A Hybrid Langrangian Model for Vehicular Traffic Flow

Salissou Moutari & Michel Rascle

Laboratoire J. A. Dieudonné

University of Nice-Sophia Antipolis

Outline

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

- Introduction
- The Models
- Link between the Models
- The Hybrid Model
- Numerical Simulations

Outline

Université Nice Sophia Antipolis
Laboratoire J. A. Dieudonné

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- Introduction
- The Models
- Link between the Models
- The Hybrid Model
- Numerical Simulations

Outline

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

- Introduction
- The Models
- Link between the Models
- The Hybrid Model
- Numerical Simulations

Outline

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- Introduction
- The Models
- Link between the Models
- The Hybrid Model
- Numerical Simulations

Outline

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- Introduction
- The Models
- Link between the Models
- The Hybrid Model
- Numerical Simulations

» Outline

Introduction
» Goal
» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

Introduction

Goal

» Outline

Introduction

» Goal

» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

To introduce a simple hybrid model for vehicular traffic flow that combines a macroscopic description away from the junctions, traffic lights etc . . . and a microscopic view near these obstacles, both in Lagrangian moving cells.

Goal

» Outline

Introduction

» Goal

» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

To introduce a simple hybrid model for vehicular traffic flow that combines a macroscopic description away from the junctions, traffic lights etc . . . and a microscopic view near these obstacles, both in Lagrangian moving cells.

- Macroscopic Model: “Aw-Rascle” macroscopic model [Aw and Rascle, SIAM J. Appl. Math., 60 (2000)]

Goal

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction
» Goal
» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

To introduce a simple hybrid model for vehicular traffic flow that combines a macroscopic description away from the junctions, traffic lights etc . . . and a microscopic view near these obstacles, both in Lagrangian moving cells.

- Macroscopic Model: “Aw-Rascle” macroscopic model [Aw and Rascle, SIAM J. Appl. Math., 60 (2000)]
- Microscopic Model: A Follow the Leader type model [Aw, Klar, Materne and Rascle , SIAM J. Appl. Math., 63 (2002)]

Motivation

» Outline

Introduction
» Goal
» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- The difficulties when building a hybrid model: compexity of the interfaces description, compatibility between the models to be coupled, . . .

Motivation

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction
» Goal
» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- The difficulties when building a hybrid model: compexity of the interfaces description, compatibility between the models to be coupled, . . .

  [Hennecke et al, 2000], [Magne et al, 2000], [Helbing et al, 2002], [Bourel et al, 2003], [Berghout et al, 2005], [Leclercq, 2005], [Leclercq and Moutari, 2006 (Preprint)].

Motivation

» Outline

Introduction
» Goal
» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- The difficulties when building a hybrid model: compexity of the interfaces description, compatibility between the models to be coupled, . . .

  [Hennecke et al, 2000], [Magne et al, 2000], [Helbing et al, 2002], [Bourel et al, 2003], [Berghout et al, 2005], [Leclercq, 2005], [Leclercq and Moutari, 2006 (Preprint)].

- In [Aw, Klar, Materne and Rascle , SIAM J. Appl. Math., 63 (2002)], a relation between the “Aw-Rascle” (AR) macroscopic model and a microscopic Follow the Leader type model is established.

Motivation

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction
» Goal
» Motivation

Models

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

- The difficulties when building a hybrid model: compexity of the interfaces description, compatibility between the models to be coupled, . . .

  [Hennecke et al, 2000], [Magne et al, 2000], [Helbing et al, 2002], [Bourel et al, 2003], [Berghout et al, 2005], [Leclercq, 2005], [Leclercq and Moutari, 2006 (Preprint)].

- In [Aw, Klar, Materne and Rascle , SIAM J. Appl. Math., 63 (2002)], a relation between the "Aw-Rascle" (AR) macroscopic model and a microscopic Follow the Leader type model is established.
- In [Bagnerini and Rascle, SIAM J. Math. Anal., 35 (2003)] is shown what (and how) can be homogenized in a multiclass version of the AR model.

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

The Models

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

We are concerned with the “Aw-Rascle” macroscopic model of traffic flow.
It consists in the conservative form (in Eulerian coordinates) of two equations

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

We are concerned with the “Aw-Rascle” macroscopic model of traffic flow.
It consists in the conservative form (in Eulerian coordinates) of two equations

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho w) + \partial_x(\rho v w) = 0, \end{cases} \tag{1}$$

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

We are concerned with the "Aw-Rascle" macroscopic model of traffic flow.
It consists in the conservative form (in Eulerian coordinates) of two equations

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho w) + \partial_x(\rho v w) = 0, \end{cases} \tag{1}$$

where,

- ρ denotes the fraction of space occupied by cars (a dimensionless local density),
- v is the macroscopic velocity of cars
- w is a Lagrangian marker and for instance $w = v + p(\rho)$.

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

For concreteness, in the sequel, we will assume that

$$p(\rho) = \begin{cases} \frac{v_{ref}}{\gamma} \left(\frac{\rho}{\rho_m} \right)^{\gamma}, & \gamma > 0, \\ -v_{ref} ln \left(\frac{\rho}{\rho_m} \right), & \gamma = 0, \end{cases} \tag{2}$$

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

For concreteness, in the sequel, we will assume that

$$p(\rho) = \begin{cases} \frac{v_{ref}}{\gamma}\left(\frac{\rho}{\rho_m}\right)^{\gamma}, & \gamma > 0, \\ -v_{ref} ln\left(\frac{\rho}{\rho_m}\right), & \gamma = 0, \end{cases} \tag{2}$$

with v_{ref} a given reference velocity and $\rho_m = 1$ is the maximal density.

