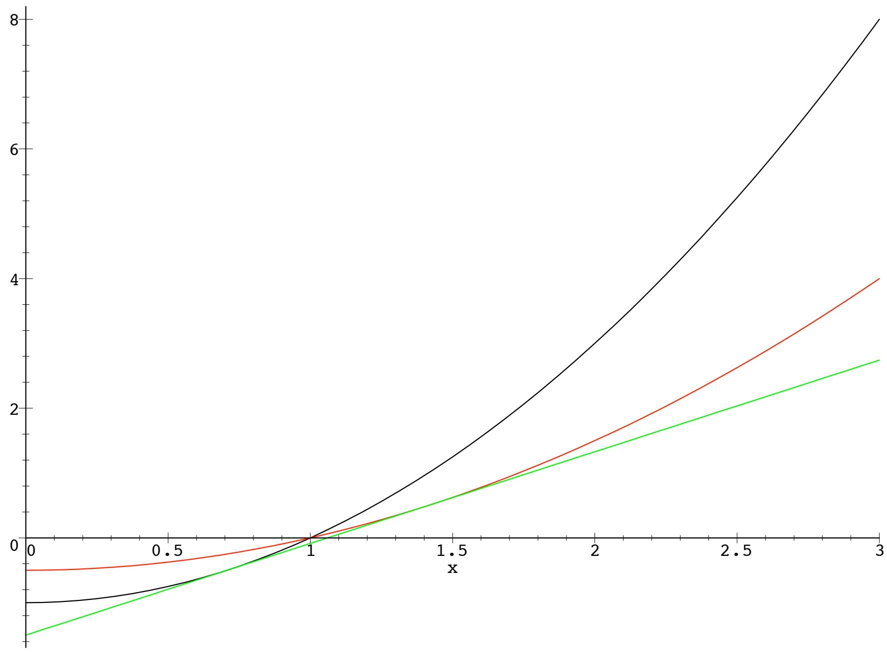
7. Optimal Design Task

Goal: Find (macroscopic) location of two materials with maximal torsion stiffness.
Model: AFERM with $W^r(Dv_h) = g_\lambda(|Dv_h|)$



for given $0 < \mu_1 < \mu_2 < \infty$, $0 < \Theta < 1$: $\mu_\Theta = \Theta\mu_1 + (1-\Theta)\mu_2$, $t_1 = \sqrt{2\lambda\mu_1/\mu_2}$, $t_2\mu_1 = \mu_2 t_1$ defines $g_\lambda(t) := \mu_2(t^2/2 - \lambda)$, $\sqrt{t_1 t_2 \mu_1 \mu_2}\, t - \lambda(\mu_1 + \mu_2)$, $\mu_1(t^2/2 - \lambda)$ for $t \le t_1$, $t_1 \le t \le t_2$, $t_2 \le t$, and

$$\mathrm{LOT}(v_h) = \int_\Omega v_h \, dx + \lambda \mu_\Theta |\Omega|.$$

Mathematical Modelling: [Murat-Tartar (1985), Kohn-Strang (1986), Godman-Kohn-Reyna (1986)]

