Mechanical Problem

[Phase Transitions for $YB_2Cu_3O_{6+x}$]

$$W(E) = \min\Big\{W_1(E), W_2(E)\Big\}$$

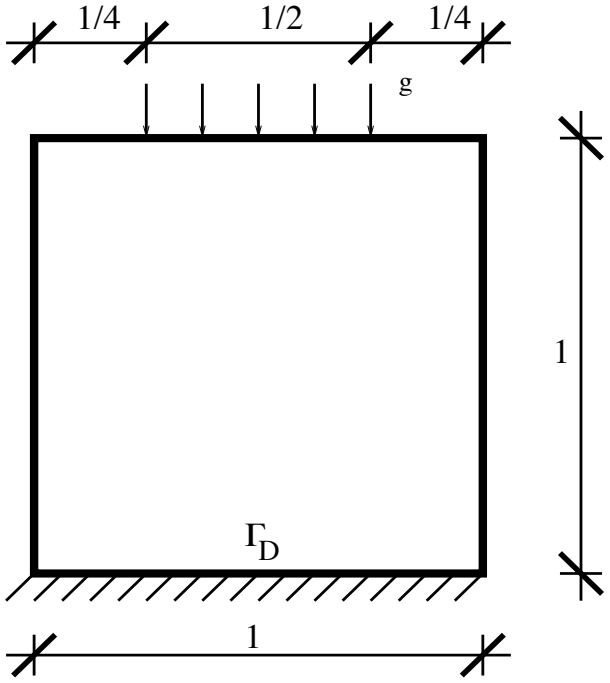
with

$$W_k(E) = \tfrac{1}{2}|\mathbb{C}^{1/2}(E - E_k)|^2 \quad \text{for } k = 1, 2$$

$$E_1 = \tfrac{-1}{100}\,\mathrm{diag}\Big(113, 102\Big),\ E_2 = \tfrac{-1}{100}\,\mathrm{diag}\Big(102, 113\Big)$$

$$\mathbb{C}\,E = 70(\mathrm{tr}E)\mathrm{Id} + 137(2E - (\mathrm{tr}E))\mathrm{Id}$$

Low-temperature superconducting material $YB_2Cu_3O_{6+x}$ undergoes tetragonal to monoclinic phase transformation



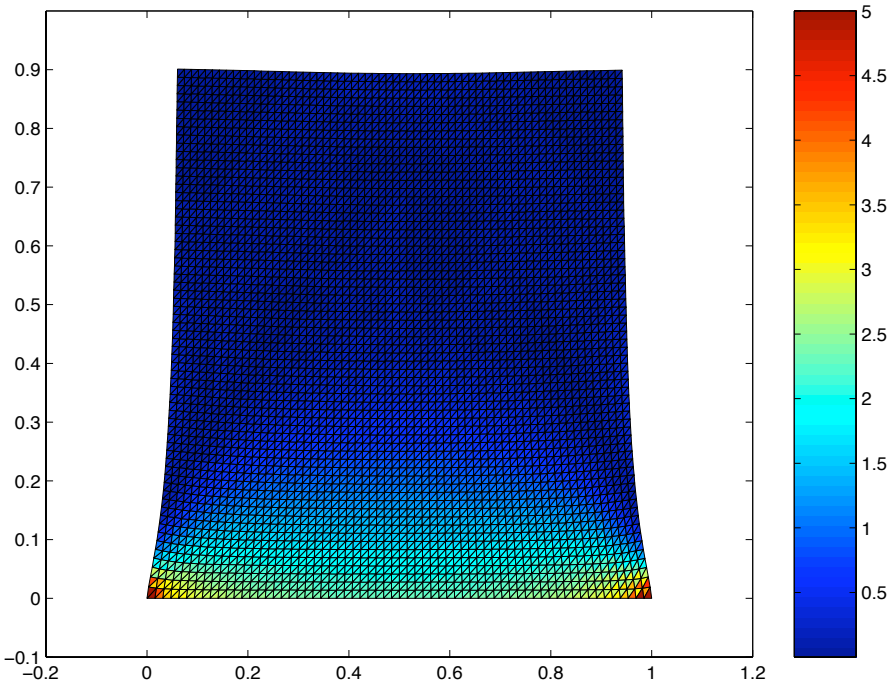
$$f \equiv 0$$
$$g = (0, -1/20)$$
$$\Omega = (0, 1)^2$$
$$\Gamma_D = [0, 1] \times \{0\}$$

