Failure-Finding Frequency for a Repairable System subject to Hidden Failures

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This paper addresses the problem of selecting a suitable failure-finding maintenance policy for repairable systems. We consider hidden failures, that do not interrupt aircraft operation when they occur, like failures of alarm devices or back-up components. We study both corrective maintenance actions, carried out after item failure, and periodic failure-finding, designed to check whether system still works. Based on our probabilistic analytic developments, the optimal maintenance policy is then obtained as a solution of an optimization problem, where the maintenance cost rate is the objective function and the risk of corrective maintenance is the constraint function. Finally, we show an application of our methodology on a real-world case provided by Airbus.

Nomenclature

T	=	Time horizon of the fleet maintenance policy. T is a given input.
k	=	Index of failure-finding interval from 0 to N within the calculation horizon T
I^k	=	Length of kth time interval between failure-finding tasks $I^k = \lfloor t^k ; t^{k+1} \rfloor, k \in [0; N]$. I^k is a given input or the output of the optimization problem..
I^{N+1}	=	Residual length of time between the last failure-finding tasks at t^{N+1} and T for optimization studies. $I^{N+1} = \lfloor t^{N+1} ; T \rfloor$.
$\lambda^j_{System}(t)$	=	Failure rate of the system which failure is hidden on the interval I^j, $j \in [0; N+1]$. $\lambda^0_{System}(t)\, \forall t \in T$ is the given failure rate of this system without any corrective or failure-finding maintenance
$\lambda_{Demand}(t)$	=	Occurrence rate of the operational demand at time t. $\forall t \in T$, $\lambda_{Demand}(t)$ is a given input.
$\lambda^j_{CM}(t)$	=	Occurrence rate of corrective maintenance actions at time t in the interval I^j, $j \in [0; N+1]$.
$Y^j(t)$	=	State variable of the system with respect to its hidden or undetected failures at time t in the interval I^j, $j \in [0; N+1]$
$NCM^j_{Aircraft}(t)$	=	Counting process of corrective maintenance actions due to unscheduled failures for one aircraft in the interval I^j from t^j to t, $\forall t \in I^j$, $j \in [0; N+1]$

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FS	=	Fleet size, given input.
NFF^j_{Fleet}	=	Mean number of detected failures on the fleet during failure-finding tasks performed at time t^{j+1}, $j \in [0; N+1]$.
$NCM^j_{Fleet}(t)$	=	Mean number of corrective maintenance actions due to unscheduled failures on the fleet from t^j to t, $\forall t \in I^j$, $j \in [0; N+1]$
P^j_{NoCM}	=	Probability to avoid corrective maintenance actions during interval I^j, $j \in [0; N+1]$
C_{Check}	=	Cost to perform the check during failure-finding maintenance
C_{CM}	=	Cost of corrective maintenance
C_{FF}	=	Cost of failure-finding maintenance to restore the hidden system following detection during check

I. Introduction

MUCH of what has been written to date on the subject of maintenance strategies[1,2] refers to predictive, preventive and corrective maintenance. Far less attention has been paid to failure-finding maintenance for hidden failure system.

A hidden failure[3] is a failure not evident to the crew or operator during the performance of normal duties. These failures occur in such a way that nobody knows that the item is in a failed state unless some other operational demand (additional failure, trigger event) also occurs. For instance, if a standby radio failed, no one would be aware of the fact because under normal circumstances the active radio would still be working. In other words, the failure of the standby radio on its own has no direct impact unless or until the active radio also fails. Generally, hidden failure affects back-up and protective systems, like safety valve (e.g. a shutdown valve or relief valve) or sensor (e.g. fire/gas detector, pressure sensor or level sensor). They are designed to be activated upon operational demands to protect people, environment or to keep a given function. These systems are common in industrial safety and protection systems, examples are presented in Ref 4 with standby devices.

Failure-finding maintenance for hidden failure falls into none of the three maintenance categories: predictive, preventive, corrective. However this kind of maintenance is far from being negligible. According to the Ref.5, if Reliability Centered Maintenance (RCM) is correctly applied to almost any modern, complex industrial system, it is not unusual to find that up to 40% of failure modes fall into the hidden category. Furthermore, up to 80% of these failure modes require failure-finding. As a result, up to one third of the tasks generated by comprehensive, correctly applied maintenance strategy development programs are failure-finding tasks. In this case, we have to deal in particular with both corrective maintenance actions and periodic failure-finding tasks.

Corrective maintenance tasks are carried out after an item has failed. The purpose of corrective maintenance is to bring the item back to a functioning state as soon as possible, either by repairing or replacing the failed item, or by switching in a redundant item. Corrective maintenance is also called breakdown maintenance or run-to-failure maintenance. To sum up, corrective maintenance refers to the actions performed, as a result of failure, to restore an item to a specified condition [3].

