Modeling and Numerical Simulations of Magnetic Field Generation in a Plasma due to Anisotropic Laser Heating

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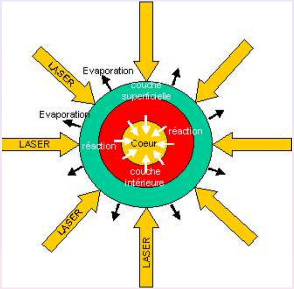
Outline

Outline

Context
Model
Numerical approximation
Numerical tests
Conclusion and perspectives

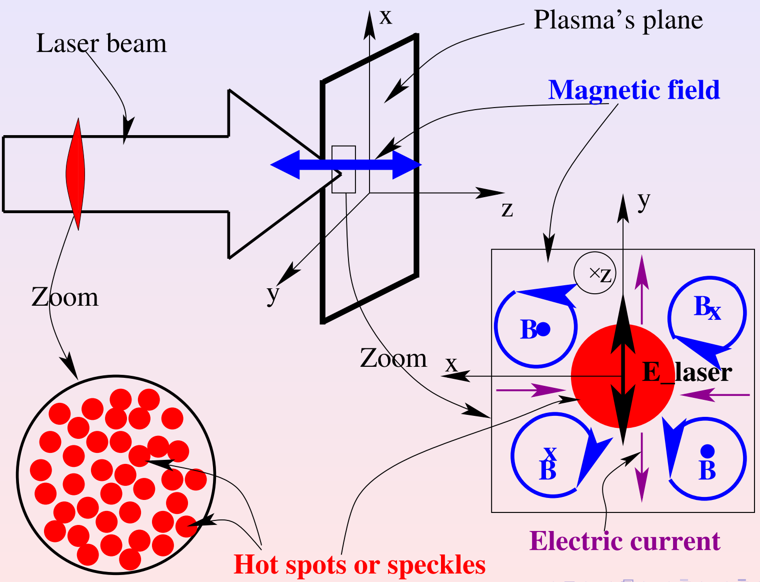
Context

- **Inertiel Confinement Fusion (ICF).**



- Framework: **Laser-Plasma interaction.**
  - Laser intensity $\mathrm{I} = 3 \times 10^{15}$–$10^{16}$ W/cm^2
  - Plasma temperature $\mathrm{T}_e = 1$–2 keV $= 1.2$–$2.3 \times 10^7\,{}^\circ$K
  - Typical time 50–100ps

Context



Laser beam
x
Plasma's plane
Magnetic field
z
y
Zoom
y
×z
B
B
x
Zoom
x
E_laser
x
B
B
Electric current
Hot spots or speckles

A mechanism to generate Magnetic Field

Faraday's Law F. L.

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}_{\mathrm{elec}}$$

Model

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$$\mathbf{E}_{\mathrm{elec}} = -\mathbf{u}_e \times \mathbf{B} - \nabla \mathrm{W}/(2e) \\ -(\nabla \cdot (n_e \mathbf{U}))/(n_e e) + \mathbf{R}_{ie}/(n_e e)$$

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Magnetic Field Evolution Equation

$$\partial_t \mathbf{B} - \nabla \times (\mathbf{V} \times \mathbf{B}) - \mu_0{}^{-1} \nabla \times (\sigma_0^{-1} \nabla \times \mathbf{B}) \\ = \nabla \times ((e n_e)^{-1} \nabla \cdot (n_e\, \mathbf{U}))$$

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Source term

$$\nabla \times ((e n_e)^{-1} \nabla \cdot (n_e\ \mathbf{U})) = 0 \Longrightarrow \mathbf{B} = \text{constante}$$

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The model

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Ten-moments approximation takes into account the anisotropy of electron pressure