Numerical Approximation

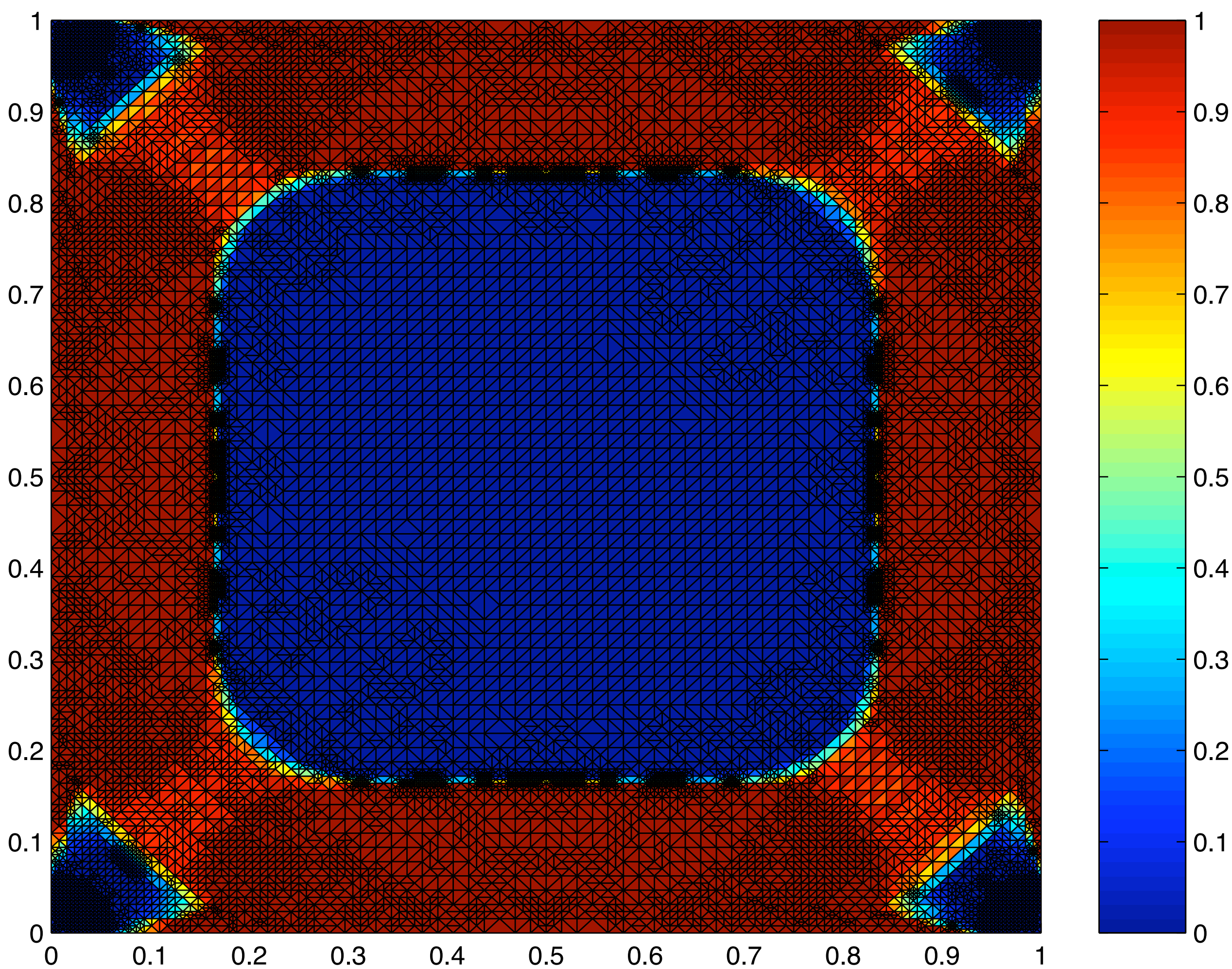
of Volume Fraction [French (1990), Kawohl-Stara-Wittum (1991) C.-Plechac (1997), C.-Bartels]:

$\sigma = g'_\lambda(|Du|)\frac{Du}{|Du|}$ and $\sigma_h = g'_\lambda(|Du_h|)\frac{Du_h}{|Du_h|}$ satisfy