Numerical Stabilization with

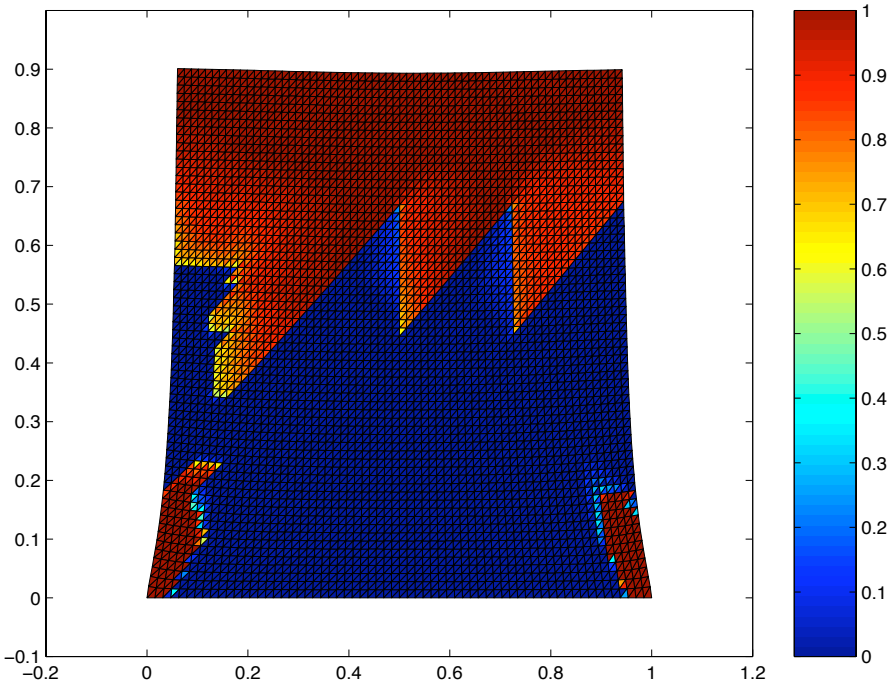
$$\frac{1}{2} \sum_{E \in \mathcal{E}_\Omega} h_E^{3/2} \int_E |[Du_h]|^2\, ds$$

