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A statistical approach for tolerancing from design stage to measurements analysis

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Abstract

Starting from the development phase, tolerance design must be accurate enough to not only hedge against various uncertainties in order to ensure assembly feasibility but also minimize production cost and avoid expensive over-quality. Once tolerances are agreed, the production allows tolerance features observations and we propose a verification and correction on initial model based on the knowledge of measurement data. The feedback consideration also enables risk evaluation of each tolerance and a more accurate limit definition knowing measures of other assembly contributors is proposed. In addition, we propose an algorithm to optimize the tolerance sharing within a stack chain based on various relevant cost criteria. Finally, an example of tolerancing industrial applications on aerostructures use-cases is detailed to illustrate the methodology from tolerance design to feedback measurement analysis.

Keywords: Tolerance design; manufacturing; tolerance analysis

Introduction

Tolerancing plays a key role in a manufacturing process. Dimensions may have some deviations from the designed value with significant impact on the quality and functional requirements of the final product. These involve different physical characteristics of parts, such as part length, hole position, pin, etc., later called features. All tolerancing issues and notations are detailed in the engineering design data set and related documentation practices [1] and [2]. In final documents describing products design, tolerance intervals are defined according to company knowledge and scientific analysis in order to determine these acceptable variations. From product development to serial phase, tolerance analysis is used as a process to manage an efficient production and profitability and also ensure safety and performance.

In recent years, global approaches on tolerancing process emerged, involving several aspects of uncertainties management through numerous tolerancing activities as detailed in [3]. In [4], relations between tolerances, functional requirements, process capabilities and product performance are addressed. Indeed, there are lot of interactions and both feed-forward and feed-back dimension should be considered. This article focuses on these aspects and details how statistical approaches can be used not only in design phase to meet functional requirements linked to product performance and safety, but also in production phase with refinement from measurement data analysis. In the context of closed loop engineering linked to process capabilities and product performance, several tolerance models for tolerancing processes are discussed in [5] and associated tools for computer aided tolerancing are presented. In [6], they also present different tools such as 3D software for computer aided design and tolerancing. The authors suggest considering the whole life cycle of the product in order to solve issues related to the focus on certain life cycle stages for the tolerance allocation.

As stated in [7], tolerancing process relies on four main aspects: tolerance representation, tolerance specification, tolerance analysis or verification and tolerance synthesis. Representation means adequacy between functional requirements and

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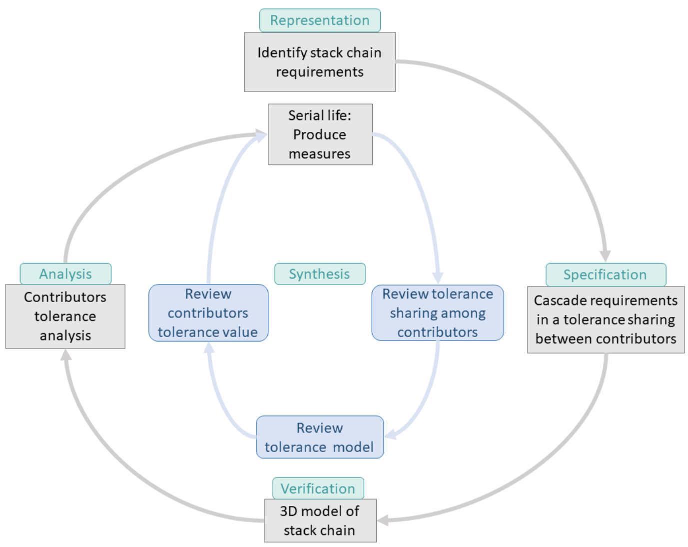

Fig. 1. Proposed tolerancing process.

targeted tolerance in output of an assembly (represented by a stack chain). For instance, targeted aerodynamics performance should be ensured thanks to proper output tolerance definition for a final assembly stage. Specification gives information on how to cascade a targeted output tolerance to contributors involved in the stack chain. This aspect is often called the “top down” approach. Conversely, tolerance analysis is about the concordance between detail part or process tolerances and functional requirements. This is a “bottom up” approach using a tolerance model. Finally, synthesis is about the correction loop performed by additional information on product. In [7] a complete model based on ISO standards and mechanical considerations is described in order to propose a complete and coherent tolerance process.