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Model

Ten-moments approximation: derivation procedure

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- Electrons Kinetic equation

  in the high frequency laser field

  in presence of the quasi-static electric field $\mathbf{E}_{\mathrm{elec}}$

  and magnetic field $\mathbf{B}$

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  in the high frequency laser field
  in presence of the quasi-static electric field $\mathbf{E}_{\mathrm{elec}}$
  and magnetic field $\mathbf{B}$
- Average this equation over laser period
  Laser's contribution contained in the tensor

$$\mathbf{W} = \varepsilon_0 \langle \mathbf{E}_{\mathrm{L}} \otimes \mathbf{E}_{\mathrm{L}} \rangle / n_c$$

where $\mathbf{E}_{\mathrm{L}}$ is the laser electric field
n_c is the critical density

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  where $\mathbf{E}_{\mathrm{L}}$ is the laser electric field

  n_c is the critical density

- Take the consecutive moments over the electron distribution function f

  assume the quasi-neutrality of the plasma

Ten-moments approximation: Notations and equations

- $$\begin{aligned} n_e &= \int f\, d\mathbf{v} \\ n_e\mathbf{u}_e &= \int f\, \mathbf{v} d\mathbf{v} \\ \mathbf{P} &= \int f\,(\mathbf{v}-\mathbf{u}_e)\otimes(\mathbf{v}-\mathbf{u}_e) d\mathbf{v} \\ \mathbf{U} &= \mathbf{P}/n_e - \mathbf{W} \\ \mathbf{V} &= \mathbf{u}_e + \mathbf{j}/(n_e e) \simeq \mathbf{u}_e \\ \mathbf{E} &= \rho\mathbf{V}\otimes\mathbf{V} + n_e\mathbf{U} \end{aligned}$$

- $$\begin{aligned} &\partial_t\rho + \nabla\cdot(\rho\mathbf{V}) = 0 \\ &\partial_t(\rho\mathbf{V}) + \nabla\cdot\mathbf{E} = -n_e\nabla \mathrm{W}/2 \\ &\partial_t\mathbf{E} + \nabla\cdot(\rho\mathbf{H}\otimes\mathbf{V})^S + \nabla\cdot\mathbf{Q} = -n_e(\nabla W\otimes\mathbf{V})^S \\ &\qquad +2\nu_T n_e\mathbf{W} + \mathbf{S}_{\mathrm{BU}} - \mathbf{S}_{\mathrm{I}} - \mathbf{S}_{\mathrm{B}} \end{aligned}$$

Model

Ten-moments approximation to Euler system

- $$\partial_t \mathbf{E} + \nabla \cdot (\rho \mathbf{H} \otimes \mathbf{V})^S + \nabla \cdot \mathbf{Q} = -n_e(\nabla W \otimes \mathbf{V})^S + 2\nu_T n_e \mathbf{W} + \mathbf{S}_{\mathrm{BU}} - \nu_P(\mathbf{P} - \mathrm{P}\mathbf{I}) - \mathbf{S}_{\mathrm{B}} \quad (1)$$
- Set $\mathrm{B} = 0$ and $\mathbf{W} = 0$
- Tend $\nu_P \longrightarrow +\infty$ and make expansion of (1) respect to ν_P
  $$\Longrightarrow \mathbf{P} = \mathrm{P}\mathbf{I}$$
- Take $\frac{1}{2}\mathrm{trace}$ of the result
- Set $\mathrm{E} = \rho(\mathbf{V}_1^2 + \mathbf{V}_2^2)/2 + 3n_e\mathrm{U}/2$ and $\mathrm{p} = n_e\mathrm{U}$
- Euler energy equation
  $$\partial_t \mathrm{E} + \nabla \cdot ((\mathrm{E} + \mathrm{p})\mathbf{V}) = 0$$

