Multiscale modeling and control of production flows

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Overview:

1. Introduction
   - Fundamental Scale of the Problem
   - Queueing theory models
   - Discrete event simulations
2. Adiabatic (quasi-static) models
   - validation against χ simulations
3. Advection diffusion equations
   - validation against factory data
   - validation against χ simulations
4. Factory Master equations for a linear factory
   - Moment expansions
   - Deriving the state equation from first principles
5. Boundary control of hyperbolic nonlinear wave equations

1) Introduction

Example: Chip production in semiconductor manufacturing.
Factory investment several billions of $$.

Issues:
1.) Hardware: how many machines, topology of production flow
2.) Software: starts policies, dispatch policies, production mix

a) Fundamental Scale of the Problem:

250 production stages
individual wafers or batches of a few wafers go through a machine
cycle time in a machine: from a few minutes to several hours
raw cycle time through the factory: 18 days
actual cycle time through the factory: 40 - 60 days

Stochasticity in the cycle time:

- in the machine - very little
- due to the physical process in the machine - rework - somewhat significant
- due to operators and operator availability - significant
- due to machine failure - major

b) Queueing theory models:
Each process in a machine is modelled as a queue;
stochastic arrival process into a buffer
stochastic exit process describing the machine and operator unit
Problem:

- Some processes are not modelled well using queueing models -e.g crowding effect, operator fatigue etc.
- queueing theory not developed for non-stationary behavior

c) Simulation of choice:
Generate a faithful representation of the factory and do simulation experiments using *Discrete Event Simulations*, e.g. χ (TU Eindhoven)
Simulation of production flows with stochastic demand and stochastic production processes requires Monte Carlo Simulations

It is not scalable.

Intermediate Scale: Fluid Models

Model production flow for each production step as an ODE.
Length of the queue (buffer) in front of the step is the dependent variable
Outflux - constant mean production rate of the machine for this step.
Link: Mass conservation:

$$\frac{dx}{dt} = \mathit{influx} - \mathit{outflux}$$

Note: continuum in products.
Problems:

- 250 ODEs to parametrize
- how to model the delay

2) Continuum Models of production flows

Fundamental Idea:
Model high volume, many stages, production via a continuum.
Basic variable:
product density (mass density) $\rho(\mathbf{x}, \mathbf{t})$.
x- is the position in the production process, $x \in [0, 1]$.- degree of completion- stage of production
Note: For a re-entrant process a machine corresponds to many positions x.

Mass conservation and state equations

Quasi-stationary model (adiabatic model): Mass conservation and state equation

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial F}{\partial x} &= 0 \\ F &= \rho v_{eq} \end{aligned}$$

Typical models for the equilibrium velocity v_{eq} (state equation) are

$$v_{LW}(\rho) = v_0(1 - \frac{\rho}{\rho_c}), \tag{1}$$

$$v_Q(\rho) = \frac{v_0}{1 + \frac{L(\rho)}{L_c}}, \tag{2}$$

$$v_{eq}(\rho) = \Phi(L), \tag{3}$$

with L the total load (Work in progress, WIP) given as

$$L(\rho) = \int_0^1 \rho(x,t)dx. \tag{4}$$

Φ maybe determined experimentally or theoretically.

Validation through χ-simulations

Simulate a network in χ, consisting of 5 machines, re-entrant with 4 production loops.
Run experiment for different influx levels
Generate average cycle time, average WIP, and average throughput.
Figure 1 shows resulting state equation.

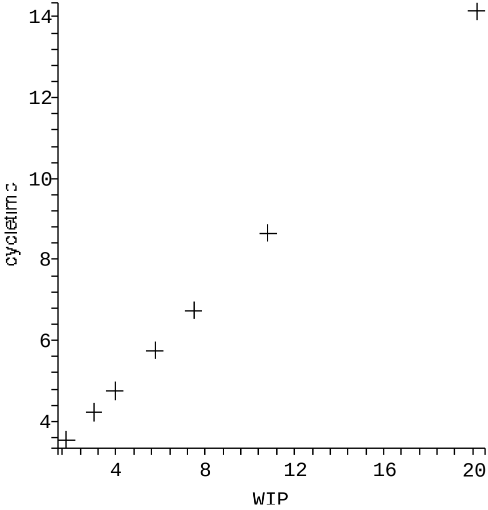


Figure: 1. Seven datapoints for a state equation describing the relationship between cycle time and WIP.