The Macroscopic Model

» Outline

Introduction

Models

» Macroscopic Model

» Microscopic Model

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

Let $\tau = 1/\rho$ be the specific volume and denote by (X, T) the Lagrangian “mass” coordinates. We have

$$\partial_x X = \rho, \qquad \partial_t X = -\rho v, \qquad T = t.$$

$X = \int^x \rho(y,t)dy$ describes the total length occupied by cars up to the point x, if they were packed “nose to tail”.

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Let $\tau = 1/\rho$ be the specific volume and denote by (X, T) the Lagrangian "mass" coordinates. We have

$$\partial_x X = \rho, \qquad \partial_t X = -\rho v, \qquad T = t.$$

$X = \int^x \rho(y,t)dy$ describes the total length occupied by cars up to the point x, if they were packed "nose to tail".

In Lagrangian "mass" coordinates, system (1) becomes

$$\begin{cases} \partial_T \tau - \partial_X v = 0, \\ \partial_T w = 0, \end{cases} \tag{3}$$

The Macroscopic Model

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction

Models
» Macroscopic Model
» Microscopic Model

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

Let $\tau = 1/\rho$ be the specific volume and denote by (X, T) the Lagrangian "mass" coordinates. We have

$$\partial_x X = \rho, \qquad \partial_t X = -\rho v, \qquad T = t.$$

$X = \int^x \rho(y,t)dy$ describes the total length occupied by cars up to the point x, if they were packed "nose to tail".

In Lagrangian "mass" coordinates, system (1) becomes

$$\begin{cases} \partial_T \tau - \partial_X v = 0, \\ \partial_T w = 0, \end{cases} \tag{3}$$

with, $w = v + P(\tau) := v + p\left(\frac{1}{\tau}\right)$.

The Macroscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Under the assumption of the CFL condition, the Lagrangian Godunov discretization of the macroscopic model (3) is

$$\begin{cases} \tau_j^{n+1} = \tau_j^n + \frac{\Delta t}{\Delta X}\left(v_{j+1}^n - v_j^n\right), \\ w_j^{n+1} = w_j^n, \end{cases} \tag{4}$$

with initial data

$$\begin{cases} \tau_j(0) = \tau_j^0 \\ v_j(0) = v_j^0 = w_j^0 + P(\tau_j^0) \end{cases} \tag{5}$$

The Microscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

We consider a microscopic *Follow-the-Leader* type model. The dynamics of a vehicle j is given by the two equations:

The Microscopic Model

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction

Models
» Macroscopic Model
» Microscopic Model

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

We consider a microscopic *Follow-the-Leader* type model.
The dynamics of a vehicle j is given by the two equations:

$$\begin{cases} \dfrac{dx_j}{dt} = v_j, \\ \dfrac{dv_j}{dt} = \dfrac{1}{\Delta X}\left(v_{j+1} - v_j\right) \dfrac{v_{ref}\,\tau_m^\gamma}{\tau_j^{\gamma+1}} \end{cases} \tag{6}$$

where
$x_j(t)$ and $v_j(t)$ are respectively the location and the velocity of the j^{th} vehicle at time t.

The Microscopic Model

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

We consider a microscopic *Follow-the-Leader* type model.
The dynamics of a vehicle j is given by the two equations:

$$\begin{cases} \dfrac{dx_j}{dt} = v_j, \\ \dfrac{dv_j}{dt} = \dfrac{1}{\Delta X}(v_{j+1} - v_j)\dfrac{v_{ref}\tau_m^\gamma}{\tau_j^{\gamma+1}} \end{cases} \tag{6}$$

where
$x_j(t)$ and $v_j(t)$ are respectively the location and the velocity of the j^{th} vehicle at time t.

Here, $\tau_j = \frac{(x_{j+1}-x_j)}{\Delta X} = \frac{1}{\rho_j}$, with ρ_j the normalized local density.

The Microscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Introducing the variable $w_j = v_j + P(\tau_j)$ in (6) (with $P(\tau_j) = p\left(\frac{1}{\tau_j}\right)$), we obtain

The Microscopic Model

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction

Models
» Macroscopic Model
» Microscopic Model

Link between the models

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

Introducing the variable $w_j = v_j + P(\tau_j)$ in (6) (with $P(\tau_j) = p\left(\frac{1}{\tau_j}\right)$), we obtain

$$\begin{cases} \frac{d\tau_j}{dt} = \frac{(v_{j+1} - v_j)}{\Delta X}, \\ \frac{dw_j}{dt} = 0. \end{cases} \tag{7}$$

with initial conditions

$$\begin{cases} \tau_j(0) = \tau_j^0, \\ v_j(0) = v_j^0 = w_j^0 - P(\tau_j^0). \end{cases} \tag{8}$$

The Microscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

The explicit Euler time discretization of system (7) is then

$$\begin{cases} \tau_j^{n+1} = \tau_j^n + \frac{\Delta t}{\Delta X}\left(v_{j+1}^n - v_j^n\right), \\ w_j^{n+1} = w_j^n \end{cases} \tag{9}$$

The Microscopic Model

» Outline
Introduction
Models
» Macroscopic Model
» Microscopic Model
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

The explicit Euler time discretization of system (7) is then

$$\begin{cases} \tau_j^{n+1} = \tau_j^n + \frac{\Delta t}{\Delta X}\left(v_{j+1}^n - v_j^n\right), \\ w_j^{n+1} = w_j^n \end{cases} \tag{9}$$

with

$$v_j^{n+1} = w_j^{n+1} - P(\tau_j^{n+1}).$$

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

The Link between the Models

Link between the Models: The scaling

Université Nice SOPHIA ANTIPOLIS Laboratoire J. A. Dieudonné

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Let us consider a large number of vehicles on a long stretch of road and introduce in the macroscopic model (3), a scaling (zoom) such that:

Link between the Models: The scaling

Université Nice Sophia Antipolis
Laboratoire J. A. Dieudonné

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Let us consider a large number of vehicles on a long stretch of road and introduce in the macroscopic model (3), a scaling (zoom) such that:

- the size of the considered domain $\longrightarrow \infty$,
- the number of vehicles $\longrightarrow \infty$ and
- the vehicles length tends to 0.