Model

Equations of the model: EMHD model

$$\partial_t \mathrm{B} + \nabla \cdot (\mathrm{B}(\mathbf{V} + \mathbf{V}_{\mathrm{N}} + \mathbf{V}_{\mathrm{H}})) - {\mu_0}^{-1} \nabla \times (\sigma_0^{-1} \nabla \times \mathbf{B})$$
$$= \nabla \times ((e n_e)^{-1} \nabla \cdot (n_e\ \mathbf{U}))$$
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0$$
$$\partial_t (\rho \mathbf{V}) + \nabla \cdot \mathbf{E} = -n_e \nabla \mathrm{W}/2$$
$$\partial_t \mathbf{E} + \nabla \cdot (\rho \mathbf{H} \otimes \mathbf{V})^S + \nabla \cdot \mathbf{Q} = -n_e (\nabla W \otimes \mathbf{V})^S$$
$$+2\nu_T n_e \mathbf{W} + \mathbf{S}_{\mathrm{BU}} - \mathbf{S}_{\mathrm{I}} - \mathbf{S}_{\mathrm{B}}$$

B. Dubroca *et al.*, *Physics of Plasmas*, **11**, 3830 (2004)

Work in preparation

Model: closure relations

Unknowns of the model

B, ρ, $\mathbf{V}$, $\mathbf{U}$ and $\mathbf{Q}$

In 2D geometry

number of unknowns $= 17$
number of equations $= 8$ $\Longrightarrow$ Crucial necessity to close the model

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Closure relations

On electronic pressure $\mathbf{P} = \mathrm{k_B}\mathbf{T}_e = \mathbf{U}+\mathbf{W}$

On heat flux tensor $\mathbf{Q} = \mathbf{Q}_{\mathrm{iso}} + \mathbf{Q}_{\mathrm{ani}}$

$$\text{where } \mathbf{Q}_{\mathrm{iso}} = -6(\kappa\nabla\mathrm{U}\otimes\mathbf{I})^S/5$$
$$\mathbf{Q}_{\mathrm{ani}} = -4\delta\mathrm{k_B}\mathrm{T}(\nabla\otimes\mathbf{\Pi})^S/(5n_e m_e \nu_{ie})$$
$$\text{with } \mathbf{\Pi} = (\mathbf{U} - \mathrm{U}\mathbf{I})$$

Input parameters for simulations

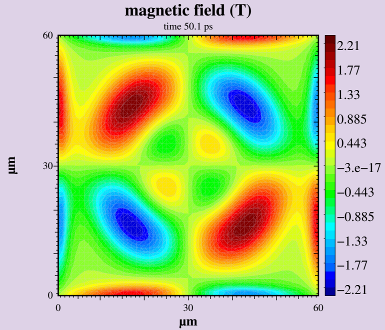
Laser

Laser intensity $\mathrm{I} = 3 \times 10^{15}$ W/cm^2
Radius of the speckle $\mathrm{R} = 10\,\mu$m
Laser wavelength $\lambda_0 = 0.35\,\mu$m

Plasma

Electronic Density $n_e = 9 \times 10^{21}$cm^{-3}
Plasma temperature $\mathrm{T}_e = 2.3 \times 10^{7\circ}$K

Structure of generated magnetic field after 50 *ps*



Model properties

Definition

The set of the physically admissible states of the above model is
$$\mathcal{E}_{pas} = \{\mathcal{U} = (\rho\ \rho\mathbf{V}\ \mathbf{E}\ \mathrm{B})^t \,/ \rho \geq 0, \text{ and } \Xi = \mathbf{E} - \rho\mathbf{V} \otimes \mathbf{V} \text{ verifies } (\Xi\,\xi, \xi) \geq 0\ \forall\,\xi\}$$

Proposition

$\mathcal{E}_{pas}$ *is a* **close cone and convex**

C. D. Levermore *et al.*, *SIAM J. of Appl. Math.*, **59**, **1**, 72 (1996)

Model properties

Compact form

$$\partial_t \mathcal{U} + \partial_x \mathcal{F}_x(\mathcal{U}) + \partial_y \mathcal{F}_y(\mathcal{U}) + \mathcal{G}(\mathcal{U}, \partial^2_{xx}\mathcal{U}, \partial^2_{yy}\mathcal{U}, \partial^2_{xy}\mathcal{U}) = \mathcal{S}(\mathcal{U}) + \mathcal{R}(\mathcal{U}) + \mathcal{C}(\mathcal{U})$$

