

# H-DISTRIBUTIONS AND COMPENSATED COMPACTNESS

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H-measures, introduced independently by Luc Tartar and Patrick Gerard, are matrix Radon measures describing the behaviour of weak limits of quadratic quantities. They proved to be very successful tool in investigations of asymptotic limits of quadratic quantities. However, they turned insufficient for nonlinear problems.

H-distributions were introduced by Antić and Mitrović as an extension of H-measures to the  $L^p-L^q$  setting. Their variants have been successfully applied to problems in velocity averaging (Lazar-Mitrović 2012) and compensated compactness with variable coefficients (Mišur-Mitrović 2015). Unlike H-measures, which are nonnegative Radon measures, H-distributions are distributions in the Schwartz sense, which follows from the standard Schwartz kernel theorem.

To give a precise description of H-distributions, we will introduce the notion of anisotropic distributions – distributions of different order with respect to different coordinate directions. In order to show that H-distributions are anisotropic distributions of finite order with respect to every coordinate direction, we will prove a variant of Schwartz kernel theorem.

In the second part of the talk, we will show a variant of compensated compactness using a variant of H-distributions. Namely, we will investigate conditions under which, for two sequences  $(\mathbf{u}_r)$  and  $(\mathbf{v}_r)$  weakly converging to  $\mathbf{u}$  and  $\mathbf{v}$  in  $L^p(\mathbf{R}^d; \mathbf{R}^N)$  and  $L^q(\mathbf{R}^d; \mathbf{R}^N)$ , respectively,  $1/p+1/q \leq 1$ , a quadratic form  $q(\mathbf{x}; \mathbf{u}_r, \mathbf{v}_r) = \sum_{j,m=1}^N q_{jm}(\mathbf{x})u_{jr}v_{mr}$  converges toward  $q(\mathbf{x}; \mathbf{u}, \mathbf{v})$  in the sense of distributions. The conditions involve fractional derivatives and variable coefficients, and they represent a generalization of the known compensated compactness theory. We will apply the developed techniques to a nonlinear (degenerate) parabolic equation.

This talk will present results of joint works with Nenad Antić, Marko Erceg and Darko Mitrović.

## REFERENCES

- [1] N. Antić, M. Erceg, M. Mišur, *On H-distributions*, in progress 25 pp.
- [2] N. Antić, M. Mišur, D. Mitrović, *On the First commutation lemma*, submitted 17 pp.
- [3] M. Mišur, D. Mitrović, *On a Generalization of Compensated Compactness in the  $L^p-L^q$  setting*, *Journal of Functional Analysis* **268** (2015) 1904–1927.

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*Key words and phrases.* H-distributions, anisotropic distributions, kernel theorem, compensated compactness.