Cont. Mechanical Problem

(M_h) uniform, stabilized, $N = 8320$



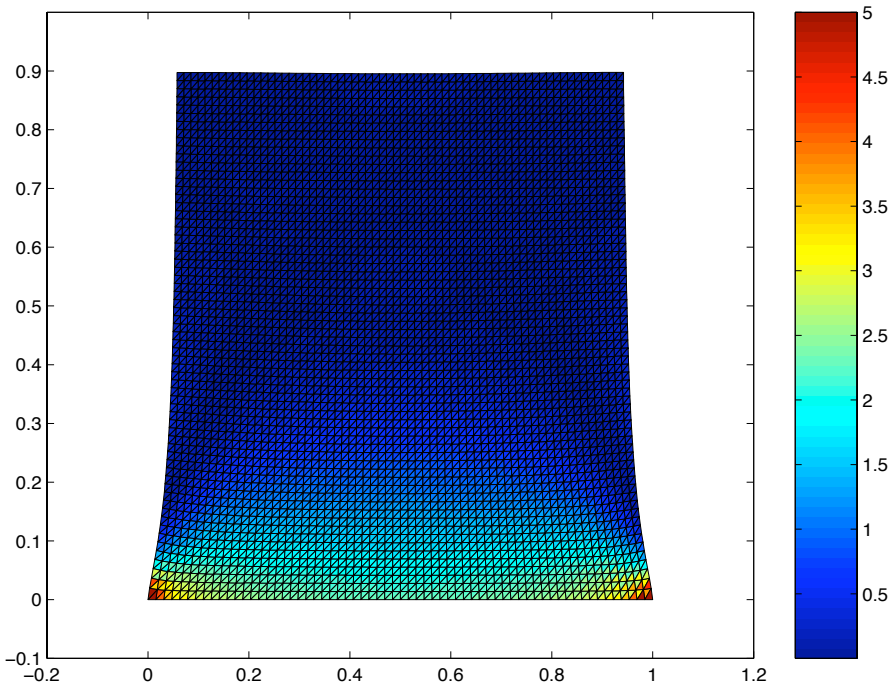
$\min\{|\sigma(\varepsilon(u_h))|, 5\}$



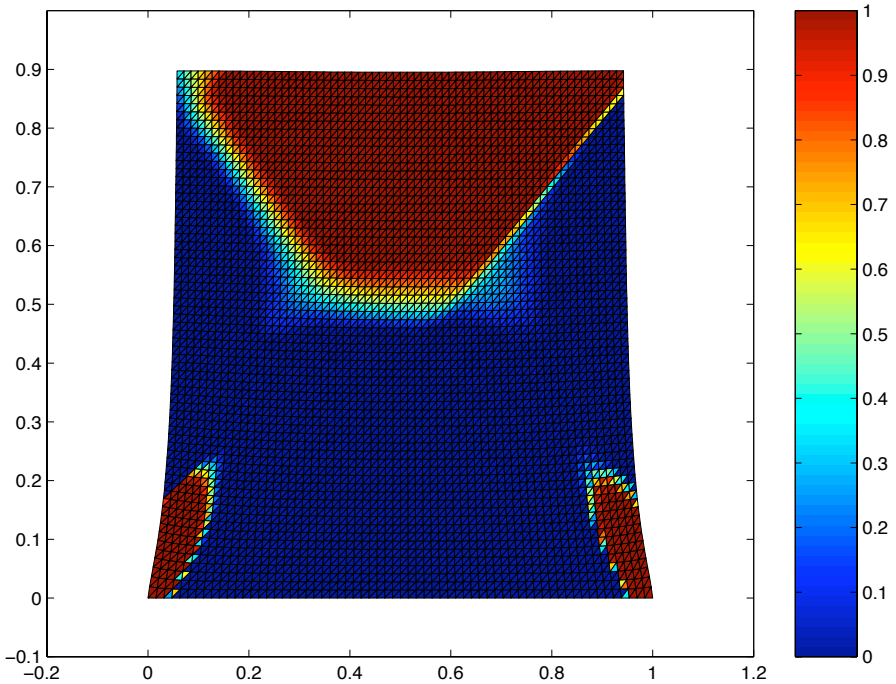
$\lambda(\varepsilon(u_h))$

Cont. Mechanical Problem

Compare with solution to $f \equiv 0$ and $g \equiv 0$



$\min\{|\sigma(\varepsilon(u_h))|, 5\}$



$\lambda(\varepsilon(u_h))$

[Computed with (Q_h) uniform, stabilized, $N = 8320$]

Cont. Mechanical Problem

Relaxed Energy Density

Kohn, Cont Mech Termodyn 1991

$$W^{**}(E) = \begin{cases} W_2(E) & \text{for } W_2(E) + \gamma \le W_1(E), \\ \frac{1}{2}\big(W_2(E) + W_1(E)\big) - \frac{1}{4\gamma}\big(W_2(E) - W_1(E)\big)^2 - \gamma/4 & \\ \qquad \text{for } |W_2(E) - W_1(E)| \le \gamma, & \\ W_1(E) & \text{for } W_1(E) + \gamma \le W_2(E), \end{cases}$$