Usage:

1. Determine steady state parameters through inter/extrapolation
2. Use as state equation for transient behavior

Figures 2, 3 show two experiments

1. Transition from a steady state with 75% utilization to 85% utilization with pull policy
2. Transition from a steady state with 67% utilization to 85% utilization with push policy

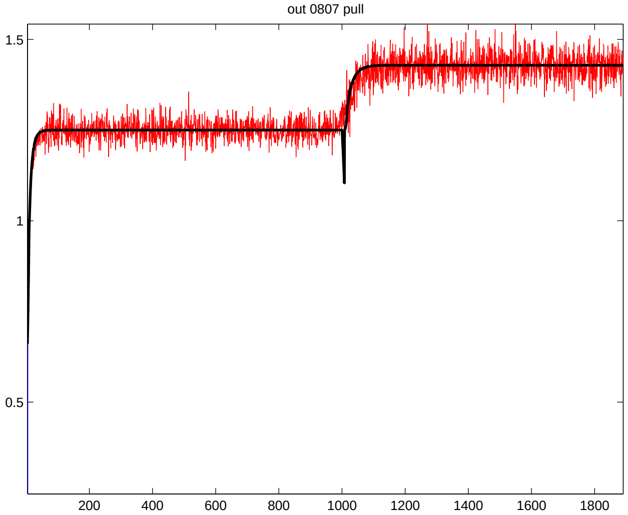


Figure: 2. Output for a pull policy

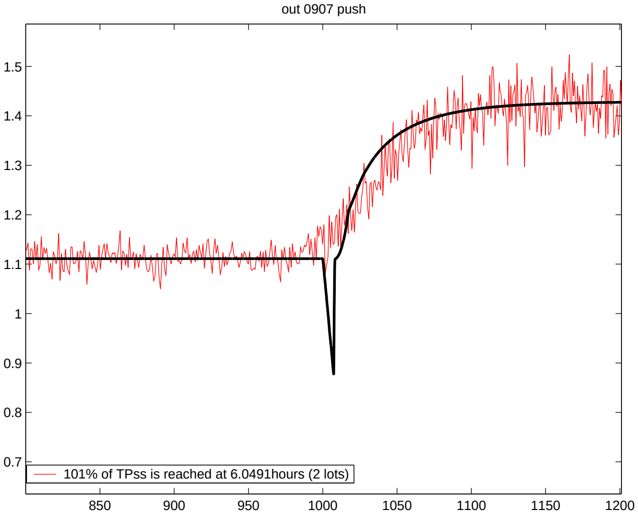


Figure: 3. Output for a push policy, zoomed

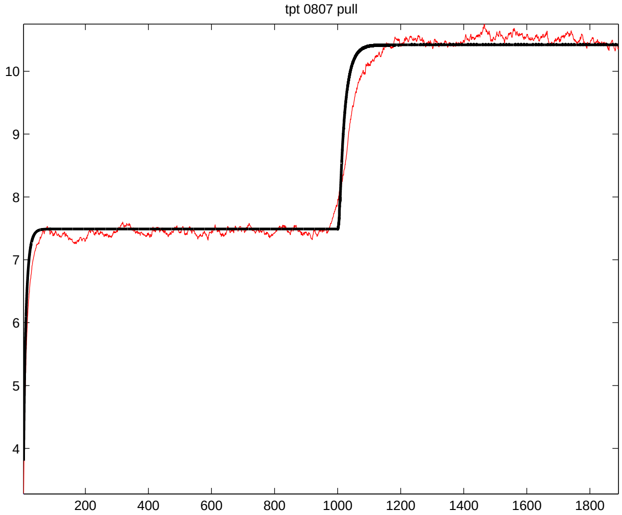


Figure: 4 Cycle time

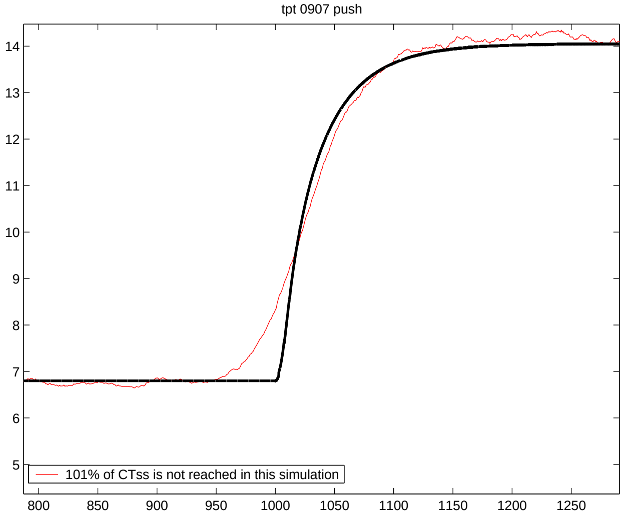


Figure: 5 Cycle time

Notice:
Inverse response in Figures 2 and 3
Due to global velocity **model**: Increase in influx leads to a change in total WIP
Result: infinite wave speed and decrease in velocity and hence decrease of output before the increase arrives at the end of the factory.
Reality: re-entrant production has this effect but much less pronounced.