Link between the Models: The scaling

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Let us consider a large number of vehicles on a long stretch of road and introduce in the macroscopic model (3), a scaling (zoom) such that:

- the size of the considered domain $\longrightarrow \infty$,
- the number of vehicles $\longrightarrow \infty$ and
- the vehicles length tends to 0.

Let ϵ be the scaling parameter.

For some given Lagrangian coordinates (X, T), we consider the rescaled coordinates

$$(X', T') = (\epsilon X, \epsilon T). \tag{10}$$

Link between the Models: The scaling

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Consequently, the lenght of a vehicle will be

$$\Delta X' = \epsilon \Delta X. \tag{11}$$

Link between the Models: The scaling

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline

Introduction

Models

Link between the models

» Scaling

Hybridization

Synchronization

BV Estimates

Simulation

Conclusion

Consequently, the lenght of a vehicle will be

$$\Delta X' = \epsilon \Delta X. \tag{11}$$

However, in the new coordinates (X', T'), the variable τ (resp. ρ) and the Riemann invariant (v, w) remain unchanged, i.e.

$$\tau' = \tau \; (\text{resp. } \rho' = \rho), \; v' = v, \; w' = w. \tag{12}$$

Link between the Models: The scaling

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Thus the system (3) becomes

$$\begin{cases} \dfrac{\partial \tau}{\partial t'} = \dfrac{\partial v}{\partial X'}, \\ \dfrac{\partial w}{\partial t'} = 0. \end{cases} \tag{13}$$

Link between the Models: The scaling

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Thus the system (3) becomes

$$\begin{cases} \frac{\partial \tau}{\partial t'} = \frac{\partial v}{\partial X'}, \\ \frac{\partial w}{\partial t'} = 0. \end{cases} \tag{13}$$

Using the same scaling for the microscopic model, (7) turns to

$$\begin{cases} \frac{d\tau_j}{dt'} = \frac{1}{\Delta X'} \left(v_{j+1} - v_j \right), \\ \frac{dw_j}{dt'} = 0. \end{cases} \tag{14}$$

Link between the Models: The scaling

Université Nice SOPHIA ANTIPOLIS
Laboratoire J. A. Dieudonné

» Outline
Introduction
Models
Link between the models
» Scaling
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Thus the system (3) becomes

$$\begin{cases} \dfrac{\partial \tau}{\partial t'} = \dfrac{\partial v}{\partial X'}, \\ \dfrac{\partial w}{\partial t'} = 0. \end{cases} \tag{13}$$

Using the same scaling for the microscopic model, (7) turns to

$$\begin{cases} \dfrac{d\tau_j}{dt'} = \dfrac{1}{\Delta X'} \left(v_{j+1} - v_j \right), \\ \dfrac{dw_j}{dt'} = 0. \end{cases} \tag{14}$$

In the rescaled coordinates, a standard explicit Euler time discretization of the microscopic model (9) $\Longleftrightarrow$ to the Godunov discretization (4) of the macroscopic model.

» Outline
Introduction
Models
Link between the models
Hybridization
» Hybrid Model
Synchronization
BV Estimates
Simulation
Conclusion

Hybridization

Hybrid Model

» Outline
Introduction
Models
Link between the models
Hybridization
» Hybrid Model
Synchronization
BV Estimates
Simulation
Conclusion

Combining a macroscopic description away from the junctions, traffic lights etc. . . and a microscopic view near these obstacles.

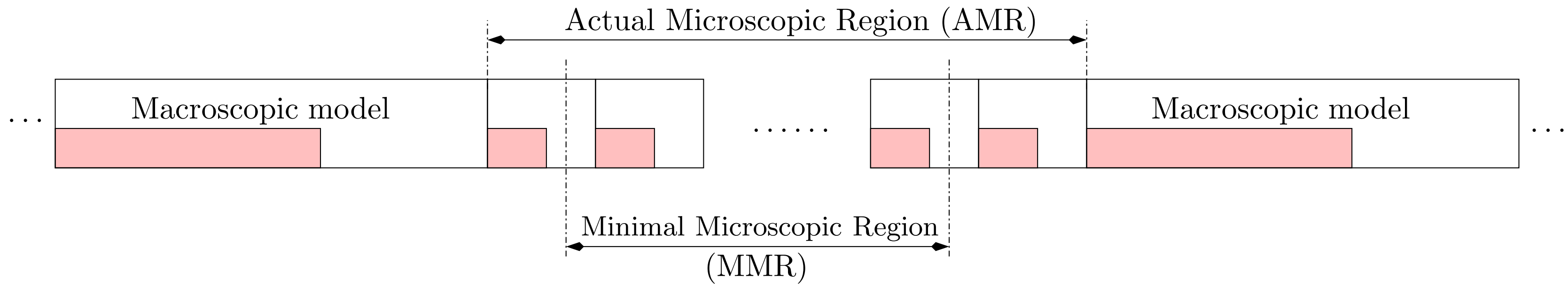


Figure 1: Hybrid Lagrangian model.

Thanks to the equivalence established above, no compatibility problem between the two models.

