Relaxation Theory

(Overview (M), (G), (Q))

From Books on Calculus of Variations (e.g. Morrey 1966, Dacorogna 1989, Roubicek 1997, Pedregal 1997, Chipot 2000, Evans 1998):

$$(M) \qquad \min_{u \in \mathcal{A}} \int_\Omega W(x, Du(x))\, dx + L(u)$$

Introduce GYM as generalized solutions

$$(G) \qquad \min_{(u,\nu)} \int_\Omega \langle \nu_x, W(x,\cdot)\rangle \; dx + L(u)$$

over $u \in \mathcal{A}$ & GYM ν_x subj. to
$\langle \nu_x, \mathrm{Id}\rangle = Du(x)$ for a.e. $x \in \Omega$

Recast minimization problem in ν_x

$$(Q) \qquad \min_{u \in \mathcal{A}} \int_\Omega W^{qc}(x, Du(x))\, dx + L(u)$$

Typical Features

- (G), (Q) have solutions;
- (M), (G), (Q) essentially equivalent;
- e.g., infimal energies, macroscopic deformation, or stress fields in (M), (G), (Q) coincide;
- difficulties in $\mathcal{A} \times GYM$ and W^{qc}.