5. Stabilisation

Difficulty: (R_h) is convex but not strictly convex and hence multiple solutions and bad condition of discrete problems are to be expected.

$(R_{h,\text{stab}})$ Minimise Stabilised Relaxed Energy

$$E_h^r(v_h) := E^r(v_h) + \frac{1}{2} a_h(v_h, v_h)$$

for FE function v_h.

Stabilisation Terms analysed for $W^r = W^c$

$$a_h(v_h, v_h) := \begin{cases} \sum_{E \in \mathcal{E}} h_E^\gamma \int_E |[Dv_h]|^2 ds & \text{(Jumps)}, \\ \int_\Omega h_\mathcal{T}^{\gamma-1} |Dv_h - ADv_h|^2 dx & \text{(Averages)}, \\ h^\gamma \|Dv_h\|^2_{L^2(\Omega)} & \text{(Gradients)}. \end{cases}$$

Theorem [Bartels-C.-Hackl-Hoppe (2003)]: Guaranteed convergence of (a_h-depending) damped quasi-Newton-Raphson Schemes for $(R_{h,\text{stab}})$.

Theorem [Bartels-C.-Prohl-Plechac (2003)]: For $u \in W^{3/2+\varepsilon,2}(\Omega)$ and quasiuniform meshsize h,

$$\lim_{h \to 0} \|D(u - u_h)\|_{L^2(\Omega)} = 0.$$