In a similar way, this article relies on a tolerancing process detailed in Figure 1 which details the different phases of a product tolerance study, starting from the product requirement identification in the beginning to measurement production and analysis at the end of the cycle. We will focus on specification, verification, analysis and synthesis of tolerances. Various methodologies are proposed at different levels of the tolerancing process.

The first section focuses on design phase: after the context statement, a robust statistical approach is proposed in order to assess about the variability of an assembly output feature based on the knowledge of contributors tolerance. This is part of the analysis phase. The second section is about enhancement of tolerance model. This is both included in the verification and synthesis of tolerances because it uses 3D model and feedback measurement data for model improvement. In the third section, feedback measurement data are used to review tolerance values through risk evaluation and contributors sharing in order to closely reflect reality. This is associated to synthesis stage as measurement data are needed and to specification phase for the tolerance sharing optimization. The last section details a use case to illustrate the proposed methodologies of the article.

1. Design phase

Before the production launch, dimension uncertainty must be managed by tolerancing analysis. During the development phase, tolerances are designed and should be as robust as possible in the limits provided by safety and performance. Indeed, parts and assemblies have not yet been produced and we set targets based on process assumptions that may be invalidated. The first step is a 3D analysis which gives parameters to build a tolerance model and define tolerance intervals.

1.1. Tolerance model and cascade

As assembly geometrical tolerancing output, we define top levels requirements. One of these is denoted Y and is associated to a stack chain composed of p contributors $X_1, \dots, X_p$ assumed to be independent. The feature Y is associated to the dimension of interest that represents one functional requirement of the final product. The contributors $X_1, \dots, X_p$ are associated to geometrical features and assumptions on the type of distribution are made in order to define their tolerances.

Each contributor impacts the top level requirement feature in its own way. For isoconstrained mechanisms, one of the common approaches assumes a linear link between contributors and top level requirement (see [8]). Tolerance model coefficients can for instance be determined thanks to specific software with 3D simulation such as 3DCS or Mecamaster. We will focus on these methods to compute coefficients $\alpha_1, \dots, \alpha_p$ of each contributor within a linear model $Y = \sum_{j=1}^{p} \alpha_j X_j$.

This model allows to cascade requirements from one assembly level to another from elementary parts to final assembly.

1.2. Tolerance definition

In design stage, there are no available dimension measurements because we focus on tolerance allocation in the design phase of a product prior production. Considering one specific assembly stage, one of the main issues for tolerancing is to assess the variability of contributors knowing the target for top level requirement. Conversely, another issue is to assess the variability of an output feature of the assembly knowing the tolerance range of the input features.

To determine the variation, several approaches based on sampling, fuzzy arithmetic or analytical procedures have been studied. These methods are reviewed in [9]. The two main analytical methods are detailed in [10] : Worst Case and statistical approaches. The Root-square Sum of Squares (RSS) gives a statistical result relying on the assumption that contributors are produced following a perfect Gaussian distribution providing a range of 6σ on Y if all contributors are within their 6σ range. In process control context, it means reaching a C_p of 1 and C_{pk} of 1 for all contributors (see [11] for indicators definition). However, and more specifically in aeronautic industry where no pre-series are done, indicators commonly used to monitor process capabilities such C_p and C_{pk} are not available during design phase as measurement are not yet produced and the normality of distributions can not be verified. As a conservative approach

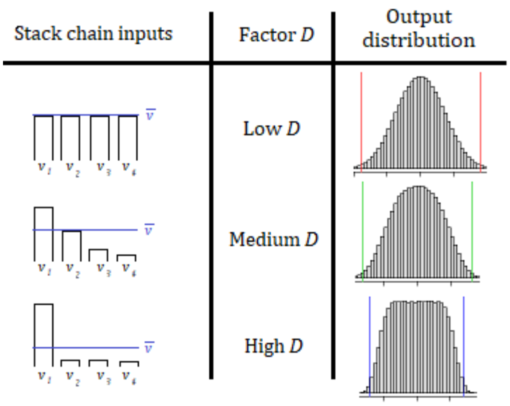

Fig. 2. Stack chain structure and balance factor D.

and with the sole knowledge of contributors tolerance bounds, we can take the less informative distributions for contributors: uniforms.