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Parabolicity

black + *magenta* = Parabolic

Problem ill-posed without the terms $\mathbf{S}_{\mathrm{BU}}$, $\mathbf{Q}_{\mathrm{ani}}$

Anisotropic filamentation instability

Model properties

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Parabolicity

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Problem ill-posed without the terms $\mathbf{S}_{\mathrm{BU}}$, $\mathbf{Q}_{\mathrm{ani}}$

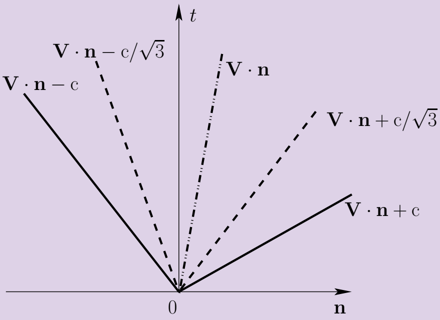
Anisotropic filamentation instability

Hyperbolicity

black + *violet* = Hyperbolic

Model properties

Hyperbolicity: waves



Hyperbolicity: waves nature

$\mathbf{V}\cdot\mathbf{n} \pm c$ *genuinely nonlinear*

$\mathbf{V}\cdot\mathbf{n}$, $\mathbf{V}\cdot\mathbf{n} \pm c/\sqrt{3}$ *linearly degenerated*

Numerical approximation: main ideas

Approximation in time

Non-linear implicit scheme designed on

- A Newton-Krylov method
- Using a non-linear GMRES

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Approximation in space

Approximate Riemann solver HLLC for Hyperbolic part

Second order accuracy extension by slope limiters (MUSCL)

Centred schemes for diffusion terms

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Mesh

Cartesian uniform mesh in 2D

Numerical approximation: Precisely

Continuous compact form

$$\partial_t \mathcal{U} + \partial_x \mathcal{F}_x(\mathcal{U}) + \partial_y \mathcal{F}_y(\mathcal{U}) + \mathcal{G}(\mathcal{U}, \partial^2_{xx}\mathcal{U}, \partial^2_{yy}\mathcal{U}, \partial^2_{xy}\mathcal{U})$$
$$= \mathcal{S}(\mathcal{U}) + \mathcal{R}(\mathcal{U}) + \mathcal{C}(\mathcal{U}) = \mathcal{RHS}(\mathcal{U}(t))$$

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Discrete compact form in the cell $\mathrm{l,m}$

$$\partial_t\mathcal{U}_{\mathrm{l,m}}(t) + \frac{\mathcal{F}_{\mathrm{l+1/2,m}}(t) - \mathcal{F}_{\mathrm{l-1/2,m}}(t)}{\Delta x}$$

$$\frac{\mathcal{F}_{\mathrm{l,m+1/2}}(t) - \mathcal{F}_{\mathrm{l,m-1/2}}(t)}{\Delta y} + \mathcal{G}_{\mathrm{l,m}}(\mathcal{U}(t)) = \mathcal{RHS}_{\mathrm{l,m}}(\mathcal{U}(t))$$

Numerical approximation: Implicit method

Non-linear equation

Discrete compact form in the cell l, m re-written as

$$\partial_t \mathcal{U}_{\mathrm{l,m}}(t) = \Theta_{\mathrm{l,m}}(\mathcal{U}(t))$$

Implicit Euler method time discretizatisation

$$\mathcal{U}^{n+1}_{\mathrm{l,m}} - \mathcal{U}^{n}_{\mathrm{l,m}} - \Delta t \Theta(\mathcal{U}^{n+1}) = 0$$