3) Advection diffusion equations

Including variance in stochastic models typically introduces diffusion. We expect that the basic mass conservation model becomes

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + v_{eq}\frac{\partial \rho}{\partial x} &= D\frac{\partial^2 \rho}{\partial x^2} && (5) \\
v_{eq}(t) &= \Phi(W(\rho(x,t))) && (6) \\
W(t) &= \int_0^1 \rho(x,t)dx && (7)
\end{aligned}
$$

a) Real Factory Data

Data analysis of sanitized data of a real INTEL factory for about 3 months production.

Details:

- 920 lots
- time log in and out at all machines
- identify 8 approximately equally spaced machines
- Determine time-in at all 8 machines
- interpolate paths
- generate histograms at different times.
- Fit the histograms to the explicit solution of the advection-diffusion equation by a least square fit for the diffusion coefficient.

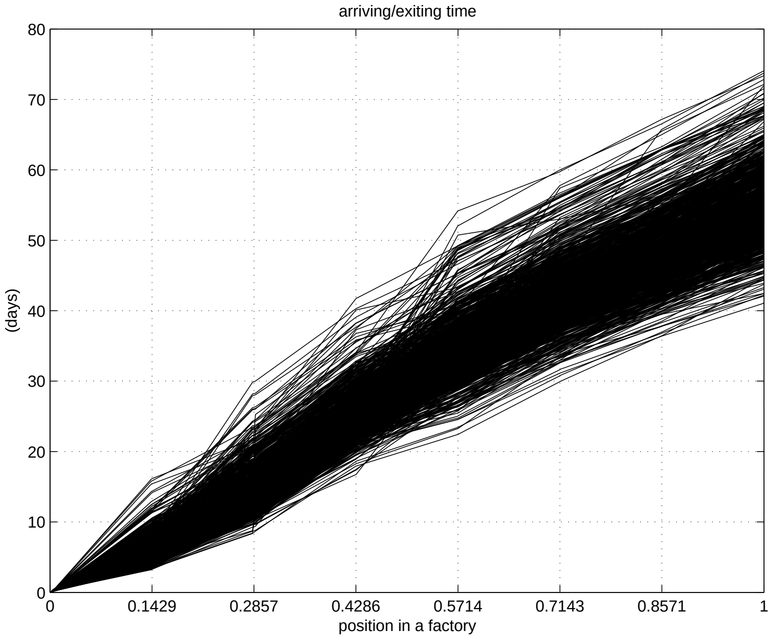
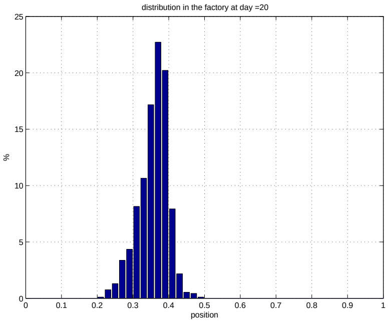
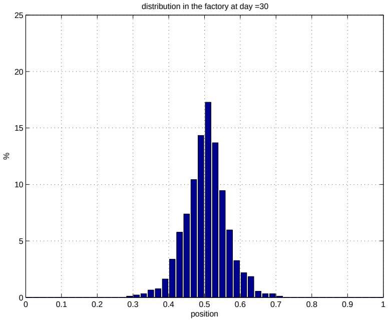