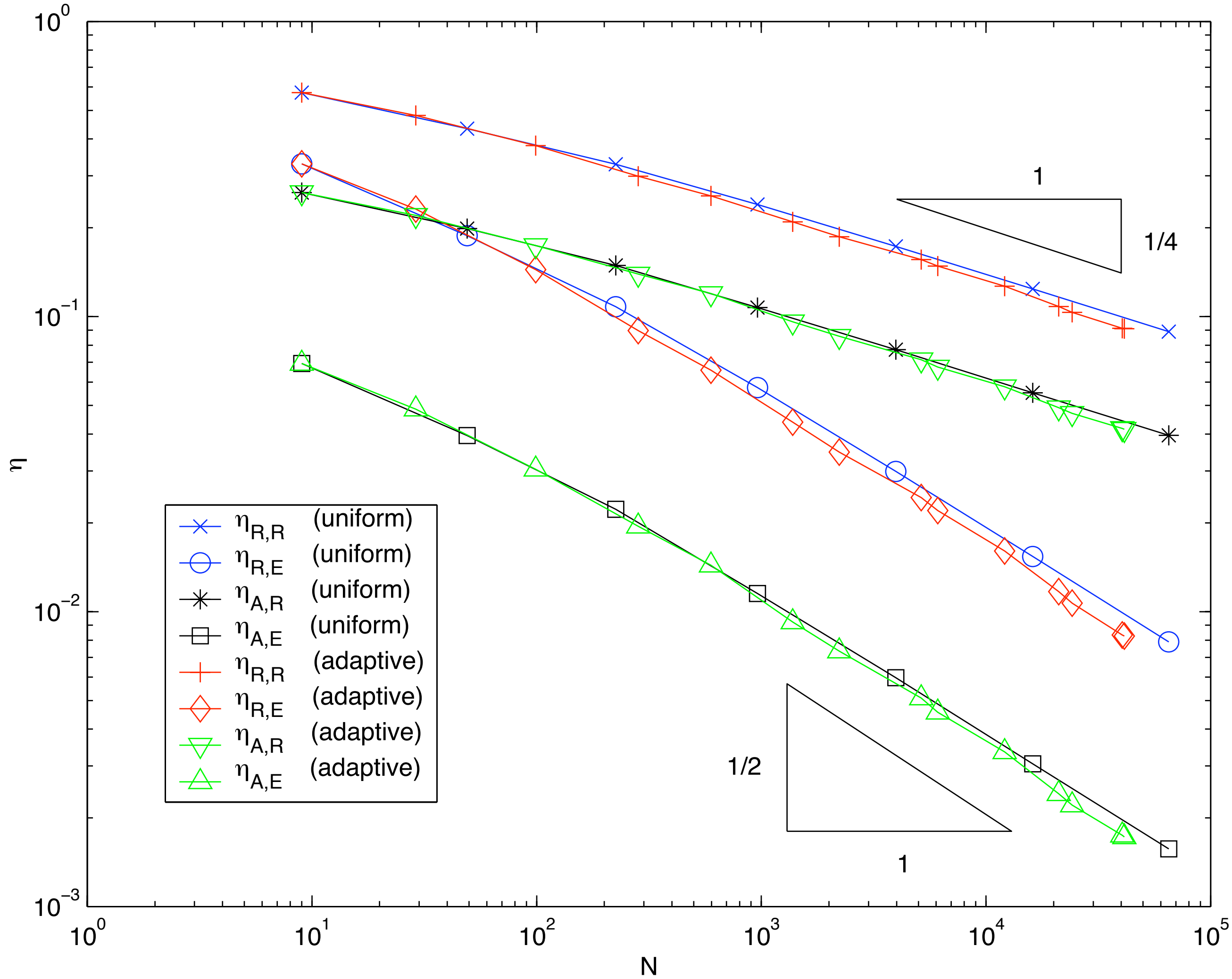
$$\|\sigma - \sigma_h\|_2 \lesssim \min\left\{\|Du - Dv_h\|_{L^4(\Omega)}, \eta^{1/2}\right\}$$

for all FE functions v_h and for standard explicit residual or averaging error estimator η.

AFERM with $\lambda = .08$ yields mesh with $N = 12089$ dof and discrete volume fraction



Cont. 7. Optimal Design Task Experimental convergence rates of a posteriori error bounds for uniform vs. adaptive mesh-refining and reliability-efficiency gap:



8. Micromagnetism [C.-Praetorius (2003)]

Goal: Compute macroscopic magnetisation vector without resolving microscopic physics. Minimise

$$E(m_h) := \int_\Omega \phi(m_h)\,dx - \int_\Omega f\cdot m_h\,dx + \frac{1}{2}\int_{\mathbb{R}^d} |\nabla u|^2\,dx$$

over piecewise constant $m_h : \Omega \to \mathbb{R}^d$ subject to $|m_h| \le 1$ a.e. and with magnetic potential u s.t.