Non-linear equation follows

$$\mathbf{F}(\mathcal{U}^{n+1}) = 0$$

That must be solved to obtain $\mathcal{U}^{n+1}$

Numerical approximation: Implicit method

Newton method to solve Non-linear equation

Numerical approximation: Implicit method

Newton method to solve Non-linear equation

- At each time solve $\mathbf{F}(\mathcal{U}) = 0$ by a Newton method $\Longrightarrow$ linear system

$$\mathcal{A}(\mathcal{U}^k)(\mathcal{U}^{k+1} - \mathcal{U}^k) = -\mathbf{F}(\mathcal{U}^k)$$

where $\mathcal{A}(\mathcal{U}^k) = \mathbf{F}'(\mathcal{U}^k)$

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    - finite difference to compute $\mathcal{A}\mathbf{z}$

      $\mathcal{A}\mathbf{z} = \mathcal{A}(\mathcal{U}^k)\mathbf{z} = \dfrac{\mathbf{F}(\mathcal{U}^k + \varepsilon\mathbf{z}) - \mathbf{F}(\mathcal{U}^k)}{\varepsilon}$ with $\varepsilon = O(\varepsilon_{machine}^{1/2})$

Numerical approximation: Implicit method

Newton method to solve Non-linear equation

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    - For one fixed ε, use preconditioners

Numerical approximation: function $\mathbf{F}(\mathcal{U})$

Discrete compact form in the cell $\mathrm{l,m}$

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Numerical approximation: $\mathbf{F}(\mathcal{U})$

$\mathcal{F}_{\mathrm{l}\pm1/2,\mathrm{m}}(t)$ and $\mathcal{F}_{\mathrm{l},\mathrm{m}\pm1/2}(t)$

Finite volume method

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Finite volume method

- 1D Riemann problem by coordinate interface

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Finite volume method

- 1D Riemann problem by coordinate interface
- Approximate Riemann solver HLLC for ten-moments

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Finite volume method

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HLLC for ten-moments properties

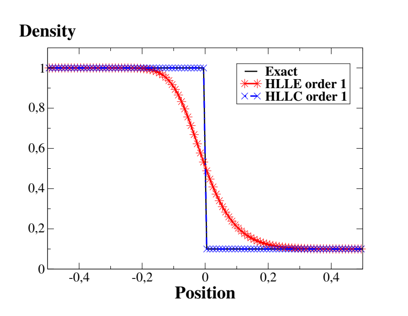
Positively conservative

Exact resolution of 1-shock and 5-shock

Exact resolution of contact discontinuity

HLLC solver: contact discontinuity

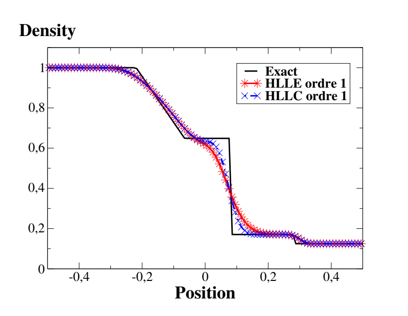
	ρ	$\mathbf{V}_1$	$\mathbf{V}_2$	$\mathbf{P}_{11}$	$\mathbf{P}_{12}$	$\mathbf{P}_{22}$	$\mathbf{P}_{33}$
left state	1	0	3	2	0	2	2
right state	0.1	0	3	2	0	2	2



Stationary contact discontinuity problem at $t = 0.2$, 100 *cells on* $(-0.5,\ 0.5)$ *and* $CFL = 0.5$

HLLC solver: Sod's shock tube problem

	ρ	$\mathbf{V}_1$	$\mathbf{V}_2$	$\mathbf{P}_{11}$	$\mathbf{P}_{12}$	$\mathbf{P}_{22}$	$\mathbf{P}_{33}$
left state	1	0	0	10^5	0	10^5	10^5
right state	0.125	0	0	10^4	0	10^4	10^4



Sod's shock tube problem at $t = 0.1$, 100 *cells on* $(-0.5,\ 0.5)$ *and* $CFL = 0.5$

