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Souslin measurable homomorphisms of topological groups

By

Dominikus Noll

1. In a recent paper [18], A. Kleppner proved that every Haar measurable homomorphism $\phi: G \to H$ between locally compact groups G, H is continuous. This result also follows from a measure theoretic result by D. H. Fremlin [8], (see also [9, 19]), which provides a technique to prove measurability of uncountable unions of measurable sets.

The purpose of this note is to present a different and new approach to continuity of measurable homomorphisms, based on Souslin and Borel rather than Haar measurability. This slight investment on part of the measurability of the homomorphism ϕ allows for weakening considerably the conditions to be imposed on the groups G, H. In particular, the presence of the Haar measure, which is essential in the approach of [18] resp. [8, 9], is no longer needed to establish our continuity result.

Our technique is completely different from the methods used in [18] (resp. [8, 9]). It gives rise also to a generalization of Banach's classical result [2] stating that Borel measurable homomorphisms between Polish groups G, H are continuous. We prove that every Souslin measurable homomorphism $\phi: G \to H$ from a paracompact Čech complete group G into any topological group H is continuous (Theorem 1). Some consequences of this result such as a theorem on joint continuity of separately measurable homomorphisms and a version of the Souslin graph theorem are included.

There is, of course, a variety of results dealing with continuity of homomorphisms of topological groups measurable in one sense or other. In contrast with our present approach, these all need countability type assumptions on at least one of the groups G, H. We just mention the following list [2, 5, 10, 13, 15, 17, 18] of references on Borel and Souslin graph theorems and continuity theorems for measurable homomorphisms. Further references are [1, 3, 5, 12, 16, 22, 23, 25, 26], where closed graph theorems are treated. An interesting paper is [27], where the continuity of sequentially continuous homomorphisms between locally compact groups is discussed.

2. In this section we state our main results and obtain some consequences. The proofs of Theorems 1, 2 below will be given in Section 3.

Theorem 1. *Let $\phi: G \to H$ be a Souslin measurable homomorphism from a paracompact Čech complete group G into a topological group H. Then ϕ is continuous.*

In the present context, a homomorphism $\phi: G \to H$ is called Souslin measurable if $\phi^{-1}(U)$ is a Souslin set in G for every open F_σ-set U in H. Recall that a Souslin set in a topological space X is a subset of X which admits a Souslin representation by closed subsets of X, i.e. a set of the form

$$\bigcup \left\{ \bigcap_{n=0}^{\infty} F_{\sigma|n} : \sigma \in \omega^\omega \right\}$$

for closed sets $F_{\sigma|n}$ in X. We refer to [4, 5, 11, 13, 14, 20] for details concerning Souslin sets. Just notice that Borel sets in metric spaces are known to be Souslin (see Section 4).

A homomorphism $\phi: G \to H$ from a locally compact group G into a topological group H will be called Haar measurable if there exists a Haar measurable subset A of G having finite positive (left) Haar measure such that $\phi^{-1}(U) \cap A$ is Haar measurable for every open set U in H. With this notation, Kleppner's result [16, Theorem 1] tells that every Haar measurable homomorphism $\phi: G \to H$ between locally compact groups G, H is continuous. Here we obtain the following parallel to this result.

Theorem 2. *Let $\phi: G \to H$ be a homomorphism from a locally compact group G into a topological group H. Suppose there exists a subset A of finite positive left Haar measure such that $\phi^{-1}(U) \cap A$ Souslin set for every open F_σ-set U in H. Then ϕ is continuous.*

An even more general result of this type could be obtained if the notion of Haar null sets introduced by Christensen [5] was used. Local compactness of the group G would then no longer be needed, but G had to be metrizable instead.

Next we obtain a version of the Souslin graph theorem for a homomorphism $\phi: G \to H$ of groups.

Corollary 1. *Let $\phi: G \to H$ be a homomorphism from a locally compact group G into a σ-compact group H. Suppose there exists a subset A of G, having $0 < \lambda(A) < \infty$ with respect to some left Haar measure λ on G, such that $G(\phi) \cap (A \times U)$ is a Souslin set in $G \times H$ for every open K_σ-set U in H. Then ϕ is continuous.*

P r o o f. First observe that we may assume the group G to be σ-compact and locally compact. Indeed, choose a compact neighbourhood V of e in G such that $\lambda(A \cap V) > 0$. It suffices to prove the continuity of $\phi_0 = \phi \mid G_0$, where G_0 is the subgroup of G generated by V. But setting $A_0 = A \cap G_0$ provides a subset of G_0 for which the assumptions of the Corollary are again met with ϕ replaced by ϕ_0.