where

$$\gamma = \frac{1}{2}|\mathbb{C}^{1/2}(E_2 - E_1)|^2.$$

Define stress

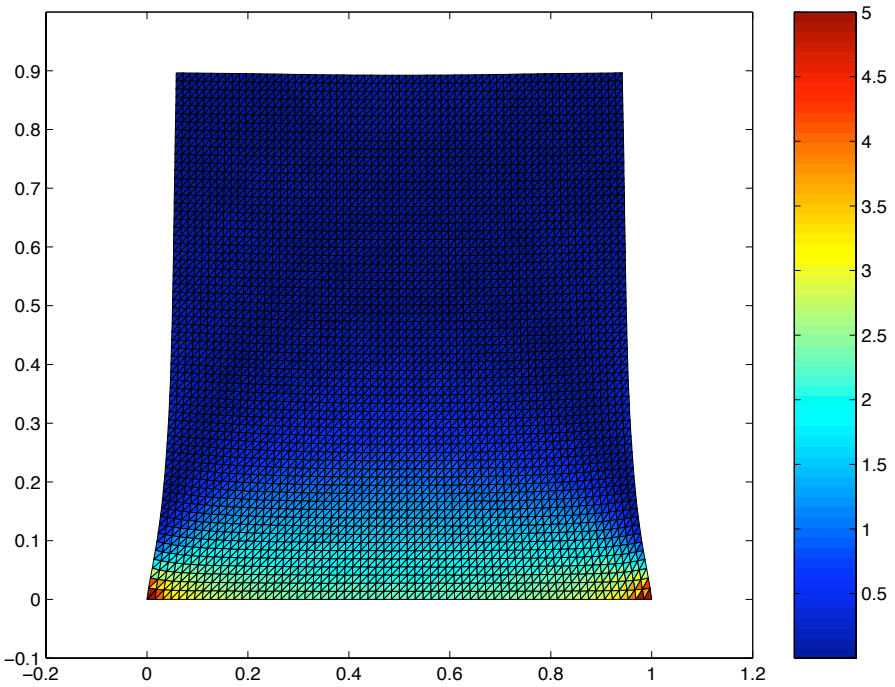
$$\sigma(E) := \frac{DW^{**}}{DE}(E)$$

and volume fraction

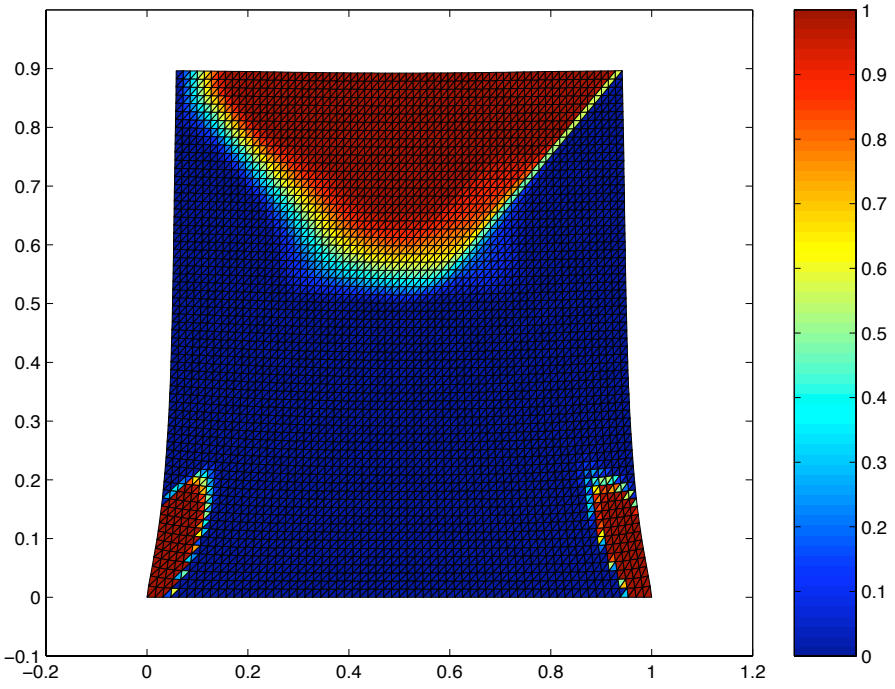
$$\lambda(E) = \begin{cases} 0 & \text{for } W_2(E) + \gamma \le W_1(E), \\ \frac{1}{2} + \frac{1}{2\gamma}\Big(W_2(E) - W_1(E)\Big) & \\ \qquad \text{for } |W_2(E) - W_1(E)| \le \gamma, & \\ 1 & \text{for } W_1(E) + \gamma \le W_2(E). \end{cases}$$