$$\nabla u \in L^2(\Omega)^d \text{ and } \operatorname{div}(-\nabla u + m_h) = 0 \text{ in } \mathcal{D}'(\mathbb{R}^d).$$

Given data are anisotropic energy density with uni-axis $e \in \mathbb{R}^d$ and

$$\phi(x) = 1 - (x\cdot e)^2$$

(e.g. Cobalt) and the exterior applied magnetic field f. Model based on Landau-Lifshitz [Brown 1963, 1966] with vanishing exchange energy for limit of large and soft magnets by DeSimone (1993).

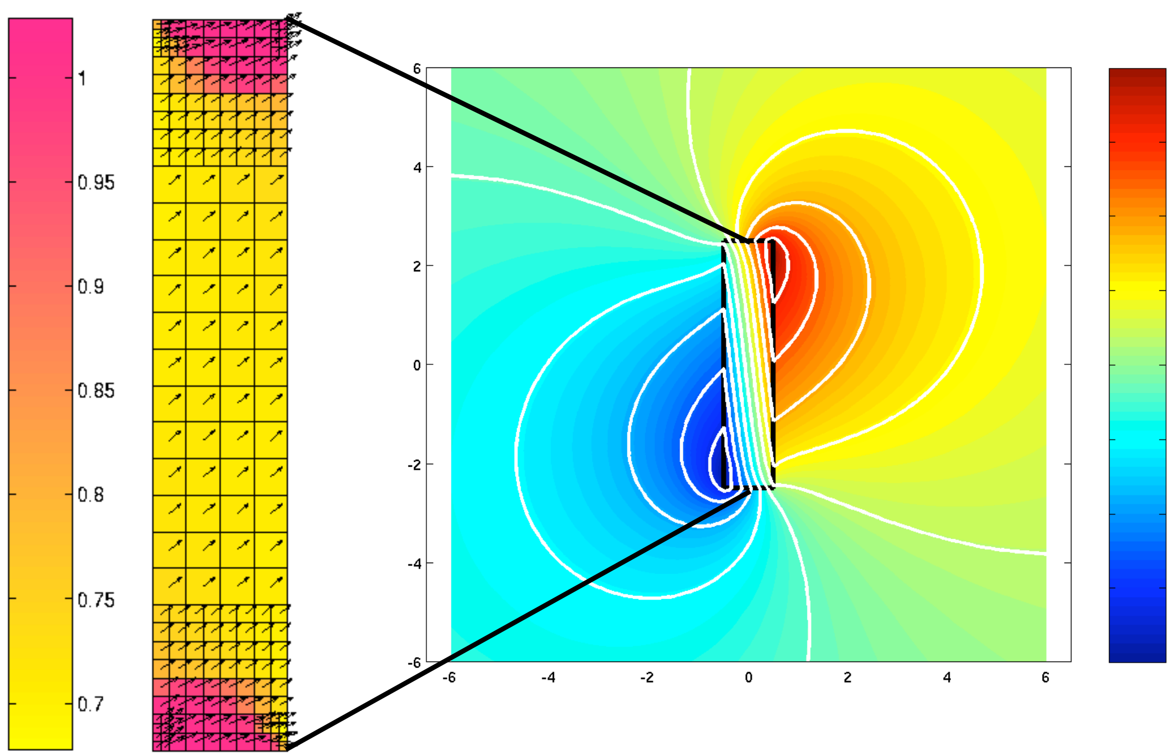
Elimination of u with Newton potential

$$G(x) := |\partial B(0,1)| \begin{cases} -\log|x| & \text{for } d=2,\\ |x|^{2-d}/(2-d) & \text{for } d>2 \end{cases}$$

$$\text{via} \quad u = G * \operatorname{div} m = \sum_{j=1}^{d} \frac{\partial G}{\partial x_j} * m_j.$$

Penalty term $\displaystyle \frac{1}{2}\int_\Omega \varepsilon^{-1}\max\{0, |m_h| - 1\}^2 dx$

Discrete macroscopic magnetisation m_h and corresponding magnetic potential u_h from AFERM with easy axis $e = (1,0)$ and applied magnetic field $f \equiv (.5,.5)$ on adaptive mesh with $N = 206$ dof and $\varepsilon = h_T^{3/2}$



☛ Quasioptimal Approximation of Magnetic Potential and of $D\phi(m) - D\phi(m_h)$ and Reliability-Efficiency Gap in A Posteriori Error Control!