Besides, failure-finding tasks are designed to check whether the system is still working. More precisely, failure-finding tasks are carried out to reveal hidden that have already occurred. For example the smoke detector is an emergency system; it only activates when smoke is present. Failure of the smoke detector, during normal operation, would constitute a hidden failure. The failed smoke detector would only be evident when smoke was present, and it failed to sound. A failure-finding task for the smoke detector would be to periodically check the fire detection circuit to see if it is operational (blowing smoke at the detector and checking if the alarm sounds). Another example is a pressure switch designed to shut down a machine when the lubricating oil pressure drops below a certain level. Switches of this type should be checked regularly by dropping the oil pressure to the required level and checking whether the machine shuts down.

Ideally, we have to find out a failure-finding maintenance which ensures 100% availability of the protective or back-up system when operational demand occurs. In this ideal case, there is none corrective maintenance task and the number of failure-finding tasks is infinite. In practice, it is impossible to achieve a 100% or very high availability of the hidden system, because of both technical feasibility and costs induced by high number failure-finding tasks. Therefore in real-world cases the problem is to determine an optimal frequency of failure-finding tasks that makes the best compromise between the cost of corrective tasks and the cost failure-finding tasks. Note that, for some systems, due to safety or operational reasons, constraints on system availability have also to be taking into account in the optimization process.

This purpose of this paper is to develop a precise framework to optimally determine failure-finding maintenance frequency for repairable systems put in operation at time t=0 for a finite time horizon. It is organized as follows. In section 2, based on in-service aircraft utilization, we introduce the modeling assumptions made on failure-finding maintenance. Section 3 is devoted to mathematical developments. We give equations for computing the mean number of both corrective and failure-finding maintenance actions over a finite time horizon. From these previous results, we state the optimization problem that defines the optimal frequency of failure-finding tasks. Section 4 deals with constant failure rate. Simple analytical formulae are then derived from those of section 3. They provide a clear understanding about the influence of each reliability parameter on the optimization process result. In section 5, a numerical example provided by Airbus is given on a pressure relief valve to illustrate how the model can be used on a real world case. Finally in section 6, we conclude and open further perspectives.

II. Assumptions

The following modelling assumptions are based on common practices of airlines.

- The operational demand is a general non-homogenous Poisson process[1]. In particular the rate of occurrence of the demand can be a function of time and there will be no more than one demand at the same time.
- The failure rate of the system with hidden failure is a function of time (ex: infant mortality, aging effect...) . Failure of all fleet systems are supposed to be statistically independent with the same probability distribution.
- If the hidden system checked during failure-finding task is in an operating state, nothing is done.
- If the hidden system checked during failure-finding task is in a failed state, it is replaced by a new component of the same type, or restored to an "as good as new" condition. It means that the failure time distribution of the repaired or the new system is identical to that previous one at t=0.
- In case of corrective maintenance, the breakdowns are minimally repaired. With minimal repair, a failed item is returned to operation with the same effective age, as it possessed immediately prior to failure.
- No time-value for money.

III. Analytic Development

In this section we define the model of failure-finding maintenance as a Markov Process. From this model, we derive the formula of the system failure rate. Then, we estimate the mean number of maintenance actions over a fleet, either corrective or during failure-finding tasks. We also assess the probability to avoid corrective maintenance actions over an interval. Finally, we state the optimization problem that allows determining the optimal failure-finding frequency minimizing maintenance costs and satisfying an acceptable level of corrective maintenance frequency.

A. Definition of System State and Failure Rate

The type of system studied here can be modeled within an interval by a simple Markov graph, as below:

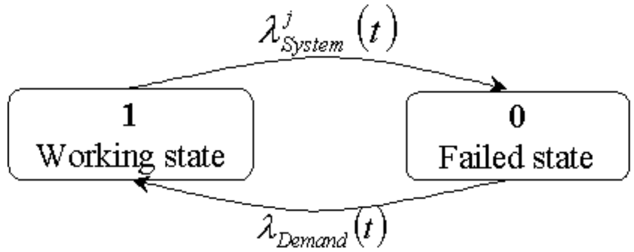

Fig. 1 Markov transition diagram.

$\forall t \in I^j$, $j \in [0; N+1]$, the state variable $Y^j(t)$ of a system with respect to its hidden failures is:

- $Y^j(t) = 1$, if the system is functioning at time t;
- $Y^j(t) = 0$, if the system item is not functioning at time t.