If we assume uniform distributions on the parts dimensions, the result would be robust against non-normal distributions up to a certain limit that is generally not reached by industrial production. Processing uniform distributions induces more complex calculations than the RSS method requiring statistical software. To avoid this, a method is used by Airbus to approximate the combination of uniform distributions in order to define robust tolerances in the design phase. It has been built based on Monte Carlo simulation data and a disproportion parameter D. As for the Gaussian case, quantile at 0.27% are observed on Monte Carlo simulations and a linear regression with respect to D is carried out to obtain the result.The expression of D is $\forall v_1, \dots, v_n > 0, D = \frac{\max_i(v_i) - \bar{v}}{\sum_{i=1}^n v_i}$ where $\bar{v}$ is the mean of values $v_1, \dots, v_n$.

Such D factor quantifies how far from the mean is the main contributor of the stack chain and has the advantage of being interpretable, as shown in Figure 2.

Let $v_1, \dots, v_n > 0$ be a set of tolerance bounds with input features balance ratio D, this rule gives an output feature tolerance interval $[-T_{Airbus}, T_{Airbus}]$ defined as : $T_{Airbus} = 1.6 \times (-0.56D + 1.04) \times T_{RSS}$ with $T_{RSS} = \sqrt{\sum_{i=1}^n v_i^2}$ and D as defined above.

A more complex and accurate approach has been designed in [12]. It mathematically defines a methodology in between worst case approach and Root-square Sum of Squares (RSS) to allocate tolerance, with the advantage of allowing the management of confidence level.

2. Tolerance model enhancement from feedback measurement data

As we discussed in Section 1, design phase induces a robust evaluation and as expected production may differ from initial assumption without impact on product safety and performance. Inconsistency between as-design tolerance model and data driven model may appear due to measurement uncertainty, unexpected distortions or other geometrical deviations. This model error is the topic of a case study for tooth contact in [13]. The impact of measurement uncertainty is detailed [14] through the role of metrology and in [15] thanks to evaluation, normalization and management of measurement uncertainties. In [16], several approaches for tolerance analysis are compared to skin shape based model in order to assess about each model performance.

The challenge of the following sections is to refine tolerance models and tolerance sharing based on feedback data in order to better reflect the industrial capabilities. Once a stack chain is used in production, some feedback measures become available. Unfortunately, it is common that not all contributors have available observations. Only a number m of contributors out of the $p \leq m$ contributors in the stack chain are measured. We might also have observations for the top level requirement of the stack chain feature Y. The knowledge of the amount of information in the m measured contributors allows to discuss the tolerance model.

2.1. Sign of influence coefficient

During the design phase, most tolerances are defined as centered around the nominal value of the characteristic as it is a clear representation of expected dimension and its tolerated variation. Indeed, a tolerance interval centered around zero is not affected by an influence sign error. Geometrical dimension engineer will not pay attention to the sign of influences knowing that it is also affected by the way the measurement are implemented years after. As a result, the implementation of tolerance influence calculation sometimes leads to erroneous sign.

A correlation study between contributors and the top level feature can solve this issue by identifying the sign of influence coefficient directly from contributors features observations. Another approach that we detail hereafter is based on linear regression relying only on coefficients signs. It differs from the traditional linear regression approach because we assume that absolute values of coefficients are known, and only signs need to be corrected. For this analysis, we need measurements on contributors to perform the regression technique.