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization
» Macro-Micro
» Micro-Macro

BV Estimates

Simulation

Conclusion

Macro-Micro and Micro-Macro Synchronization

From Macro to Micro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

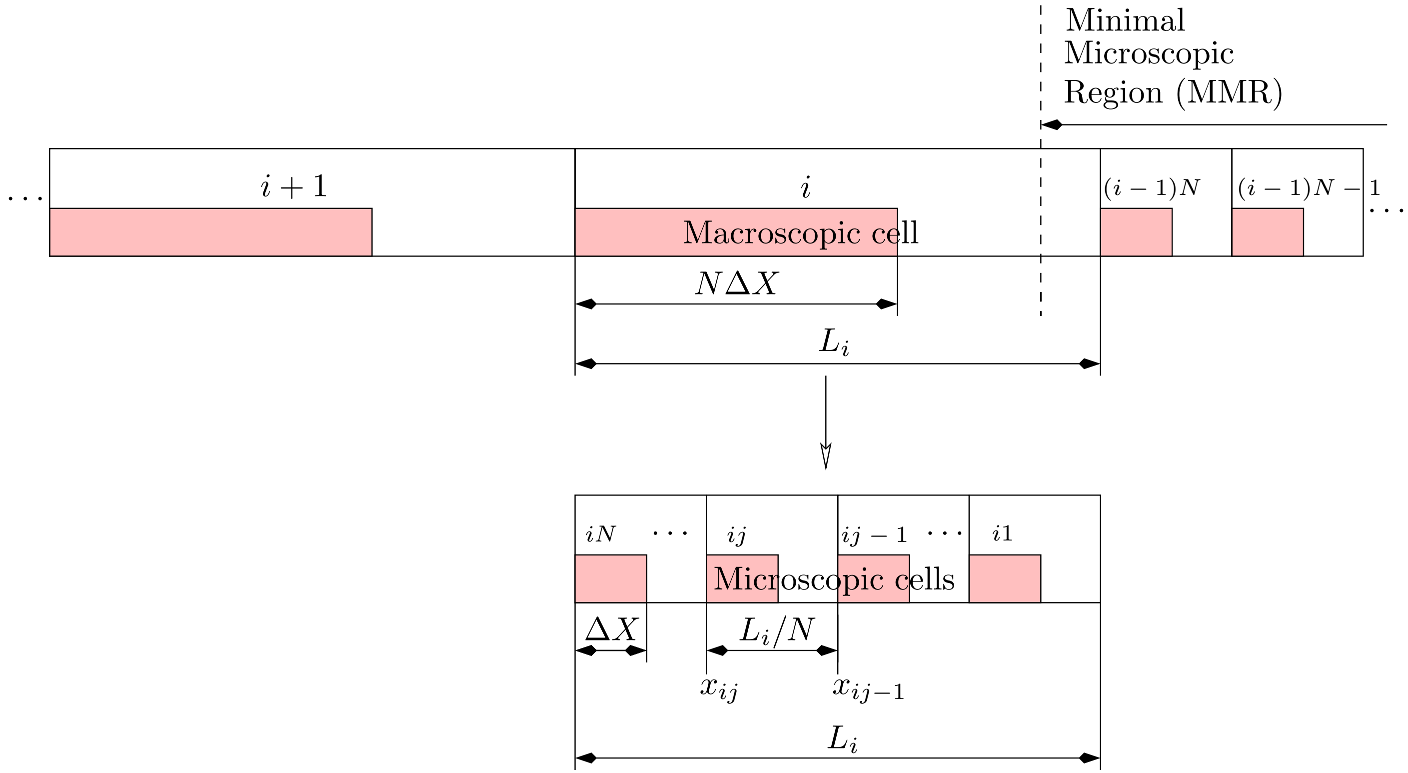


Figure 2: From the macroscopic to the microscopic model: before (above) and after (below) the synchronization.

From Macro to Micro

Université Nice Sophia Antipolis
Laboratoire J. A. Dieudonné

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

» Macro-Micro

» Micro-Macro

BV Estimates

Simulation

Conclusion

- This transformation does not modify the specific volume τ. Indeed, in the macroscopic cell i,

$$\tau_i = \frac{L_i}{N\Delta X} = \tau_{mac}. \tag{15}$$

From Macro to Micro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

- This transformation does not modify the specific volume τ. Indeed, in the macroscopic cell i,

$$\tau_i = \frac{L_i}{N\Delta X} = \tau_{mac}. \qquad (15)$$

- When this cell i becomes microscopic, the distance between two successive cars (i, j) (the follower) and $(i, j-1)$ (the leader) is

$$(x_{i,j-1} - x_{i,j}) = \frac{L_i}{N}. \qquad (16)$$

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

From Macro to Micro

- Therefore the microscopic specific volume in each of these smaller cells is

$$\tau_{i,j} = \tau_{mic} = \frac{L_i/N}{\Delta X} = \tau_{mac}. \tag{17}$$

So the specific volume does not change when passing from the macroscopic to the microscopic model.

From Macro to Micro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

- Therefore the microscopic specific volume in each of these smaller cells is

$$\tau_{i,j} = \tau_{mic} = \frac{L_i / N}{\Delta X} = \tau_{mac}. \tag{17}$$

So the specific volume does not change when passing from the macroscopic to the microscopic model.

- The Lagrangian variable w is conserved i.e.

$$w_{i,j} = w_i. \tag{18}$$

From Macro to Micro

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

» Macro-Micro

» Micro-Macro

BV Estimates

Simulation

Conclusion

- Therefore the microscopic specific volume in each of these smaller cells is

$$\tau_{i,j} = \tau_{mic} = \frac{L_i / N}{\Delta X} = \tau_{mac}. \tag{17}$$

So the specific volume does not change when passing from the macroscopic to the microscopic model.