$\forall t \in I^j$, with the assumption of minimal repair during corrective maintenance, we have the following differential equation:

$$\dot{\overline{\begin{pmatrix} P(Y^j(t)=1) \\ P(Y^j(t)=0) \end{pmatrix}}} = \begin{pmatrix} -\lambda^j_{System}(t) & \lambda_{Demand}(t) \\ \lambda^j_{System}(t) & -\lambda_{Demand}(t) \end{pmatrix} \times \begin{pmatrix} P(Y^j(t)=1) \\ P(Y^j(t)=0) \end{pmatrix} \tag{1}$$

With

$$P(Y^j(t)=1) + P(Y^j(t)=0) = 1 \tag{2}$$

And the initial conditions:

$$\begin{pmatrix} P(Y^j(t^j)=1) \\ P(Y^j(t^j)=0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

Let us define $A^j(t) = \begin{pmatrix} -\lambda^j_{System}(t) & \lambda_{Demand}(t) \\ \lambda^j_{System}(t) & -\lambda_{Demand}(t) \end{pmatrix}$

Then the analytical solution of the previous equations (1), (2) and (3) is given by:

$$\forall t \in I^j,\ \begin{pmatrix} P(Y^j(t)=1) \\ P(Y^j(t)=0) \end{pmatrix} = \exp\left(\int_{t_j}^{t} A^j(\tau)d\tau\right) \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

To solve it, we can use discretization schemes, such as Euler, Runge Kutta[6].

Because of assumptions made on corrective and failure- finding maintenance, it is now straightforward to formulate the system failure rate as:

$$1 \le j \le N,\ \forall t \in I^j,\ \lambda^j_{System}(t) = \lambda^{j-1}_{System}(t) \times P\left(Y^{j-1}(t^j) = 1\right) + \lambda^0_{System}(t - t^j) \times P\left(Y^{j-1}(t^j) = 0\right) \tag{5}$$

With $\lambda^0_{System}(t)\ \forall t \in T$ is the given failure rate of this system without any corrective or failure-finding maintenance and $\lambda_{Demand}(t)\ \forall t \in T$ is the given occurrence rate of the operational demand.

B. Mean Numbers of Maintenance Actions

Basically the system studied in this article can fail to operate:

- In the presence of an operational demand, generating corrective maintenance;
- In the absence of operational demand, generating failure-finding maintenance.

The probability to detect a failure for one aircraft during failure-finding tasks performed at time t^{j+1} $j \in [0; N+1]$ is given by:

$$NFF^j_{Aircraft} = P(Y^j(t^{j+1}) = 0) \tag{6}$$

Besides $NCM^j_{Aircraft}(t)$, the mean number of corrective maintenance actions on $\lfloor t^j; t \rfloor$ per aircraft in the interval I^j, is a renewal process. Its rate of occurrence can be easily computed from, $\lambda^j_{CM}(t) = \lambda_{Demand}(t) \times P\left(Y^j(t) = 0\right)$ $\forall t \in I^j$.

Then we have:

$$\forall t \in I^j,\ NCM^j_{Aircraft}(t) = \int_{t^j}^{t} \lambda^j_{CM}(\tau)d\tau \tag{7}$$

These results are directly expanded to a fleet through the following formulae $\forall j \in [0; N+1]$:

$$NFF^{j}_{Fleet} = FS \times NFF^{j}_{Aircraft} \tag{8}$$

$$\forall t \in I^{j},\ NCM^{j}_{Fleet}(t) = FS \times NCM^{j}_{Aircraft}(t) \tag{9}$$

C. Probability to Avoid Corrective Maintenance Actions between Two Failure-Finding Tasks

The event "no corrective maintenance action during Interval I^{j} $\forall j \in [0; N+1]$ is divided in two distinct events:

- No system failure within interval I^{j};
- System failure within I^{j}, but no operational demand within I^{j} after system failure.

Therefore we can write

$$\begin{aligned} P^{j}_{NoCM} &= P\left(No\ system\ failure\ within\ I^{j}\right) \\ &+ P\begin{pmatrix} \text{System failure within } I^{j} \\ \cup \\ \text{No process demand within } I^{j} \text{ after system failure} \end{pmatrix} \end{aligned} \tag{10}$$

We are going to detail each of these two probabilities.

To begin with, the probability of no system failure within I^{j} is given by:

$$P\left(No\ system\ failure\ within\ I^{j}\right) = \exp\left(-\int_{t^{j}}^{t^{j+1}} \lambda^{j}_{System}(\tau)d\tau\right) \tag{11}$$

A first order approximation of the probability of first system failure between t and t+dt at time t is given (here) by the density function multiply by dt. It is well known that the density function of a first system failure is equal to the failure rate multiplied by the survival function:

$$\forall t \in I^{j},\ \lambda^{j}_{System}(t) \times \exp\left(-\int_{t^{j}}^{t} \lambda^{j}_{System}(\tau)d\tau\right) \tag{12}$$

Now we assess the probability of no system demand after system failure at time t.