Let us denote m the number of contributors in an assembly, $\tilde{X}_1, \dots, \tilde{X}_m$ the measured contributors, $\tilde{Y}$ the measured top level feature, $a_1, \dots, a_m > 0$ the absolute values of coefficients and $s_1, \dots, s_m \in \{-1, 1\}$ the signs of regression coefficients. The result of this study gives $\hat{s}_1, \dots, \hat{s}_m \in \{-1, 1\}$ the estimated best solution for the contributors influence coefficients signs:

$$(\hat{s}_1, \dots, \hat{s}_m) \in \underset{(\hat{s}_1, \dots, \hat{s}_m) \in \{-1,1\}^m}{\operatorname{argmin}} \left\| \tilde{Y} - \sum_{i=1}^{m} \left(s_i a_i \tilde{X}_i \right) \right\|^2 \qquad (1)$$

The final result for influence coefficients of the tolerance model denoted $\alpha_1, \dots, \alpha_p$ are derived as follows $\forall i = 1, \dots, m,\ \alpha_i = \hat{s}_i a_i$.

Influence coefficients with correct sign are particularly important when we have to process a real value on an assembly item.

2.2. Integration effect

Knowing the regression coefficients and their proper signs, we denote $\tilde{X}_1, \ldots, \tilde{X}_m$ the observed contributors. We introduce the partial residuals e to compensate the difference between observations and initial theoretical features $X_1, \ldots, X_m$: $Y = \sum_{j=1}^{m} \alpha_j \tilde{X}_j + \sum_{j=m+1}^{p} \alpha_j X_j + e$. If we assume all non-measured features to be centered, we have an information about the mean μ_e of e given by $\mu_e = \mu_Y - \sum_{j=1}^{m} \alpha_j \mu_j$ where μ_Y is the mean of observations of the top level requirement Y and $\mu_j = \frac{1}{n} \sum_{i=1}^{n} \tilde{x}_j^i$ is the mean of the contributor j.

This μ_e actually represents integration effects, as gravity effects that happen during the assembly or any shift errors during the measurement process or due to non-measured features. These effects are not captured by the 3D model used for regression coefficients estimation.

3. Risk analysis and tolerance sharing optimization

3.1. Risk evaluation and acceptance criteria

When the product is in design stage, tolerances both on contributors and top level requirement are set. The risk to be out of specification at the top level feature of an assembly is therefore fixed in accordance with the contributors tolerances and the selected value as top level target which should not be exceeded. During production phase, distributions of the contributors features are essential to value the risk at the assembly level. Knowing the tolerance model corrected and enhanced, the risk to be out of tolerance at the top level requirement can be re-evaluated. A convolution product of as-designed unmeasured contributors distributions and observed contributors distributions is carried out to estimate top level feature distribution and compute the risk to be out of requirement target. This approach can be completed with the consideration of the probability for a feature to be out of tolerance to determine acceptance criteria as a risk management.

3.2. Tolerance sharing identification before optimization

Another way to refine the tolerancing approach is to identify assemblies with capability disparities between contributors. Indeed, if a contributor is better produced than expected when tolerance has been designed, another contributor can benefit of this positive margin to enlarge its tolerances, as in the Figure 3 right. Several criteria have to be taken into account in order to identify the best candidate for tolerance sharing. For instance, the number of drawings impacted by a contributor or a requirement gives an information on how difficult it would be to initiate a change of design, and the number of recorded nonconformities for a tolerance feature gives an information on how valuable would be the change of design. Moreover, capabilities indicators such as C_p and C_{pk} (see [11]) can be used if sufficient feedback is available in order to define if a tolerance optimization is conceivable. Once assemblies opportunities for tolerance re-sharing are identified, optimization can be performed.

3.3. Multi criteria optimization

Optimization techniques have been considered for tolerance allocation, verification and variation management during product life cycle. In [17] and [18], the aim is to determine the optimal set of tolerances through the minimization of costs related to a part production and eventually meet the imposed restraint conditions. In [19], an approach based on genetic algorithm is proposed in order to minimize manufacturing cost, taking into account the interrelation of stack chains.In design phase, the verification cost (measurement tools, conformity assessment, …) are considered in [20] in order to define best tolerances considering inspection cost. The selection of the best assembly technique for an optimal tolerances allocation is tackled in [21]. Another parameter which is the product degradation and time value of money is taken into account in [22] for the the minimization of both cost and loss. In [23], an objective function is defined combining cost of manufacturing activities, inspection, product scraping, recycling, reliability and conformity. Our approach in this paper is to considered cost related to non-quality and to the process for changing a tolerance value. We focus on the objective to reduce money spent when nonconformities occur without spending too much for tolerances change in drawings.