Let us therefore assume that G is locally compact and σ-compact. The result is now a consequence of Theorem 2 above, when we prove that $\phi^{-1}(U) \cap A$ is Haar measurable for every σ-compact open set U in H. (See also [15, (22.18)].)

Observe that for any such U we have

$$\phi^{-1}(U) \cap A = \mathrm{proj}_G(G(\phi) \cap (A \times U)),$$

where proj_G denotes the projection operator $G \times H \to G$, and where $G(\phi)$ is the graph of ϕ. By assumption, the set $G(\phi) \cap (A \times U)$ is Souslin in $G \times U$, a σ-compact space, hence its image under the continuous projection proj_G onto G is Souslin in G (cf. [10, Lemma 1], [4]). It is known, however, that Souslin sets are universally measurable with respect to the Borel σ-algebra (see [24, § 3] or [5, Theorem 1.5]), which means that for every Borel measure μ on G they are elements of the Carathéodory completion of the Borel σ-algebra with respect to μ. Choosing as a special Borel measure the Haar measure λ proves that $\phi^{-1}(U) \cap A$ is Haar measurable. This ends the proof. $\square$

Remarks. 1) The countability assumption on the group H cannot be omitted in the statement of the Souslin graph theorem. This may be seen by considering the following example. Let G be a locally compact group, and let $H = G_d$ be the group G with discrete topology. Choosing for $\phi: G \to H$ the identity mapping provides a homomorphism whose graph $G(\phi)$ is even closed in $G \times H$, but which is certainly not continuous in general.

2) The above example also shows that Theorems 1, 2 above resp. Kleppner's results [18] are no longer valid if the corresponding measurability assumptions on $\phi^{-1}(U)$ resp. $\phi^{-1}(U) \cap A$ are required for the elements U of a basis of H only. Indeed, in the above example, basic open sets for H are all singletons, whose preimages are clearly measurable in either sense. Notice also that preimages of σ-compact open subsets of H are measurable here, since σ-compact sets in H are countable. This proves, in particular, that the measurability hypothesis for preimages of open F_σ-sets U in Theorem 2 cannot be weakened to pertain to preimages of open K_σ-sets only.

3) We do not know whether the statement of the Corollary remains valid if Souslin measurability of $G(\phi) \cap (A \times U)$ is replaced by Haar measurability or even universal measurability (concerning Borel measurability see Section 4). The reason is that in these cases it is not clear whether the image of a measurable set under the projection proj_G is again measurable in an appropriate sense.

We conclude this section with the following result on joint continuity of separately measurable homomorphisms.

Corollary 2. *Let G_1, G_2 be paracompact Čech complete groups and let $\phi: G_1 \times G_2 \to H$ be a mapping into a topological group H such that $\phi(x, \cdot): G_2 \to H$ and $\phi(\cdot, y): G_1 \to H$ are Souslin measurable homomorphisms for all $x \in G_1$, $y \in G_2$. Then ϕ is (jointly) continuous.*

Proof. It follows from Theorem 1 that the homomorphisms $\phi(x, \cdot)$, $\phi(\cdot, y)$ are actually continuous, i.e. ϕ is separately continuous. But now we deduce the joint continuity of ϕ using a result of Namioka [21]. Fix a left-invariant pseudo-metric σ on H. It suffices to show that ϕ is continuous as a mapping $G_1 \times G_2 \to (H, \sigma)$, for the topology of H is generated by these σ. Next observe that Čech complete spaces are so-called k-spaces, which means that the continuity of ϕ follows from the continuity of $\phi \mid C: C \to (H, \sigma)$ for every compact subset C of $G_1 \times G_2$. Let us therefore fix a compact subset K of G_1. It suffices to prove the continuity of $\phi \mid K \times G_2: K \times G_2 \to (H, \sigma)$.

Now Namioka's result [21, Theorem 1.2] tells that $\phi \mid K \times G_2$ is continuous at the points of $K \times A$, where A is some dense G_δ subset of G_2. As $\phi(x, \cdot)$ is a homomorphism for every x, this implies that $\phi \mid K \times G_2$ is continuous everywhere on $K \times G_2$. As K was chosen arbitrarily, this completes the proof. $\square$

Remark. Using the same reasoning combined with Theorem 2, we might obtain a result on joint continuity of a separately Haar measurable ϕ defined on a product of locally compact groups. The reader might compare these with the corresponding results [6, Cor. 8] and [21, Theorem 3.1].