The number of operational demands $\{N_{Demand}(t), t \geq 0\}$ is assumed to be a Non-Homogeneous Poisson Process (NHPP) with rate function $\lambda_{Demand}(t)$ for $t \geq 0$. From the properties of NHPP[7], we have the following equality given:

$$\forall t,\tau \in I^j \ \ with\ \tau \leq t,\ P(N_{Demand}(t) - N_{Demand}(\tau) = 0) = \exp\left(-\int_{\tau}^{t} \lambda_{Demand}(u)du\right) \tag{13}$$

To sum up, the probability that no operational demand after system failure writes:

$$P\begin{pmatrix} \text{System failure within } I^j \\ \cup \\ \text{No process demand within} \\ I^j \text{ after system failure} \end{pmatrix} = \int_{t^j}^{t^{j+1}} \lambda_{System}^{j}(t) \times \exp\left(-\int_{t^j}^{t} \lambda_{System}^{j}(\tau)d\tau\right) \times \exp\left(-\int_{t}^{t^{j+1}} \lambda_{Demand}(\tau)d\tau\right)dt \tag{14}$$

As a result the probability to avoid corrective maintenance actions over an interval I^j is the sum of equation (11) and (14):

$$P_{NoCM}^{j} = \exp\left(-\int_{t^j}^{t^{j+1}} \lambda_{System}^{j}(\tau)d\tau\right) + \int_{t^j}^{t^{j+1}} \lambda_{System}^{j}(t) \times \exp\left(-\int_{t^j}^{t} \lambda_{System}^{j}(\tau)d\tau\right) \times \exp\left(-\int_{t}^{t^{j+1}} \lambda_{Demand}(\tau)d\tau\right)dt \tag{15}$$

D. Optimization of Failure-Finding Frequency

The problem here is to determine the optimal interval length I^k between failure-finding tasks over a finite time horizon $[0,T]$ (e.g. the time an airline holds an aircraft). Under a risk constraint on the probability of avoiding corrective maintenance, the optimal object is to minimize the maintenance cost of the system by balancing failure-finding tasks and corrective actions. A model to optimize maintenance policies by minimizing system cost rate with availability constraint is presented in Ref. 8.

Compare to corrective maintenance, failure-finding maintenance are planned and hence cheaper, while failure during operation might be costly and dangerous. However high numbers of failure-finding checks can also conduct to prohibitive costs. Therefore the problem is to determine the best optimal length I^k that insure the best compromise.

For the sake of simplicity and because in real word cases of aircraft maintenance the intervals are constant, we suppose in this section that the intervals between two successive scheduled tasks are constant (i.e. $I^k = I$ and $t^k = k \times I$). Note that the maintenance optimization problem can be extended in a straightforward manner to variable interval length. In that case, we have to deal with a multivariable optimization problem.

The objective function of this optimization problem is the expected maintenance cost rate on the horizon period T. Its numerator is the sum of the maintenance costs on each interval $I^k = I$ plus the residual length interval I^{N+1}. For each interval maintenance costs takes into account

- The cost of corrective maintenance to restore an item following functional failure;
- The cost of failure-finding maintenance to restore an item following detection during check;
- The cost to perform the check during failure-finding task to identify any potential failure.

The denominator term is the T horizon interval length. There are two constraints to this optimization problem. This interval length must be higher or equal to zero and the probability to have no corrective maintenance must be higher than a given threshold δ . This threshold corresponds to the maximum unscheduled event that the airline tolerates. Note that $\delta = 0$ means that the only criterion is economic.

The optimization problem can be expressed as:

$$\text{Min}\, C(I) \tag{16}$$

Subject to:

$$I \geq 0 \tag{17}$$

$$\min_{0 \leq j \leq N+1} P_{NoCM}^{j} \geq \delta \tag{18}$$

With:

$$C(I) = \begin{cases} \dfrac{\sum_{j=0}^{N} \left(C_{CM} \times NCM_{Fleet}^{j}(j \times I + I) + C_{FF} \times NFF_{Fleet}^{j} + C_{Check} \right)}{T}, & if\ I^{N+1} = 0 \\ \dfrac{\sum_{j=0}^{N} \left(C_{CM} \times NCM_{Fleet}^{j}(j \times I + I) + C_{FF} \times NFF_{Fleet}^{j} + C_{Check} \right)}{T - I^{N+1}} + \\ \dfrac{C_{CM} \times NCM_{Fleet}^{N+1} + C_{FF} \times NFF_{Fleet}^{N+1} + C_{Check}}{I^{N+1}}, & if\ I^{N+1} > 0 \end{cases} \tag{19}$$

This optimization problem has generally no analytical solution. The I* optimal solution can be obtain by using adequate non-linear programming methods[9].