- The Lagrangian variable w is conserved i.e.

$$w_{i,j} = w_i. \tag{18}$$

- Consequently, the velocity also does not change:

$$v_{i,j} = w_{i,j} - P(\tau_{i,j}) = w_i - P(\tau_i) = v_i, \tag{19}$$

for all microscopic car j in this cell i.

From Micro to Macro

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

» Macro-Micro

» Micro-Macro

BV Estimates

Simulation

Conclusion

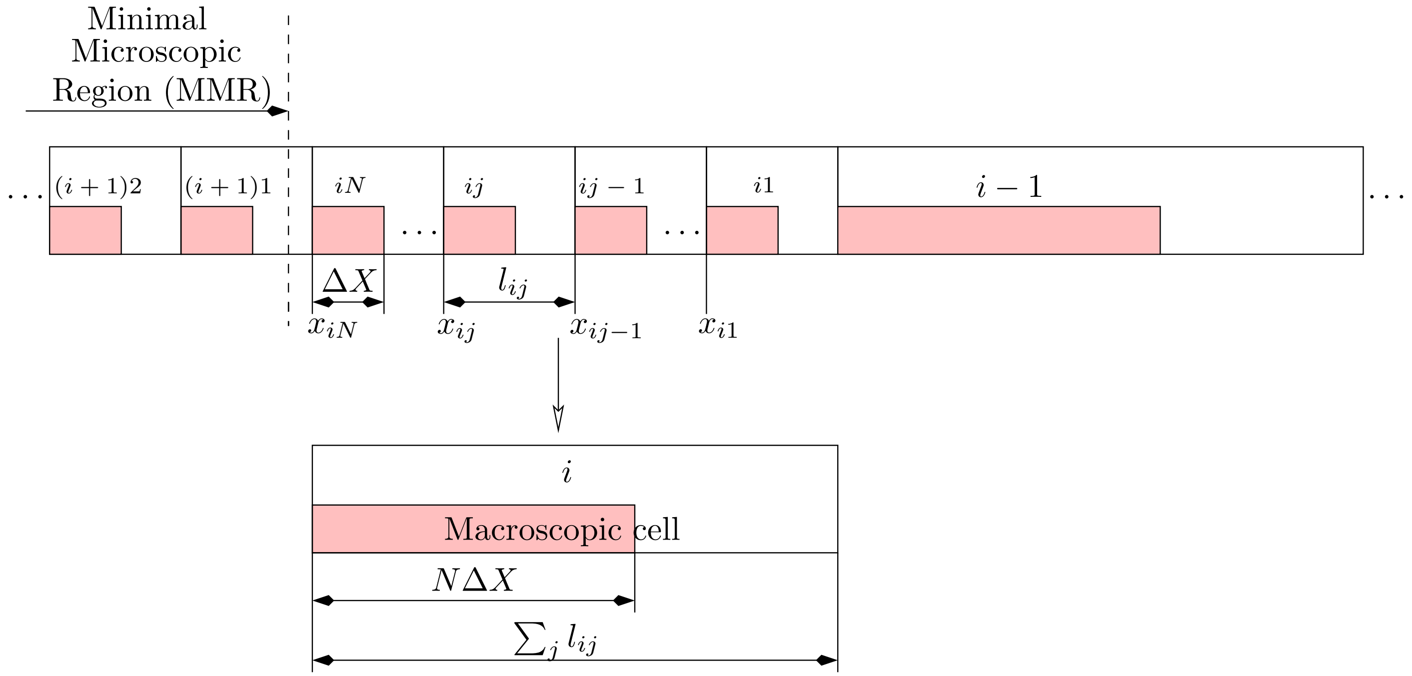


Figure 3: From the microscopic to the macroscopic model: before (above) and after (below) the synchronization.

From Micro to Macro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

- The macroscopic specific volume will be

$$\bar{\tau}_i = \frac{\sum_{j=1}^{N} l_{i,j}}{N\Delta X} = \frac{1}{N}\sum_{j=1}^{N}\frac{l_{i,j}}{\Delta X} = \frac{1}{N}\sum_{j=1}^{N}\tau_{i,j}. \qquad (20)$$

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

From Micro to Macro

- The macroscopic specific volume will be

$$\bar{\tau}_i = \frac{\sum_{j=1}^{N} l_{i,j}}{N\Delta X} = \frac{1}{N}\sum_{j=1}^{N}\frac{l_{i,j}}{\Delta X} = \frac{1}{N}\sum_{j=1}^{N}\tau_{i,j}. \tag{20}$$

- The Lagrangian variable $w_{i,j}$ is conserved i.e.

$$w_{i,j} = w_i. \tag{21}$$

Thus, averaging in Lagrangian coordinates, we have

$$\frac{1}{N}\sum_{j=1}^{N} w_{i,j} = \frac{1}{N}\sum_{j=1}^{N} w_i = w_i. \tag{22}$$

From Micro to Macro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

- Therefore, the corresponding macroscopic velocity is

$$\bar{v} = w - P(\bar{\tau}). \tag{23}$$

From Micro to Macro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

- Therefore, the corresponding macroscopic velocity is

$$\bar{v} = w - P(\bar{\tau}). \tag{23}$$

- In this case, the macroscopic model does not inherit exactly the microscopic parameters but only the average for τ and w, and the above corresponding velocity.

From Micro to Macro

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
» Macro-Micro
» Micro-Macro
BV Estimates
Simulation
Conclusion

- Therefore, the corresponding macroscopic velocity is

$$\bar{v} = w - P(\bar{\tau}). \tag{23}$$

- In this case, the macroscopic model does not inherit exactly the microscopic parameters but only the average for τ and w, and the above corresponding velocity.