In a first approach for tolerance optimization, let us focus on one assembly with a number p of contributors, represented by features $X_1, \ldots, X_p$. We might have feedback measurement data on these contributors (or not). Individually, the definition of tolerance will induce costs of out of tolerance. The tighter the contributor tolerance, the more expensive the price is. If a contributor is observed, we are able to assess the out of tolerance rate that we expect of this contributor whatever its tolerance bounds are. This is a criteria that will be taken into account in the stack chain optimization. A perfect stack chain should ensure the consistency between the tolerance bounds defined for contributors and the tolerated interval for the top level requirement. Tolerance model used by tolerancing specialists allows to have a prediction of the top level requirement distribution when we have observations for contributor input features. Considering the targeted limit for this top level feature, we are able to assess about the nonconformity rate for the top level requirement thanks to quantile functions. The article [24] also gives some other methods to estimate such a scrap rate in order to perform tolerance sharing optimization. When in the production phase, a change of tolerance design involves costs related to this modification. However, no matter how different is the new design from the old stack chain, the cost remains the same. The criteria which is relevant is a constant cost if a contributor tolerance interval is modified within a stack chain. Let $(x, t) = \big((x_1, \ldots, x_p), (t_1, \ldots, t_p)\big) \in \mathbb{R}^p \times \{0, 1\}^p$ where p is the number of contributors, $\big(x_1, \ldots, x_p\big)$ are the proposed tolerance bounds for contributors and $\left(t_1, \ldots, t_p\right)$ are binary indicators to assess about the change or not of contributors tolerance bounds. The indicator is equal to 1 if the initial tolerance is changed for the contributor, and 0 otherwise. Let us denote $c_1 : \mathbb{R}^p \times \{0,1\}^p \longrightarrow \mathbb{R}$, $c_2 : \mathbb{R}^p \times \{0,1\}^p \longrightarrow \mathbb{R}$ and $c_3 : \mathbb{R}^p \times \{0,1\}^p \longrightarrow \mathbb{R}$ the cost functions representing respectively non-quality for contributors, non-quality for top level feature and cost of a contributor tolerance change.

The non-quality for inputs is expressed as the sum of probabilities to be out of designed tolerance interval for each contributor. $c_1(x,t) = \sum_{i=1}^{p} \left[t_i \mathbb{P}(|X_i| > x_i) + (1-t_i)\mathbb{P}(|X_i| > v_i)\right]$. The second cost is the non-quality for the top level requirement. We consider it as the probability for the top level feature to be out of its targeted interval, it is highly dependent of tolerance bounds applied to contributors via the linear tolerance model. $c_2(x,t) = \mathbb{P}\left(|Y(x,t)| > v_y\right)$ The last cost is the cost of change, represented as a unit cost $c_3(x,t) = \sum_{i=1}^{p} t_i$. The optimization problem can then be formulated as the following non-linear mixed integer programming (MINLP) problem:

$$\min_{(x,t)\in\mathbb{R}^p\times\{0,1\}^p} \lambda_1 c_1(x,t) + \lambda_2 c_2(x,t) + \lambda_3 c_3(x,t) \tag{2}$$

where $\lambda_1, \lambda_2, \lambda_3 > 0$ are tuning parameters to be defined according to engineering judgment about importance of costs. For instance, non-quality for the output feature might be more detrimental than the cost of change of contributors tolerances. The user should adjust what is the most important criteria for the re-sharing optimization through the calibration of those coefficients.

4. Application - Use case

Let us introduce a very simple assembly to illustrate methodologies presented in this paper. The first part of this section presents initial assembly and tolerance model. The second part focuses on the various approaches to improve tolerancing process and finally the third part summarizes results.

4.1. Assembly description and data

In the following assembly, the top level feature is Y and the $p = 3$ contributors are X_1, X_2 and X_3 as described in Figure 3 left.

Table 1 summarizes information available for the assembly example. The second column gives the initial contributors influences that are all equal to 1 in this case. The third column gives the initial contributors tolerances intervals that have been designed and the fourth column gives observed tolerance intervals from measurement data, with the assumption that 99.73% of observations should be within the range presented in the table.