IV. Application for Constant Failure Rate

In this section, the formulae are developed for system whose hidden failure can occur randomly, i.e. constant failure rate[10,11]. This assumption is widely taken in real aeronautical world when we have to deal with system during their useful life period. With constant failure for both system and operational demand, simple analytical formulae are derived from those of section 3. They provide a clear understanding of the influence of each input parameter on the optimization process.

A. Mean Numbers of Maintenance Actions

The system with equations (1), (2) and (3) can be resolved analytically. The probabilities that the item is functioning at time t is:

$$\forall t \in I^j\,,\; P(Y^j(t)=1) = \frac{\lambda_{Demand}}{\lambda_{Demand}+\lambda^0_{System}} + \frac{\lambda^0_{System}}{\lambda_{Demand}+\lambda^0_{System}} \times \exp\left(-\left(\lambda_{Demand}+\lambda^0_{System}\right)\times\left(t-t^j\right)\right) \tag{20}$$

The probabilities that the item is not functioning at time t is:

$$\forall t \in I^j\,,\; P(Y^j(t)=0) = \frac{\lambda^0_{System}}{\lambda_{Demand}+\lambda^0_{System}} - \frac{\lambda^0_{System}}{\lambda_{Demand}+\lambda^0_{System}} \times \exp\left(-\left(\lambda_{Demand}+\lambda^0_{System}\right)\times\left(t-t^j\right)\right) \tag{21}$$

We can now write analytically the formulae (8) and (9). The mean number of detected failures on the fleet during failure-finding tasks performed at time t^{k+1} and T are given respectively by:

$$\forall k \in [0;N]\;\; NFF^k_{Fleet} = \frac{FS\times\lambda^0_{System}}{\lambda_{Demand}+\lambda^0_{System}}\left[1-\exp\left(-\left(\lambda_{Demand}+\lambda^0_{System}\right)\times I^k\right)\right] \tag{22}$$

$$NFF^{N+1}_{Fleet} = \begin{cases} \dfrac{FS\times\lambda^0_{System}}{\lambda_{Demand}+\lambda^0_{System}}\left[1-\exp\left(-\left(\lambda_{Demand}+\lambda^0_{System}\right)\times I^{N+1}\right)\right] & if\ I^{N+1}>0 \\ 0 & if\ I^{N+1}=0 \end{cases} \tag{23}$$

with $I^{N+1} = T - \sum_{k=0}^{N} I^k$.

The mean number of corrective maintenance actions due to unscheduled failures on the fleet from t^j to t, $\forall t \in I^j$.

$$NCM_{Fleet}^{j}(t) = \frac{FS \times \lambda_{Demand} \times \lambda_{System}^{0}}{\lambda_{Demand} + \lambda_{System}^{0}} \int_{t^j}^{t} \left[1 - \exp\left(-\left(\lambda_{Demand} + \lambda_{System}^{0}\right) \times \left(\tau - t^j\right)\right)\right] d\tau \tag{24}$$

It is straightforward to have the total number of corrective maintenance actions over interval I^k and I^{N+1}.

$$NCM_{Fleet}^{k}(t^{k+1}) = \frac{FS \times \lambda_{Demand} \times \lambda_{System}^{0}}{\lambda_{Demand} + \lambda_{System}^{0}} \left[I^k + \frac{\exp\left(-\left(\lambda_{Demand} + \lambda_{System}^{0}\right) \times I^k\right) - 1}{\lambda_{Demand} + \lambda_{System}^{0}} \right] \tag{25}$$

$$NCM_{Fleet}^{N+1} = \begin{cases} \dfrac{FS \times \lambda_{Demand} \times \lambda_{System}^{0}}{\lambda_{Demand} + \lambda_{System}^{0}} \left[I^{N+1} + \dfrac{\exp\left(-\left(\lambda_{Demand} + \lambda_{System}^{0}\right) \times I^{N+1}\right) - 1}{\lambda_{Demand} + \lambda_{System}^{0}} \right] & if\ I^{N+1} > 0 \\ 0 & if\ I^{N+1} = 0 \end{cases} \tag{26}$$

One should notice the special cases when there is no failure-finding task. In this case, the equipment deliberately runs to failure and the previous probabilities become:

$$\lim_{t \to +\infty} P(Y(t) = 1) = \frac{\lambda_{Demand}}{\lambda_{Demand} + \lambda_{System}^{0}} \tag{27}$$

$$\lim_{t \to +\infty} P(Y(t) = 0) = \frac{\lambda_{System}^{0}}{\lambda_{Demand} + \lambda_{System}^{0}} \tag{28}$$

$$\lim_{t^{j+1} \to +\infty} NFF_{Fleet}^{j} = \frac{FS \times \lambda_{System}^{0}}{\lambda_{Demand} + \lambda_{System}^{0}} \text{ with } t^j = 0 \tag{29}$$

$$\lim_{t^{j+1} \to +\infty} NCM_{Fleet}^{j}(t^{j+1}) = +\infty \text{ with } t^j = 0 \tag{30}$$