- However the total variation in space and time of v and τ are controlled.

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
» Theorem
» Proof
Simulation
Conclusion

Estimates on the Total Variation

Estimates on the Total Variation

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
» Theorem
» Proof
Simulation
Conclusion

Theorem 1
Assume that the sequences (v_h^0, τ_h^0), respectively the initial data for v and τ (and therefore for w), are in $BV(\mathbb{R})$, and the the CFL condition is satisfied both in the macro and micro parts. Then, in the hybid model

a) *the total variation in x of $v_h(.,t)$ (resp. in t of $v_h(.,.)$) is non increasing in time (resp. is bounded on $\mathbb{R} \times [t,t']$);*

b) *the total variation in x of $\tau_h(.,t)$ on $\cup_{j\in\mathbb{Z}} I_j$ (resp. in t of $\tau_h(.,.)$ on $\mathbb{R} \times [t,t'])$) is bounded (resp. is bounded).*

Proof of the Theorem

Université Nice Sophia Antipolis
Laboratoire J. A. Dieudonné

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
» Theorem
» Proof
Simulation
Conclusion

Lemma 1 *Let* $U = (u_1, u_2, \dots u_n) \in \mathbb{R}^n$ *and* $\bar{U} \in \mathbb{R}$ *such that*

$$m = \min_i (u_i) \leq \bar{U} \leq \max_i (u_i) = M.$$

Then,

$$\forall\, \alpha, \beta \in \mathbb{R}, |\alpha - \bar{U}| + |\bar{U} - \beta| \leq |\alpha - u_1| + \sum_{i=1}^{n-1} |u_i - u_{i+1}| + |u_n - \beta|$$

Proof of the Theorem

» Outline

Introduction

Models

Link between the models

Hybridization

Synchronization

BV Estimates

» Theorem

» Proof

Simulation

Conclusion

(i) In the macroscopic model (resp. the microscopic model) (see Aw, Klar, Materne, Rascle, SIAM J. Appl. Math., 63 (2002) and
Bagnerini & Rascle, SIAM J. Math. Anal., 35 (2003)),
we have:

Proof of the Theorem

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
» Theorem
» Proof
Simulation
Conclusion

(i) In the macroscopic model (resp. the microscopic model) (see Aw, Klar, Materne, Rascle, SIAM J. Appl. Math., 63 (2002) and
Bagnerini & Rascle, SIAM J. Math. Anal., 35 (2003)),
we have:

a) the total variation in x of $v_h(.,t)$ (resp. in t of $v_h(.,.)$) is non increasing in time (resp. is bounded on $\mathbb{R} \times [t,t']$);

Proof of the Theorem

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
» Theorem
» Proof
Simulation
Conclusion

(i) In the macroscopic model (resp. the microscopic model) (see Aw, Klar, Materne, Rascle, SIAM J. Appl. Math., 63 (2002) and
Bagnerini & Rascle, SIAM J. Math. Anal., 35 (2003)),
we have:

a) the total variation in x of $v_h(.,t)$ (resp. in t of $v_h(.,.)$) is non increasing in time (resp. is bounded on $\mathbb{R} \times [t,t']$);

b) the total variation in x of $\tau_h(.,t)$ on $\cup_{j\in\mathbb{Z}} I_j$ (resp. in t of $\tau_h(.,.)$, on $\mathbb{R} \times [t,t']$) is bounded (resp. is bounded).

Proof of the Theorem

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
» Theorem
» Proof
Simulation
Conclusion

(i) In the macroscopic model (resp. the microscopic model) (see Aw, Klar, Materne, Rascle, SIAM J. Appl. Math., 63 (2002) and
Bagnerini & Rascle, SIAM J. Math. Anal., 35 (2003)), we have:

a) the total variation in x of $v_h(.,t)$ (resp. in t of $v_h(.,.)$) is non increasing in time (resp. is bounded on $\mathbb{R} \times [t,t']$);

b) the total variation in x of $\tau_h(.,t)$ on $\cup_{j\in\mathbb{Z}} I_j$ (resp. in t of $\tau_h(.,.)$, on $\mathbb{R} \times [t,t'])$) is bounded (resp. is bounded).