3. In this section we give the proofs of Theorems 1, 2. The following lemma turns out to be a crucial step towards both results.

Lemma 1. *Let H be a topological group and let V be a neighbourhood of e in H. Then there exists an open cover $\mathscr{V} = \bigcup_{n=1}^{\infty} \mathscr{V}_n$ of H refining the cover $\mathscr{A} = \{Vy : y \in H\}$ such that every $\mathscr{V}_n$ is disjoint (i.e. $V \cap V' = \emptyset$ for $V, V' \in \mathscr{V}_n$, $V \neq V'$). Moreover, $\mathscr{V}$ can be found completely F_σ-additive in H.*

Proof. Let (V_n) be a sequence of symmetric open neighbourhoods of e in H satisfying

$$V_n \cdot V_n \subset V_{n-1}, \quad V_1 \cdot V_1 \subset V,$$

and let $S = \bigcap_{n=1}^{\infty} V_n$. Then S is a subgroup of H. Let ψ denote the natural mapping $H \to H/S$ onto the homogeneous right coset space H/S. We consider two topologies on H/S, the quotient topology, and the coarser metric topology induced by the right-invariant pseudo-metric σ on H arising from the sequence (V_n), resp. the corresponding quotient metric σ^* on H/S (cf. [15, pp. 68, 76]). Notice that the sets $\psi(V_n)$, $n \in \mathbb{N}$, form a base of neighbourhoods of $\psi(e)$ in H/S for the metric topology. Let W be the interior of $\psi(V_1)$ with respect to the metric topology. Then W is as well open in the quotient topology, hence $\psi^{-1}(W)$ is open in H. But observe that $\psi^{-1}(W)$ is contained in V in view of

$$(*) \qquad \psi^{-1}(W) \subset \psi^{-1}\psi(V_1) = V_1 \cdot \operatorname{Ker}\psi \subset V_1 \cdot V_1 \subset V.$$

Now consider the open cover $\mathscr{O} = \{\tau_y W : y \in H\}$ of H/S in the metric topology, where τ_y denotes the translation operator $Hx \to Hxy$ on H/S. Notice that W is a neighbourhood of $\psi(e)$ and that τ_y is a homeomorphism of H/S with respect to the metric topology mapping $\psi(e)$ into $\psi(y)$. This proves that $\mathscr{O}$ is actually an open cover of H/S. Using the fact that metric spaces are paracompact, we obtain an open cover $\mathscr{W} = \bigcup_{n=1}^{\infty} \mathscr{W}_n$ of H/S refining $\mathscr{O}$ such that each $\mathscr{W}_n$ is discrete. But $\psi^{-1}(\mathscr{O})$ refines $\mathscr{A}$ by $(*)$ and the fact that τ_y commutes with the right translation $x \to xy$ on H over ψ. Hence $\mathscr{V} = \psi^{-1}(\mathscr{W})$ is a disjoint open refinement of $\mathscr{A}$ the $\mathscr{V}_n = \psi^{-1}(\mathscr{W}_n)$ being disjoint open families in H refining $\mathscr{A}$. Moreover, as $\mathscr{W}_n$ is a discrete family of open sets in a metric space, it is completely F_σ-additive, which means that $\bigcup \mathscr{W}'$ is an F_σ-set in H/S for *every* subfamily $\mathscr{W}'$ of $\mathscr{W}_n$. Hence the same is true for $\mathscr{V}_n = \psi^{-1}(\mathscr{W}_n)$, and consequently also for $\mathscr{V}$. This completes the proof of the lemma. $\square$

Let us now give the proof of Theorem 1. Let U be a neighbourhood of e in H. Choose a symmetric neighbourhood V of e having $V \cdot V \subset U$. According to the lemma above let