9. Outlook

Open Tasks:

☛ (Further) Design and Analysis of Effective Multilevel Algorithms for (R_h).

☛ Error Analysis of (R_h) for General Non-Convex Relaxed Problems.

[C.-Dolzmann: An a priori error estimate for finite element discretizations in nonlinear elasticity for polyconvex materials under small loads. *Numer. Math.* (2003) accepted]

☛ Numerical Quasiconvexification (e.g. in Finite Plasticity).

[Bartels-C.-Hackl-Hoppe: Effective Relaxation for Microstructure Simulations (2003)],

[Dolzmann: Variational Methods for Crystalline Microstructure–Analysis and Computation, 2003, Lecture Notes in Mathematics 1803]

∃ Time-Evolving Microstructures?

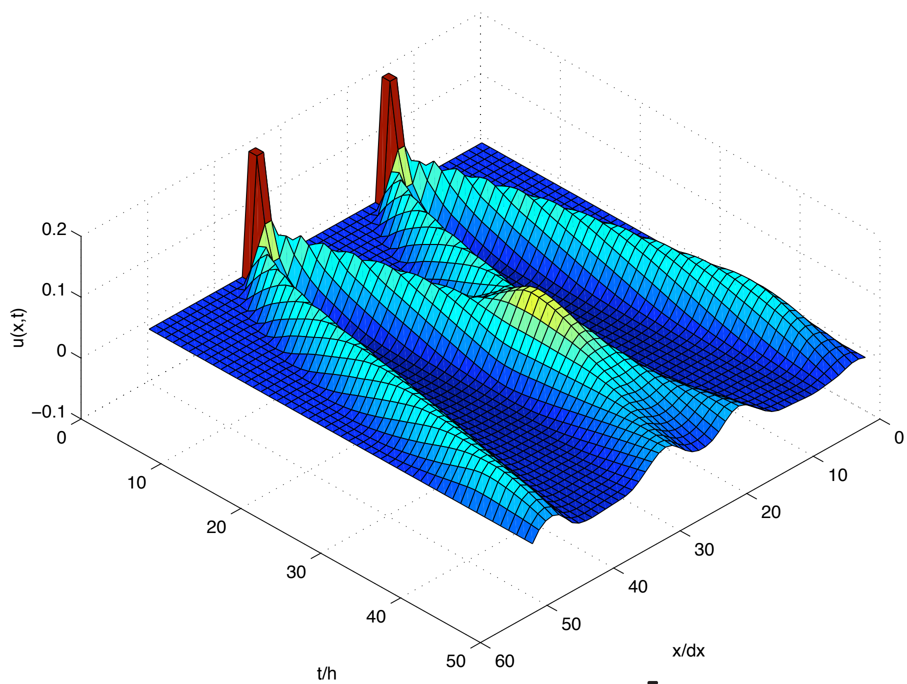
Model with non-monotone hyperbolic system: Seek $u(t,\cdot) : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ with

$$u_{tt} = \text{div}(\sigma(Du) + \nu\, Du_t) \text{ in } \Omega \times (0,T)$$

subject to the boundary and initial conditions

$$\begin{aligned} u &= 0 && \text{on } \partial\Omega \times (0,T), \\ u &= u_0 && \text{in } \Omega \times \{0\}, \\ u_t &= v_0 && \text{in } \Omega \times \{0\}. \end{aligned}$$

☛ Strong damping $\nu = 1$ in [C.-Dolzmann (2004)]
☛ No damping $\nu = 0$



with YM approximation [C.-Rieger (2002)]

☛ Propagation of microstructures observed but its development for smooth I.C. is not.