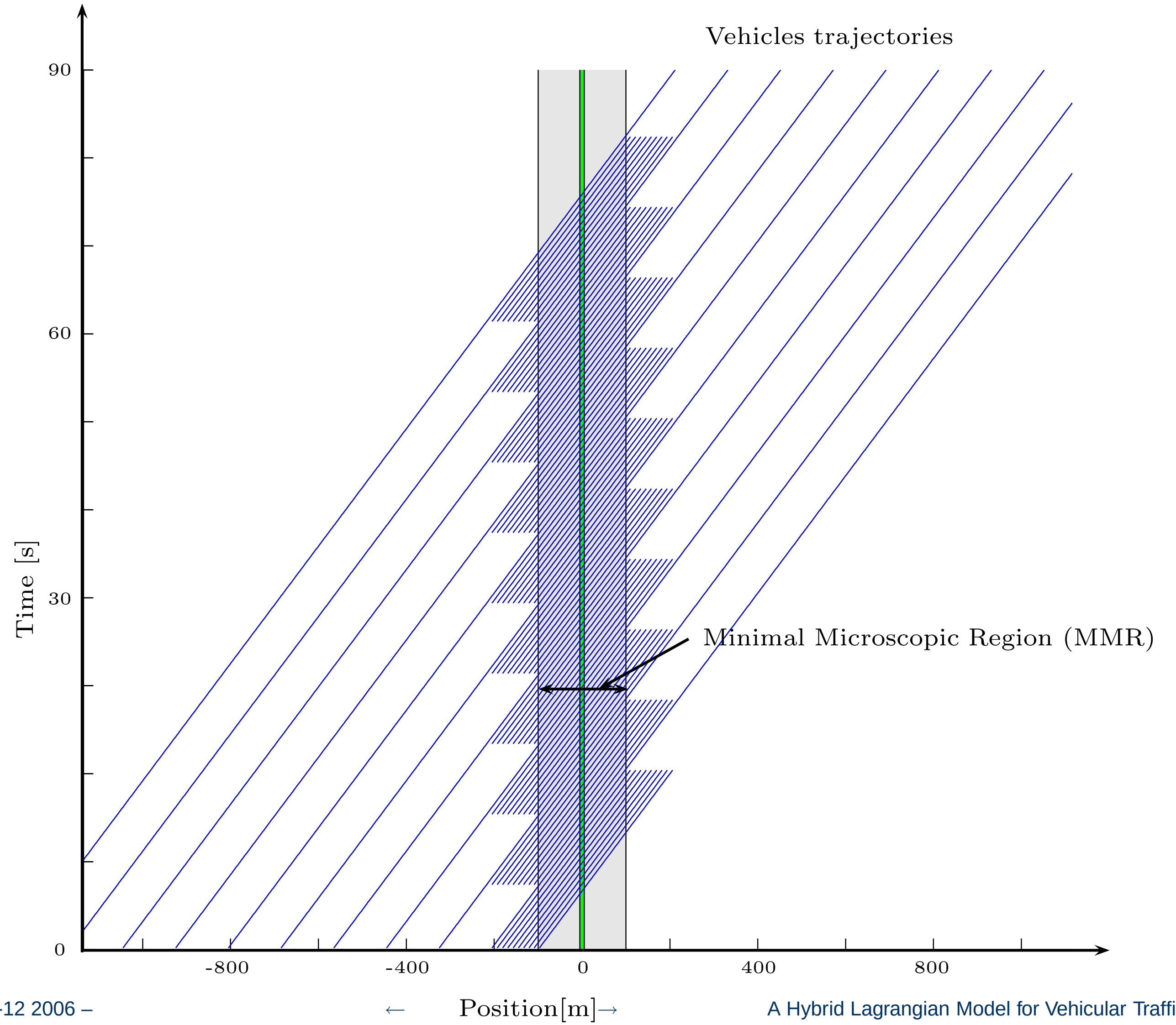
(ii) During the synchronization process, at each time t_n, the total variations in x of v_h and τ_h do not increase and their total variations in time are controlled thanks to Lemma 1.

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
» Scenario 1
» Scenario 2
» Scenario 3
Conclusion

