Differential systems

Exercise 1. 1. Compute the solution (y_1, y_2) to the system

$$\begin{cases} y_1' = 2y_1, & y_1(0) = 1, \\ y_2' = y_2, & y_2(0) = 1. \end{cases}$$

Prove that $y_1(t) = y_2(t)^2$ and give a graphical representation in the plan of $t \mapsto (y_1(t), y_2(t))$ (called the trajectory of the solution).

2. Compute the solution (y_1, y_2) of the system

$$\begin{cases} y_1' = 2y_1, & y_1(0) = 1, \\ y_2' = -y_2, & y_2(0) = -1. \end{cases}$$

and represent the trajectory.

Exercise 2. 1. Give the matrix diagonalization of $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$.

2. Deduce the general expression of the solutions to the system

$$\begin{cases} y_1' = 3y_1 - 2y_2, \\ y_2' = y_1. \end{cases}$$

3. Compute the solution corresponding to the initial conditions $y_1(0) = 3$, $y_2(0) = 2$ and represent its trajectory.
4. ** Using the same strategy employed in the previous questions compute and represent in the plan the solution of

$$\begin{cases} y_1' = 5y_1 - 6y_2, & y_1(0) = 3, \\ y_2' = 3y_1 - 4y_2, & y_2(0) = 2. \end{cases}$$

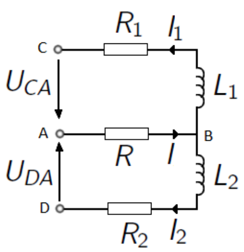
Exercise 3. We consider the system

$$(\mathrm{S}) \begin{cases} y_1' = y_2, & y_1(0) = 0, \\ y_2' = y_3, & y_2(0) = 1, \\ y_3' = 2y_1 + y_2 - 2y_3, & y_3(0) = -3. \end{cases}$$

1. Give the matrix A such that the system (S) is formulated under the form $Y' = AY$. Then give the diagonalization of A.
2. Compute the general solutions of (S).
3. Give the expression of the solution corresponding to the initial conditions $y_1(0) = 0$, $y_2(0) = 1$ and $y_3(0) = -3$.
4. ** Using the same strategy employed in the previous questions, compute the solution of

$$\begin{cases} y_1' = y_2 + y_3, & y_1(0) = 1, \\ y_2' = y_1 + y_3, & y_2(0) = 1, \\ y_3' = y_1 + y_2, & y_3(0) = 1. \end{cases}$$

Exercise 4. We consider an electrical circuit given by



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The intensities I_1 and I_2 are solutions of the differential system

$$\begin{cases} \dfrac{dI_1}{dt} = -\left(\dfrac{R+R_1}{L_1}\right) I_1 - \left(\dfrac{R_2}{L_1}\right) I_2 + \dfrac{U_{CA}}{L_1}, \\ \dfrac{dI_2}{dt} = -\left(\dfrac{R_1}{L_2}\right) I_1 - \left(\dfrac{R+R_2}{L_2}\right) I_2 + \dfrac{U_{DA}}{L_2}. \end{cases}$$

In the following, we consider the case where $L_1 = L_2 = 1$, $R = 1$, $R_1 = 2$, $R_2 = 3$.

1. Prove that the system verifies by the couple (I_1, I_2) is

$$\begin{cases} \dfrac{dI_1}{dt} = -3I_1 - 3I_2 + U_{CA}, \\ \dfrac{dI_2}{dt} = -2I_1 - 4I_2 + U_{DA}. \end{cases}$$

2. Compute the associated general solution of the homogenous system (i.e. for $U_{CA} = 0$ and $U_{DA} = 0$).
3. We suppose that the tensions U_{CA} and U_{DA} are constants and equal to $U_{CA} = 3$ and $U_{DA} = 8$. Moreover we suppose that the initial intensities $I_1(0)$ and $I_2(0)$ are equal to 0. Compute the corresponding solution (I_1, I_2).