B. Probability to Avoid Corrective Maintenance Actions between Failure-Finding Tasks

With the assumption of constant failure rate, the formulae (15) can be rewrite:

$$P_{NoCM}^{j} = \exp\left(-\int_{0}^{I^j} \lambda_{System}^{0} d\tau\right) + \int_{t^j}^{t^{j+1}} \lambda_{System}^{0} \times \exp\left(-\int_{0}^{t-t^j} \lambda_{System}^{0} d\tau\right) \times \exp\left(-\int_{t-t^j}^{I^j} \lambda_{Demand} d\tau\right) dt \tag{31}$$

The formulae (31) can be integrated in a straightforward way. The probability to face no risk of corrective maintenance over I^k is given by:

$$P_{NoCM}^{k} = \exp\left(-I^k \times \lambda_{System}^{0}\right) + \frac{\lambda_{System}^{0}}{\lambda_{Demand} - \lambda_{System}^{0}} \times \left[\exp\left(-I^k \times \lambda_{System}^{0}\right) - \exp\left(-I^k \times \lambda_{Demand}\right)\right] \quad (32)$$

$$P_{NoCM}^{N+1} = \begin{cases} \exp\left(-I^{N+1} \times \lambda_{System}^{0}\right) + \frac{\lambda_{System}^{0}}{\lambda_{Demand} - \lambda_{System}^{0}} \times \left[\exp\left(-I^{N+1} \times \lambda_{System}^{0}\right) - \exp\left(-I^{N+1} \times \lambda_{Demand}\right)\right] & if\ I^{N+1} > 0 \\ 0 & if\ I^{N+1} = 0 \end{cases} \quad (33)$$

When there is no failure-finding task, i.e. $t^j = 0$ and $t^{j+1} \to +\infty$, the limit of P_{NoCM}^{j} is:

$$\lim_{t^{j+1} \to +\infty} P_{NoCM}^{j} = 0 \text{ with } t^j = 0 \quad (34)$$

C. Optimization of Failure-Finding Frequency

Thanks to the previous analytic results, we are now able to compute analytic sensitivity of the objective and constraint functions of the optimization problem.

Note that when there is no failure-finding task, i.e. $I \to +\infty$, the limit of $C(I)$ is:

$$\lim_{I \to +\infty} C(I) = C_{CM} \times \frac{FS \times \lambda_{Demand} \times \lambda_{System}^{0}}{\lambda_{Demand} + \lambda_{System}^{0}} \quad (35)$$

V. Numerical Example

A. Case Studied Definition

To illustrate the formulae developed, we consider a pressure relief valve. The component is a classic failure-finding scenario because it serves as a protective device. Its conditional property of failure is unrelated to age and it does not exhibit infant mortality.

Pressure relief valves are designed to provide protection from over-pressure in steam, gas, air and liquid lines. As a result the failure of the valve stays hidden if there is no overpressure. Moreover there is a process demand of the valve every time safe pressures are exceeded. In this case, the valve lets off steam to drop the pressure to a preset level. The corrective maintenance actions will be performed every time the valve fails to ensure that the system does not exceed the preset level.

We consider the following numerical data:

- Mean Time Between Failure=5,000 flight hours, hence $\lambda^0_{System} = 1/5{,}000$
- Valve used once every 900 flight hour, hence $\lambda_{Demand} = 1/900$
- 10 aircrafts in the fleet, hence $FS = 10$
- Cost to perform the check during failure-finding maintenance $C_{Check} = 15\$$
- Cost of failure-finding maintenance $C_{FF} = 60\$$
- Cost of corrective maintenance $C_{CM} = 150\$$
- Computation period T=10,000 flight hours

B. Impact of Failure-Finding Frequency

In this section we illustrate the formulae presented in section 4. The failure-finding interval length varies from to 100 flight hours to T.

The first shape is the maintenance cost rate C(I) as a function of I. It shows how the length of the failure-finding interval impacts the maintenance cost. This is a U-curve.

