies.

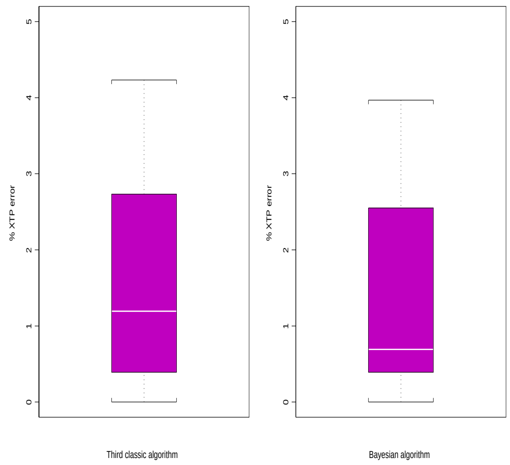

Figure 15: Boxplots of errors of the two algorithms estimated in percents of the X_{TP} calculated with the raw wind. The resuls are comparable. In the two cas, the maximum error is less than 4.5%.

Let compare this results with the ones obtained with Bayesian Curve Fitting.

Comparison between the third algorithm and the Bayesian Curve Fitting

For the same subset of 10 winds as in Section 4.2, error between X_{TP} touch-down obtained with the split-linear Bayesian fit and the raw wind is estimated. The comparison protocol is applied here. For each wind, K_{class} is estimated and λ_p is chosen such as $|K_{class} - K_{Bayesian}| \leq 3$.

Touch-down simulations are computed and then the error between X_{TP} obtained with the raw wind and the split-linear fit is estimated in percent. Figure 15 depicts the boxplot of errors of the third classic algorithm and the Bayesian one. The performance of these two methods are similar.

5 Discussion

The purpose was the split-linear fit of wind curves. Four algorithms have been detailled in this article, tested and compared on real-life data issued from wind measurement during flight tests. We aimed at determining every significant slope changes in wind curves. The criterion to met was the achievement of the same simulated touch-down with the raw wind and its linear fit. Two kinds of approach have been considered. First of all, classic algorithms based on slope or smoothing comparisons have been tested. Then the best one has been compared to a Bayesian curve fitting algortihm. This involved particularily the choice of best values for the parameters of each method. For the best classic algorithm, the smoothing bandwidth and parameter had to be set while the prior distribution of the number of slope changes was the main parameter to consider for the Bayesian algorithm. The main features of this article is the interest of a prior smoothing of the data and the setting protocol for all parameters. This settings have to be carried out empirically in our case because of the specificity of the satisfactory criterion to met. The performance of these two methods are anyway satisfactory and similar. The choice can only be made according to the difficulty to arrange the parameters. One could be interested to consider a cross-validation to refine this settings or even automate the choice of these parameters and carry on the comparison.

Acknoledgements This research was carried on in collaboration between AEROSPATIALE-MATRA-AIRBUS, the ANRT and the Probability and Statistics laboratory of Paul Sabatier University in Toulouse,FRANCE. We are grateful to C.BES, J.IRVOAS from AEROSPATIALE-MATRA-AIRBUS for their suggestions and comments. We thank V.COUALLIER, Paul Sabatier University for his help in the search of references, D.DENISON for his help in the use of his algorithm.

References

Denison, D. G. T., Mallick B. K. & Smith, A. F. M. (1998) Automatic Bayesian curve fitting, *J. R. Statistical Soc. B*, **60**, 333-350.

Green, P. (1995) Reversible jump Markov chain Monte Carlo computation and Bayesian model determination, *Biometrika*, **82**, 711-732.

Hall, P. & Titterington, D.M. (1992) Edge-Preserving and Peak-Preserving Smoothing, *Technometrics*, **34**, 429-440.

Loader, C. R. (1996) Change point estimation using nonparametric regression, *Annals of Statistics*, **24**, 1667-1678.

Müller, H. G. & Song, K. S. (1997) Two-stage change-point estimators in smooth regression models, *Statistics* & *Probability Letters*, **34**, 323-335.

S-Plus (1997) *S-PLUS 4 Guide to Statistics*, Mathsoft.

Wu, J.S. & Chu, C. K. (1993) Kernel-type estimators of jump points and values of a regression function, *Annals of Statistics*, **21**, 1545-1566.