$\mathscr{V} = \bigcup_{i=1}^{\infty} \mathscr{V}_i$, with $\mathscr{V}_i = \{V_{\alpha i} : \alpha \in I_i\}$, be a σ-disjoint completely F_σ-additive open refinement of $\{Vy : y \in H\}$ covering H. Now let

$$G_n = \bigcup \{\phi^{-1}(V_{\alpha n}) : \alpha \in I_n\},$$

then G is the union of the countably many sets G_n, hence some G_n is of the second category in G. The main step of the proof now consists in checking that some of the sets $\phi^{-1}(V_{\alpha n})$, $\alpha \in I_n$, is as well of the second category in G. Suppose for a moment that this has been shown for the set $S = \phi^{-1}(V_{\alpha n})$. Then S is a second category Souslin subset of G. As Souslin sets are known to have the Baire property (cf. [20, p. 94]), Pettis' Lemma (cf. [5, p. 86] or [17, p. 92]) gives that $W = S \cdot S^{-1}$ is a neighbourhood of e in G. By the construction of $\mathscr{V}$ we have $V_{\alpha n} \subset Vy$ for some $y \in H$. This gives

$$\phi(W) = \phi(S \cdot S^{-1}) \subset V_{\alpha n} \cdot V_{\alpha n}^{-1} \subset Vy \cdot (y^{-1} V^{-1}) = V \cdot V^{-1} \subset U,$$

proving that ϕ is continuous.

It remains to show that some $\phi^{-1}(V_{\alpha n})$ is of the second category. Now observe that the family $\mathscr{S} = \{S_{\alpha n} : \alpha \in I_n\}$ with $S_{\alpha n} = \phi^{-1}(V_{\alpha n})$ is a completely Souslin additive family, which means that $S_I = \bigcup \{S_{\alpha n} : \alpha \in I\}$ is a Souslin set in G for every subset I of I_n. Indeed, this follows from the fact that

$$S_I = \phi^{-1}(\bigcup \{V_{\alpha n} : \alpha \in I\})$$

is the preimage of an open F_σ-subset of H, hence is Souslin in G. We claim that this property of the *disjoint* family $\mathscr{S}$ implies that $\mathscr{S}$ has a refinement $\mathscr{X} = \bigcup_{i=1}^{\infty} \mathscr{X}_i$ covering $G_n = \bigcup \mathscr{S}$ such that each family $\mathscr{X}_i$ is discrete in G_n. Suppose this has been checked. Then

$$G_{ni} = \bigcup \{X : X \in \mathscr{X}_i\}$$

is a sequence of sets covering the second category set G_n, so some G_{ni} is as well of the second category in G. But now $\mathscr{X}_i$ is a discrete family covering the second category set G_{ni}. So some element X of $\mathscr{X}_i$ must be of the second category in G, for the Banach category theorem implies that a discrete union of first category sets is still of the first category. Now as this set X is contained in some $S_{\alpha n} = \phi^{-1}(V_{\alpha n})$, the latter is as well a second category set, and that is what we claimed to be true. Hence we are left to find a σ-discrete refinement for the disjoint completely Souslin additive family $\mathscr{S}$.

Such a σ-discrete refinement for $\mathscr{S}$ is provided by combining results of Frolik and Holicky [11, Theorem 2] and Hansell, Jayne and Rogers [14]. According to the terminology of the latter paper, a topological space G is called K-analytic if G is the image of some strongly zero dimensional metric space κ^ω under a compact-valued upper semi-continuous set-valued correspondence $K : \kappa^\omega \to 2^G$ such that the family $\{K(I(t)) : t \in \kappa^n\}$ is σ-discrete in G (see [14, Theorem 1]). The authors of [14] also prove in their Corollary 1 that every paracompact Čech complete space G is K-analytic in this sense. They further notice that in the class of paracompact spaces their notion of K-analyticity coincides with Frolik and Holickys' notion of analyticity for uniform spaces when the paracompact space under consideration is given the fine uniformity (compare [11]). In our case this means that the paracompact Čech complete group G is analytic in the sense of Frolik and Holicky. But then [11, Theorem 2] yields the desired σ-discrete refinement for the disjoint completely Souslin additive family $\mathscr{S}$ on G. This settles the proof of Theorem 1.

Let us next give the proof of Theorem 2. Using the same reasoning as in the proof of Corollary 1, we may assume that the group G is locally compact and σ-compact. Clearly also the set A from the statement of the theorem may then be chosen to be σ-compact. Now let U be a neighbourhood of e in H, and let V be a neighbourhood of e in H having $V \cdot V^{-1} \subset U$. Let $\mathscr{V} = \bigcup_{i=1}^{\infty} \mathscr{V}_i$ be the σ-disjoint completely F_σ-additive open cover of H refining $\{Vy : y \in H\}$ guaranteed by Lemma 1. Now let $\mathscr{A}_i = \{\phi^{-1}(W) \cap A : W \in \mathscr{V}_i\}$, then $\mathscr{A}_i$ is a disjoint completely Souslin addivite family in G. Let $A = \bigcup_{k=1}^{\infty} A_k$ for compact sets A_k in G. Then the family $\mathscr{A}_{ik} = \{\phi^{-1}(W) \cap A_k : W \in \mathscr{V}_i\}$ is again disjoint and completely Souslin additive in the compact space A_k. But now [11, Lemma 2] says that $\mathscr{A}_{ik}$ actually has to be countable. This implies that the family $\mathscr{A}' = \bigcup_{i=1}^{\infty} \bigcup_{k=1}^{\infty} \mathscr{A}_{ik}$ is countable. Since $\mathscr{A}'$ covers A and A has finite positive measure, it follows that some $\phi^{-1}(W) \cap A_k$, $W \in \mathscr{V}_i$, must have positive measure. But now [15, (20.17)] implies that