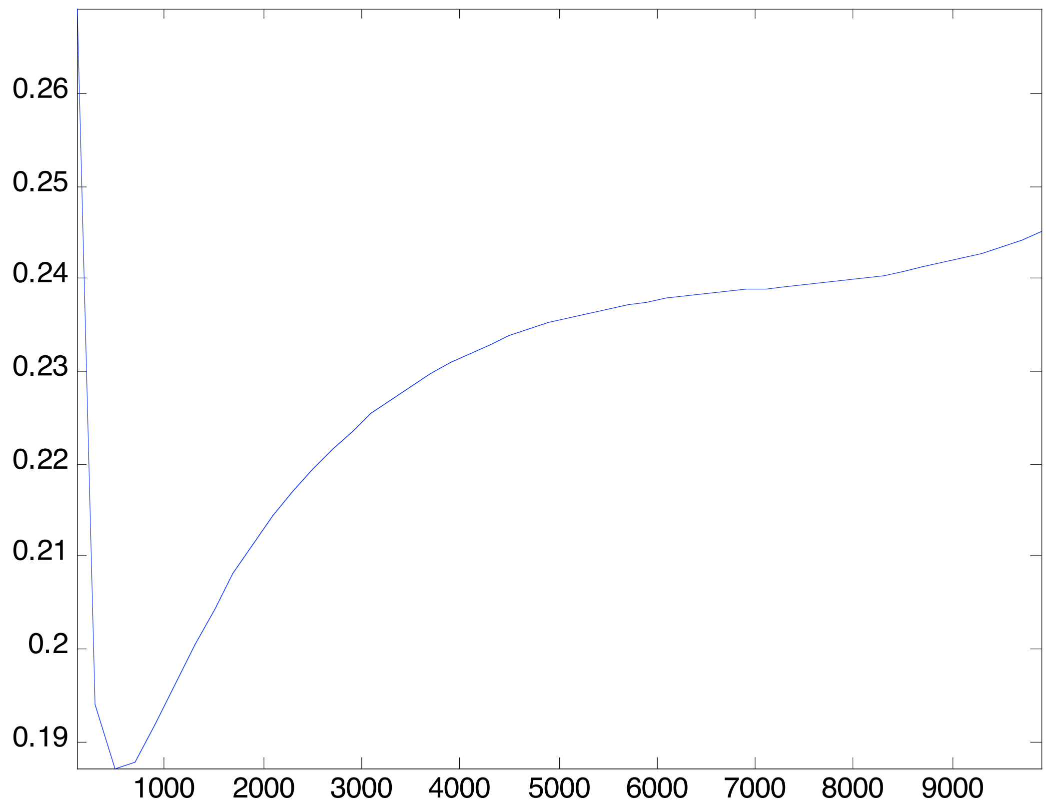

Fig. 2 Maintenance cost as a function of failure-finding interval length.

The following curve presents as a function of interval length the probability to avoid corrective maintenance actions during a failure-finding interval. This curve shows that when the interval length increases, the probability to have corrective maintenance also increases, which is logical. In fact, the main purpose of failure-finding task is to prevent or at least reduce the risk of associated failure leading to corrective maintenance actions

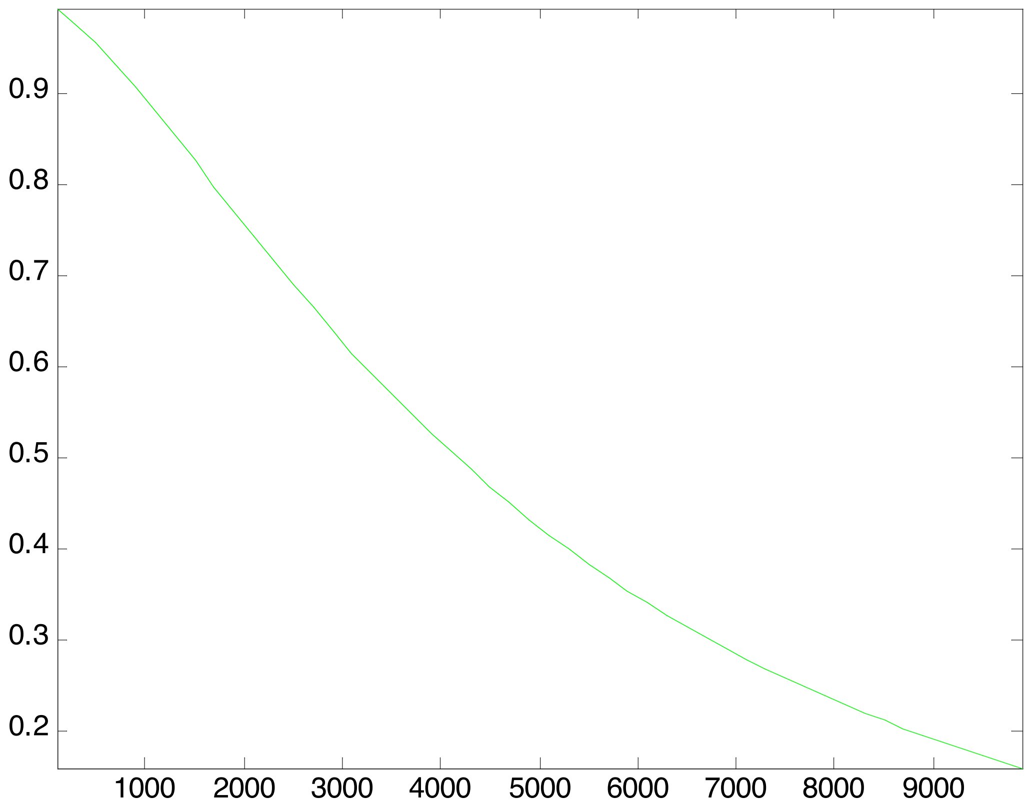

Fig. 3 Probability of no corrective maintenance actions as a function of failure-finding interval length.