Figure: 6. Paths of all 920 lots


distribution in the factory at day =20
%
position
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25


distribution in the factory at day =30
%
position
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25

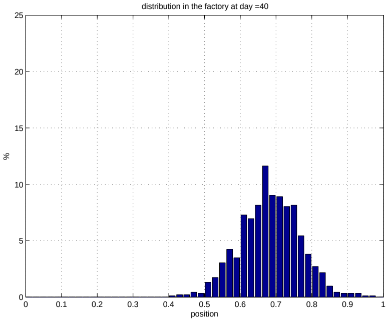


Figure: 7 Histograms at $t=20, 30$ and 40

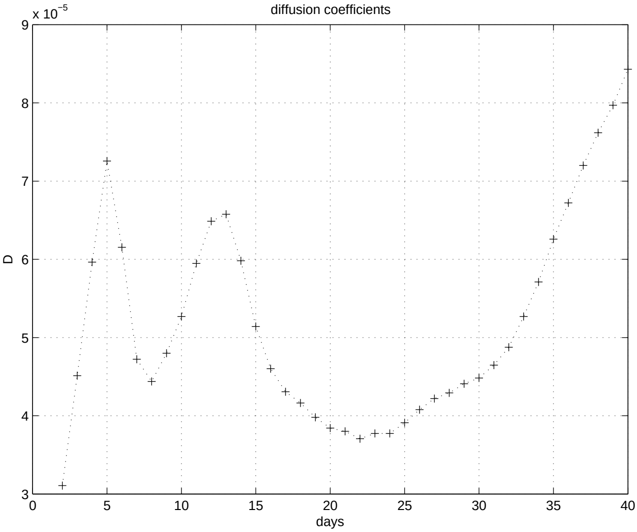


Figure: 8. Diffusion coefficient

Result:

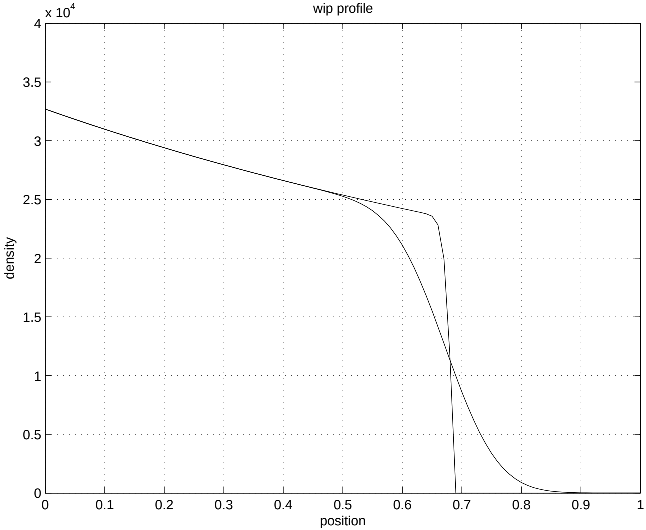


Figure: 9 WIP profile after a step up in influx, with and without diffusion

b) χ **Simulations** - Emiel v.d. Rijt

100 identical machines
characterized through mean process time t_e and squared coefficient of variation c_e^2.
Arrival process: average interarrival time t_a and its squared coefficient of variation c_a^2
Goal: Determine the dependence of the WIP profile on the ratios

