Error Control

[A Priori & A Posteriori in 2-Well Example]

[C & Plechac, Math Comp 1997]

$$\begin{aligned}\text{ERROR} \;=\; & \|\sigma - \sigma_h\|_{L^{4/3}(\Omega)} + \|\xi - \xi_h\|_{L^2(\Omega)} \\ & + \|\text{positive weight} \times \\ & \qquad \text{Dist}(S_\pm(\nabla u); S_\pm(\nabla u_h))\|^2_{L^2(\Omega)} \\ & + \|(\xi + \xi_h)^{1/2}\nabla(u - u_h)\|_{L^2(\Omega)} \\ \\ \le\; & \text{constant} \times \text{BOUND}\end{aligned}$$

$$\text{A priori BOUND} = \inf_{v_h \in \mathcal{A}_h} \|u - v_h\|_{W^{1,4}(\Omega)}$$

A posteriori BOUND

$$\eta_A^{1/2} := \left(\min_{\tau_h \in \mathcal{S}^1(\mathcal{T}_h)^n} \|\sigma_h - \tau_h\|_{L^{4/3}(\Omega)} \right)^{1/2} + \text{h.o.t.}$$

$$\eta_R^{1/2} := \left(\sum_{T \in \mathcal{T}} \eta_T \right)^{3/8}$$

where $\quad \eta_T = \|\sigma_h - A\sigma_h\|^{4/3}_{L^{4/3}(T)} \quad$ or

$$\begin{aligned}\eta_T \;=\; & h_T^{4/3} \int_T |\text{div } \sigma_h + 2(f - u_h)|^{4/3}\, dx \\ & + \int_{\partial T} h_E |J(\sigma_h \cdot n_E)|^{4/3}\, ds\end{aligned}$$