$$O = (\phi^{-1}(W) \cap A_k) \cdot (\phi^{-1}(W) \cap A_k)^{-1}$$

is a neighbourhood of e in G. Since W is contained in some Vy, it follows that

$$\phi(O) \subset Vy \cdot (y^{-1} V^{-1}) \subset U,$$

proving that ϕ is continuous. This settles the case of Theorem 2.

Remark. We do not know whether Theorem 1 remains valid if Souslin measurability of ϕ is replaced by universal measurability, while a version of Theorem 2 with respect to Haar measurability follows from Fremlin's result [8, 9] in tandem with [15, (20.17)]. However, a corresponding result giving σ-discrete refinements for disjoint completely additive families with respect to these notions of measurability cannot be expected to be true, as the following example indicates.

Let C be a Cantor set in $\mathbb{R}$ having Lebesgue measure $\lambda(C) = 0$. Then the family $\mathscr{F}$ of singleton subsets of C is a completely Haar additive family, which means that $F' = \bigcup \mathscr{F}'$ is Haar measurable in $\mathbb{R}$ for every subfamily $\mathscr{F}'$ of $\mathscr{F}$. Indeed, every such F' is a subset of a Borel nullset, hence is Haar measurable. But clearly the family $\mathscr{F}$ does not admit a σ-discrete refinement, for any such refinement had to consist of singleton subsets of C, so σ-discreteness of the refinement would imply σ-discreteness of C as a set, a contradiction with the fact that C is dense in itself. The example can be modified by taking as C any dense in itself or second category Haar null set in a locally compact group G.

4. It is well-known that the Souslin operation, when applied to the family of closed sets in a topological space, provides a family of sets, called the Souslin sets (or sometimes Souslin-$\mathscr{F}$ sets), which is closed under countable unions and intersections. Consequently, open sets need not be Souslin sets in general, but are Souslin if they are F_σ-sets.

In a metric space, therefore, all Borel sets are Souslin. Anyway, the class of Souslin sets is not too small even in the general case, for it contains all Baire sets. These observations lead to immediate Corollaries to Theorems 1, 2 stated for Baire measurable homomorphisms resp. for Borel measurable homomorphisms on metric groups. In the case of a metrizable group G we could even obtain continuity results for homomorphisms which are Borel measurable in the extended sense (compare [13] for details concerning extended Borel sets). Notice, however, that there is no ad hoc method providing Borel measurable versions of Theorems 1, 2, at least not without imposing anything additional on the group G or restricting the measurability assumption in some sense. Indeed, this follows from

Proposition 1. *Let G be a paracompact Čech complete group in which every Borel set is Souslin. Then G is metrizable.*

It follows from [14, Corollary 1] that G is K-analytic in the sense of this paper. But then the assumption on G means that every open set in G is K-analytic in G. Combining Lemmas 10, 11 of [14] now proves that then every open set in G has to be an F_σ, hence every singleton is a G_δ. This means that G is metrizable.

We conclude with a set-theoretic result. If Fleissner's axiom **P** is added to the usual axioms of set theory, then it is true that any completely Borel additive family in a metric space is σ-discretely decomposable, hence has a σ-discrete refinement (see [7]). This leads to the following

Theorem 3. *Assume axiom* **P**. *Then every Borel measurable homomorphism ϕ from a second category metrizable group G into any topological group H is continuous.*

Clearly neither axiom **P** nor the metrizability of G are needed here if the group H satisfies some countability type assumption such as σ-boundedness in the sense of Pettis [25] or the condition that disjoint open families are at most countable.

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Anschrift des Autors:

Dominikus Noll
Mathematisches Institut B
Universität Stuttgart
Pfaffenwaldring 57
DW-7000 Stuttgart 80