Cont. Mechanical Problem

(Q_h) uniform, stabilized, $N = 8320$



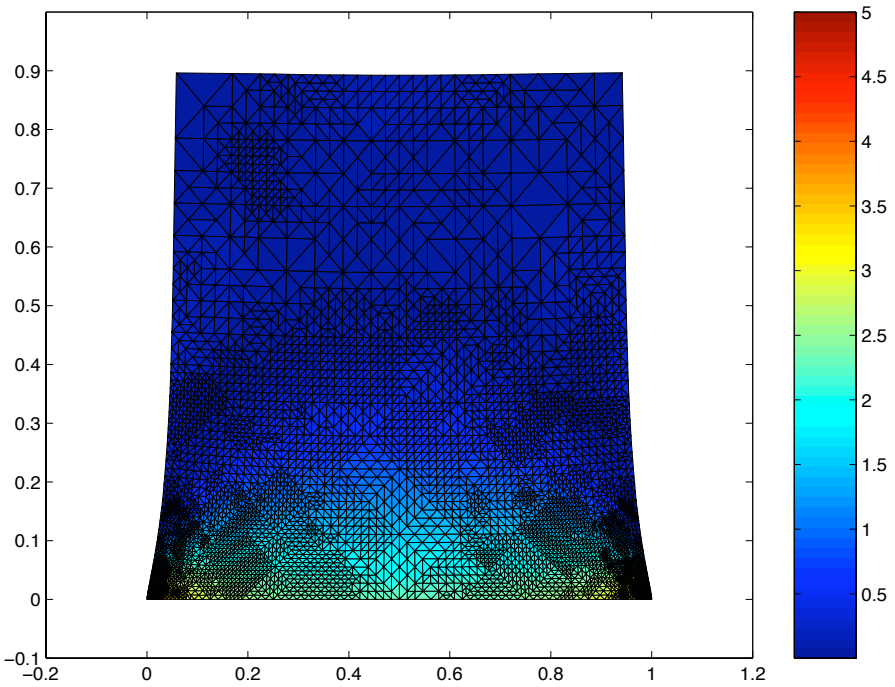
$\min\{|\sigma(\varepsilon(u_h))|, 5\}$



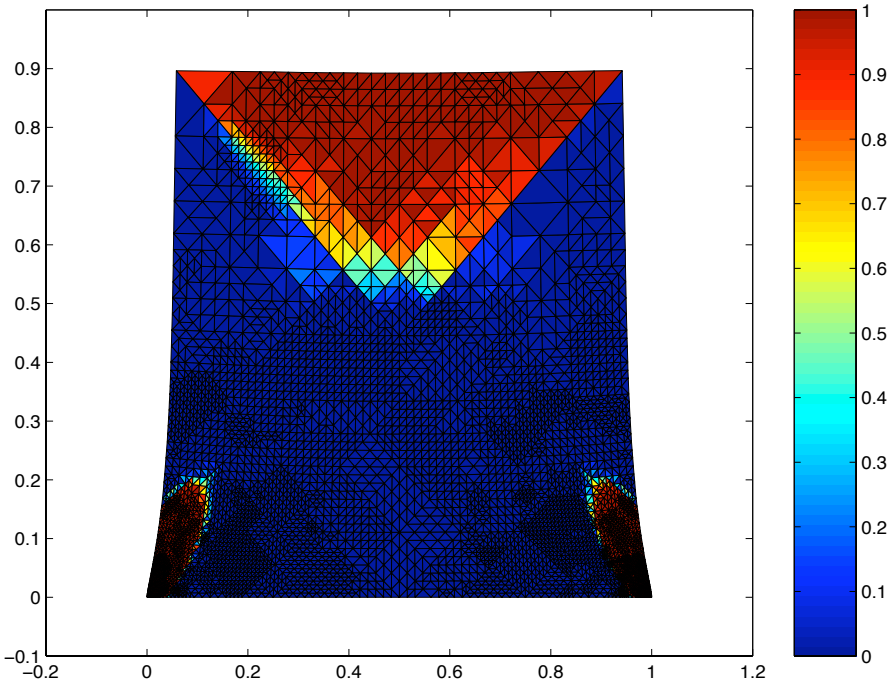
$\lambda(\varepsilon(u_h))$

Cont. Mechanical Problem

(Q_h) η_R adapted, stabilized, $N = 9404$



$\min\{|\sigma(\varepsilon(u_h))|, 5\}$



$\lambda(\varepsilon(u_h))$