Magnetic field generation: feedbacks neglected

$$\partial_t \mathrm{B} + \nabla \cdot (\mathrm{B}\mathbf{V}_{\mathrm{mag}}) - {\mu_0}^{-1} \nabla \times ({\sigma_0}^{-1} \nabla \times \mathbf{B}) = \nabla \times ((e n_e)^{-1} \nabla \cdot (n_e \; \mathbf{U}))$$

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\partial_t (\rho \mathbf{V}) + \nabla \cdot \mathbf{E} = -n_e \nabla \mathrm{W}/2$$

$$\partial_t \mathbf{E} + \nabla \cdot (\rho \mathbf{H} \otimes \mathbf{V})^S + \nabla \cdot \mathbf{Q}_{\mathrm{iso}} = -n_e (\nabla W \otimes \mathbf{V})^S + 2\nu_T n_e \mathbf{W} - \mathbf{S}_{\mathrm{I}}$$

Numerical tests

Movie on magnetic field generation: feedbacks neglected

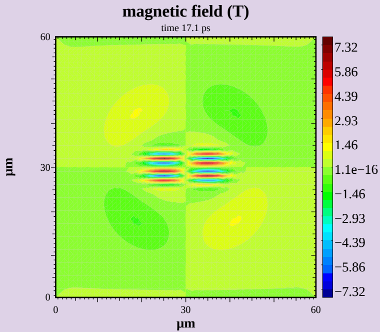
Laser	$\mathrm{I} = 3 \times 10^{15}$ W/cm^2
	$\mathrm{R} = 10\ \mu$m
	$\lambda_0 = 0.35\ \mu$m
Plasma	$n_e = 9 \times 10^{21}$cm^{-3}
	$\mathrm{T}_e = 2.3 \times 10^{7}\,^\circ$K

Magnetic field generation: feedbacks accounted for

$$\partial_t \mathrm{B} - {\mu_0}^{-1}\nabla \times ({\sigma_0}^{-1}\nabla \times \mathbf{B}) = \nabla \times ((en_e)^{-1}\nabla \cdot (n_e\ \mathbf{U}))$$
$$\partial_t \rho = 0$$
$$\partial_t (\rho \mathbf{V}) = 0$$
$$\partial_t \mathbf{E} + \nabla \cdot \mathbf{Q}_{\mathrm{iso}} = 2\nu_T n_e \mathbf{W} - \mathbf{S}_{\mathrm{I}} - \mathbf{S}_{\mathrm{B}}$$

where $\mathbf{S}_{\mathrm{B}} = 2n_e e(\mathbf{U} \times \mathbf{B})^S / m_e$ is the rotation of $\mathbf{U}$

Magnetic field structure: feedbacks accounted for



Magnetic field develops small scale perturbations, this is an anisotropic filamentation-type instability, after 17 ps. The computation cell length is 1 μm

Numerical tests

Instability analysis: $\mathbf{U}$ splitting

$$\mathbf{U} = \mathbf{\Lambda} + \mathbf{\Pi}$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \mathrm{U} & 0 & 0 \\ 0 & \mathrm{U} & 0 \\ 0 & 0 & \mathrm{U} \end{bmatrix}, \quad \mathbf{\Pi} = \begin{bmatrix} \Pi_\perp & \Pi_\wedge & 0 \\ \Pi_\wedge & -\Pi_\perp & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Adequate form for reduced model

$$\begin{aligned} \partial_t \mathrm{B} - \eta(\partial_x^2 + \partial_y^2)\mathrm{B} &= \frac{1}{e}(\partial_x^2 - \partial_y^2)\Pi_\wedge - \frac{2}{e}\partial_{xy}^2 \Pi_\perp \\ \partial_t \Pi_\perp &= -\nu_P \Pi_\perp + (\nu_T - \frac{1}{2}\nu_P)\mathrm{W} - \frac{2e}{m_e}\Pi_\wedge \mathrm{B} \\ \partial_t \Pi_\wedge &= -\nu_P \Pi_\wedge + \frac{2e}{m_e}\Pi_\perp \mathrm{B} \end{aligned}$$