C. Optimization of Failure-Finding Frequency

If we suppose that the threshold δ is equal to 79.73%, then according to the following sketch, the interval must be lower than 1,700FH. As a result to minimize the cost rate, the optimal interval I* is equal to 500 FH. For this value the cost rate is 0.1872 $/FH.

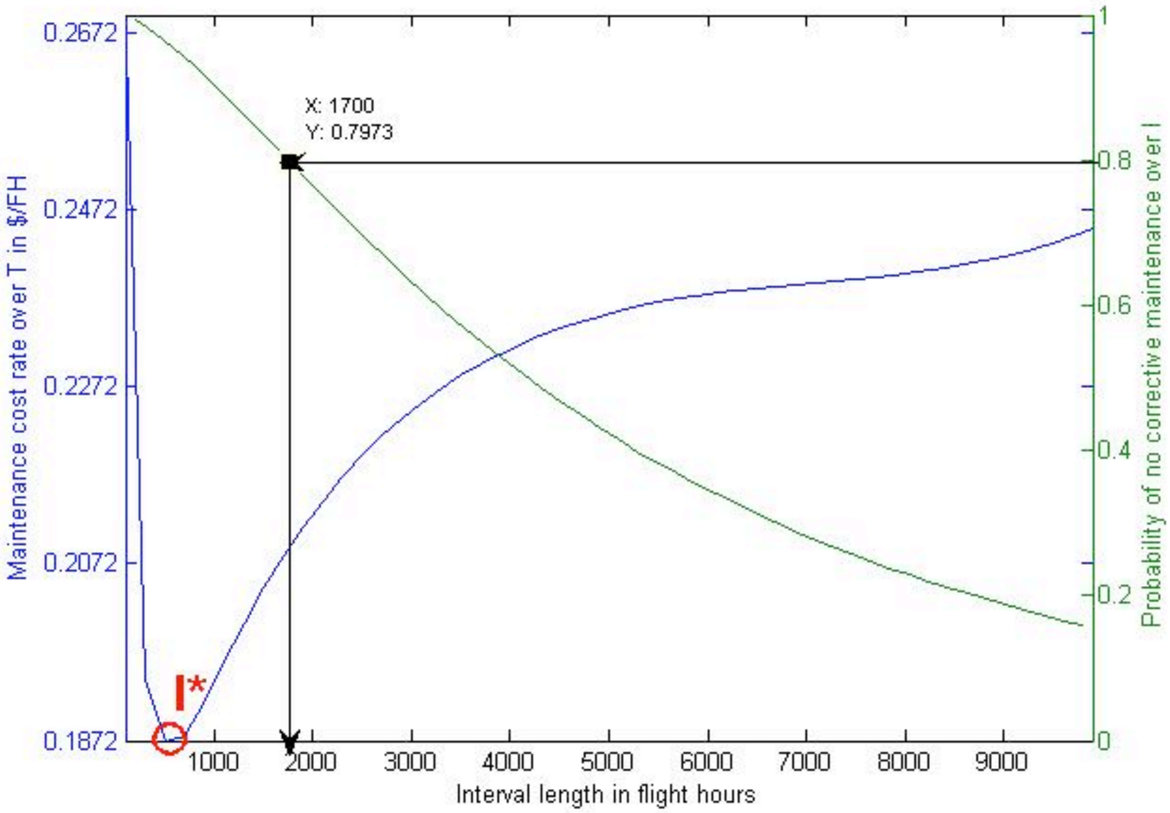

Fig. 8 Interval optimization as a function of failure-finding interval length.

VI. Conclusion

In this paper, we introduced a new methodology for modeling the combination of failure-finding and corrective maintenance for a hidden failure system. From this model, we provided cost and risk formulae and we derived an optimization problem to determine the optimal length of failure-finding task interval. Moreover we provided exact analytical formulae in the case of constant failure rate. We illustrated these results from a classic failure-finding scenario on a pressure relief valve.

Future related work will attempt to extend our approach by relaxing one or more of the original assumptions. Our first attempt will be to address a broader problem that includes condition parameters directly linked to the system degradation. Condition parameters could be any characteristic such as temperature, wear, humidity, sound, pressure, chemical concentration, and shock. As in health management system, information provided by sensors about the change on condition parameters can be used to optimize the next failure-finding task.

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