In this example we assume that some measurement data are available for the three contributors (Table 1) and for the top level feature. According to the Airbus calculation, the targeted

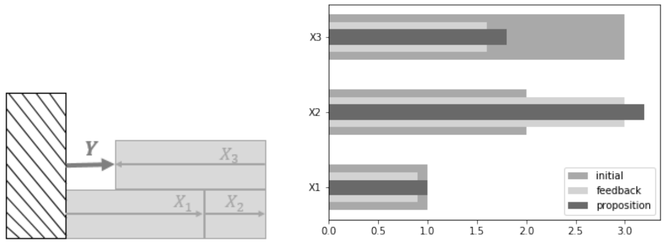

Fig. 3. Left :Application example assembly. Right:Application of tolerance sharing optimization on the example assembly.

Contributor	**Influence**	**Tol. int.**	**Obs. interval**	**Mean**
X_1	1	± 1	$\{-0.8, 1.0\}$	0.1
X_2	1	± 2	$\{-3.0, 3.0\}$	0
X_3	1	± 3	$\{-2.1, 1.1\}$	-0.5

Table 1. Assembly contributors information: initial contributors influences, initial contributors tolerance bounds and feedback

Contributor	**Influence**	**Tol. information**
X_1	1	interval ± 1
X_2	1	interval ± 2
X_3	**-1**	interval ± 3
Integration e	1	$\mu_e = -0.6$

Table 2. Assembly contributors information: corrected contributors influences, initial contributors tolerance bounds

tolerance interval for the top level feature Y of this assembly is ± 5.6 and we assumed feedback is centered.

4.2. Methodology roll-out

4.2.1. Design phase

For this study, the first step is to assess the target tolerance interval for the top level requirement. Whether it is a specification or a verification of an already defined tolerance, the proposed methodology gives a reliable value to be considered. Common approaches such as worst case and RSS respectively give $T_{WC} = 6$ and $T_{RSS} = 3.7$. With the Airbus rule, the balance indicator is $D = 0.17$ and the result for tolerance bounds of top level feature would be $T_{airbus} = 5.6$ from the computation presented in Section 1.2.

4.2.2. Model enhancement

With available measurement data, the correlation analysis between contributors and top level requirement lead to the sign correction of the influence coefficient of the third contributor : it is corrected in -1 instead of 1 initially. Still, with measurement data, the analysis detailed in Section 2.2 gives an integration stack with an offset of $\mu_e = -0.6$. Finally, the enhanced tolerance model is as described in Figure 2.

4.2.3. Tolerance sharing optimization

Based on criteria detailed in Section 3, a new tolerance sharing is proposed. Figure 3 right shows the result for the optimal tolerance sharing according to our criteria.

Based on this new tolerance sharing, non-quality for each contributor individually will be lower with this new limits. As for the assembly output, the Airbus calculation applied to the tolerance sharing obtained from optimization gives a resulting interval of ±5.6, which is in line with the initial target for the top level functional requirement. This tolerance sharing also has the economical advantage to avoid the change of tolerance for X_1 that was close to its feedback interval thanks to an accurate set-up of $\lambda_1, \lambda_2, \lambda_3$.

5. Conclusion

This article deals with various tolerancing design steps that are contained to propose a complete tolerance analysis process. First, we focus on tolerance allocation and we propose a simple formula to assess about the variability of an assembly output knowing tolerance intervals of contributors. Then, a smart use of feedback measurement data is proposed to enhance known models established in definition phase. It involves tolerance model enhancement by correcting coefficients signs of a linear tolerance model and the consideration of an integration effect estimated from measures on contributors and top level feature. This enhancement allows to better reflect the real situation in plants and supports robust and reliable approaches for risk analysis and tolerance sharing optimization. They allow to manage tolerances issues in serial life phase, either by out of tolerance risk estimation based on assembly measurements, or by reviewing tolerance sharing involving feedback analysis and optimization based on several relevant criteria.

This work provides a global view on tolerancing process and various axes of improvement are proposed, involving statistical methods about assumed features distributions or based on feedback measurement analysis and optimization techniques.

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