Numerical tests

Instability analysis

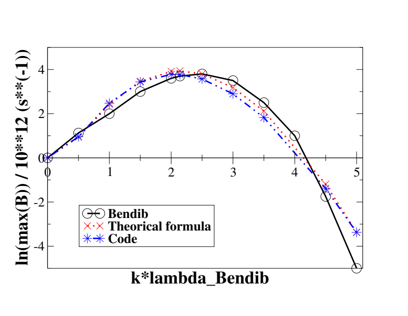
- Wave form for analysis $\mathcal{A}\exp(\omega t)\cos(\mathbf{k}_x x + \mathbf{k}_y y)$
- Dispersion relation
  $(\omega + \nu_p)(\omega + \eta|\mathbf{k}|^2) = -(\frac{2\nu_T}{\nu_P} - 1)\frac{\mathrm{W}}{m_e}|\mathbf{k}|^2\cos 2\theta$
- Asymptotically $\omega \sim |\mathbf{k}|^2$
  $\implies$ grid size is unstable

Numerical tests

Instability analysis: stabilisation

- Following terms stabilize the model
  $$\mathbf{S}_{\mathrm{BU}} = 2\delta_{visc}\mathrm{U}(\nabla \otimes (\nabla \times \mathbf{B}))^S/(e\mu_0)$$
  $$\mathbf{Q}_{\mathrm{ani}} = -4\delta \mathrm{k_B T}_e(\nabla \otimes \mathbf{\Pi})^S/(5 n_e m_e \nu_{ie})$$
- New dispersion relation follows:
  $$(\omega + \nu_p + \delta\kappa|\mathbf{k}|^2)(\omega + \eta|\mathbf{k}|^2) = \left[\left(\frac{2\nu_T}{\nu_P} - 1\right)\frac{W}{m_e} - \delta_{visc}\frac{\mathrm{U}}{m_e}\frac{c^2}{\omega_{P_e}^2}|\mathbf{k}|^2\right]k^2$$
- Asymptotically $\omega \sim |\mathbf{k}|^2 - |\mathbf{k}|^4$
  $\Longrightarrow$ cutoff that must be respected by the mesh size

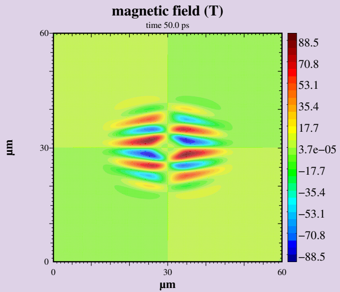
Magnetic field structure: feedbacks accounted for, stabilisation

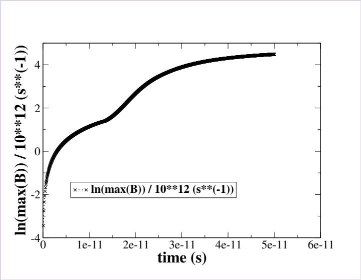


Comparison maximum growth rate of magnetic field between kinetic theory, theorical formula and Code. The computation cell length is $0.2\,\mu$m. $|\mathbf{k}| = 2\pi/\lambda$

A. Bendib *et al.*, *Physical Review E.*, **55**, 7522 (1997)

Magnetic field structure: feedbacks accounted for, stabilisation





Magnetic field structure after 50 ps and the corresponding maximum growth rate. The computation cell length is 0.2 μm

Conclusion and Perspectives

Conclusion

- Obtaining of a model EMHD capable of predicting and of reproducing the generation of auto-generated magnetic fields
- Validation
  we reproduce the analytical solutions
- Complicated and complex problem
  equations are stiff and can be unstable $\Longrightarrow$ need of robust schemes in time and in space

Perspectives

- Study mathematically the stability of the model problem with abstract theory of PDE
- Improve the closure on heat fluxes
- Introduce the equation of laser propagation