Numerical Simulations

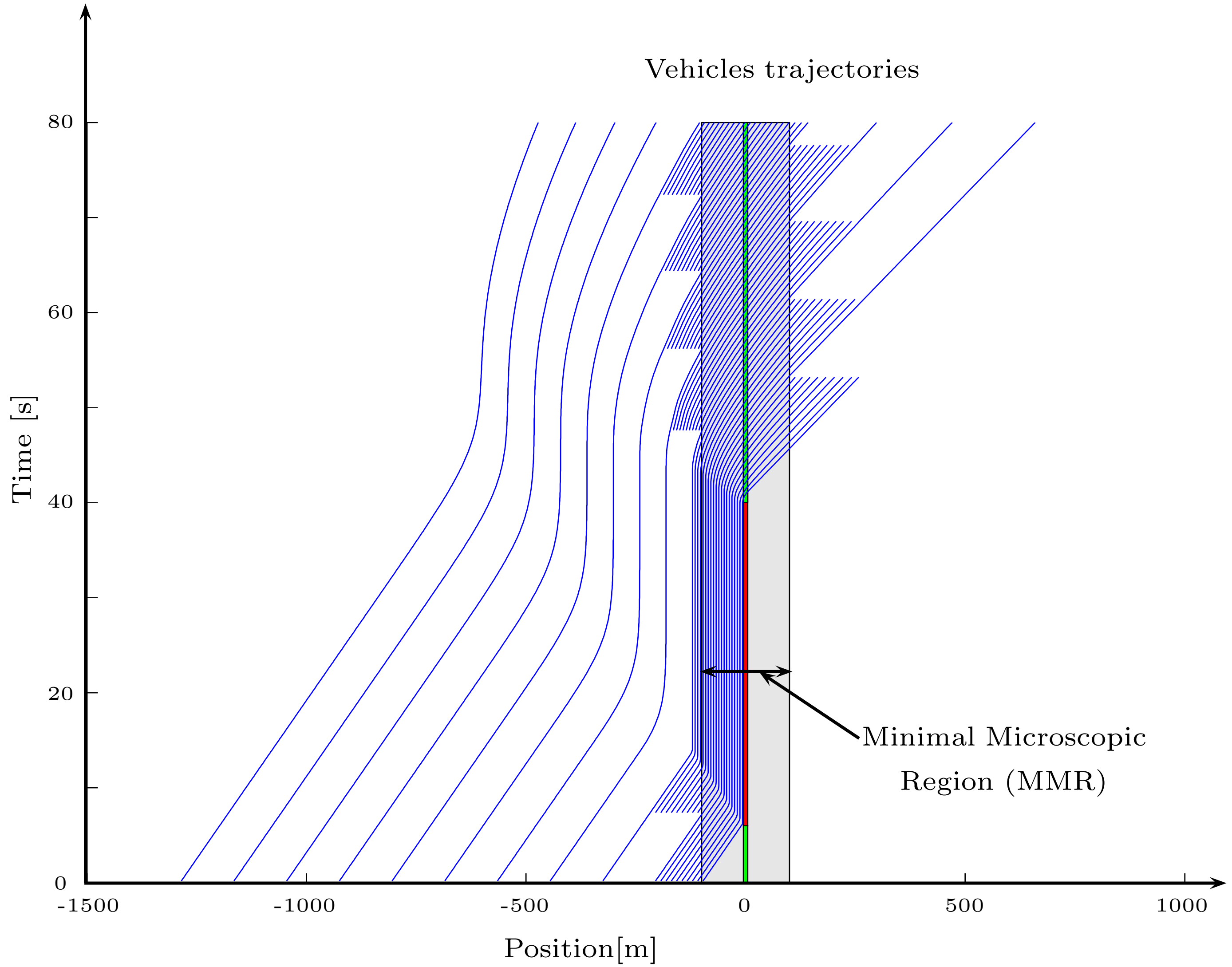
Scenario 1

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
» Scenario 1
» Scenario 2
» Scenario 3
Conclusion



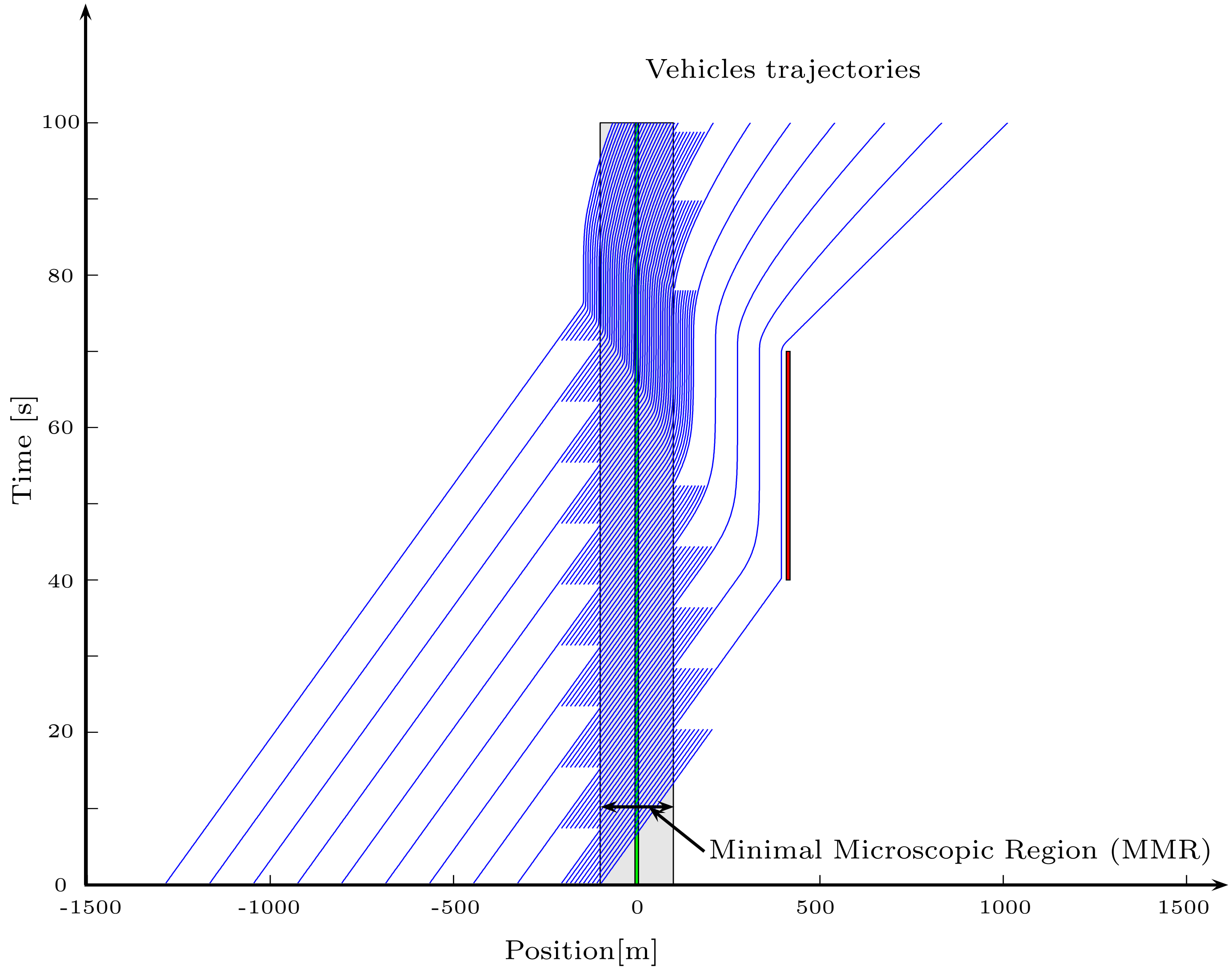
Scenario 2

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
» Scenario 1
» Scenario 2
» Scenario 3
Conclusion



Scenario 3

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
» Scenario 1
» Scenario 2
» Scenario 3
Conclusion



» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Conclusion and Outlook

Conclusion and Outlook

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion

Hybrid Model Properties

- Simple Macro-Micro and Micro-Macro synchronization;
- Mass is easily conserved through the interfaces;
- Nice wave propagation;
- Total variation controlled.

Further works

- Intersection modeling, Implementation on a road network, . . .

» Outline
Introduction
Models
Link between the models
Hybridization
Synchronization
BV Estimates
Simulation
Conclusion
Thank you!

Thank you!