$$\frac{c_a^2}{c_e^2} \quad \text{and} \quad u = \frac{t_e}{t_a}$$

Simulations - Wip profiles:

utilization $u = 0.75$,

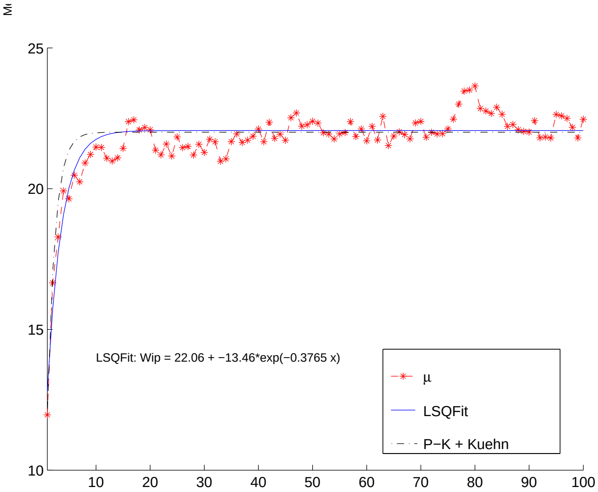


Figure: 10 WIP profile for $\frac{c_a^2}{c_e^2} = \frac{1}{9}$

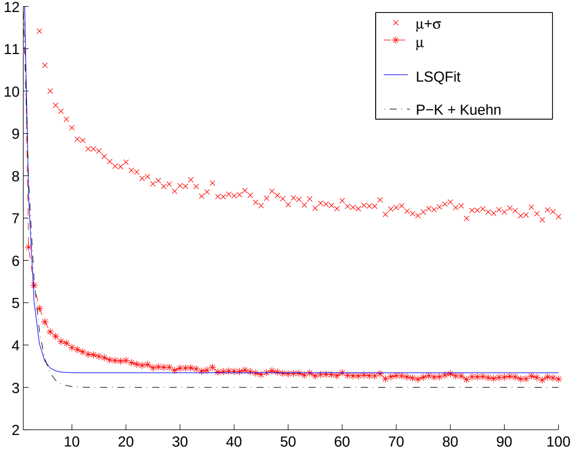


Figure: 15 WIP profile for $\frac{c_a^2}{c_e^2} = \frac{9}{1}$

Boundary layer is utilization dependent: $u = 0.99$

The BL has reduced to one machine.

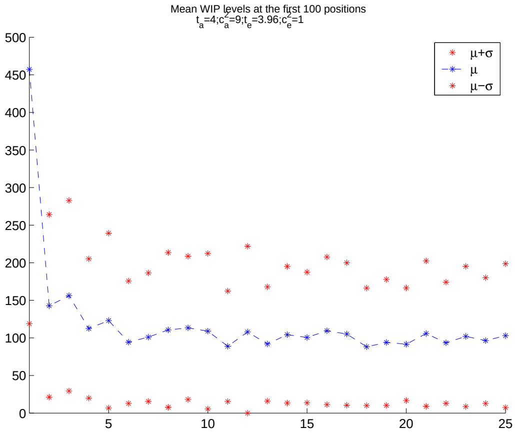


Figure: 12 WIP profile for $\frac{c_a^2}{c_e^2} = \frac{9}{1}$

PDE model

Consider the steady state of the advection diffusion equation:

$$\rho v_{eq} + D\frac{d\rho}{dx} = \lambda \tag{8}$$

with boundary condition $\rho(0)$ e.g. given by

$$\rho(0) = \frac{1}{2}(c_a^2 + c_e^2)\frac{u^2}{1-u} + u$$

The solution to Eq.(8) is

$$\rho(x) = \frac{\lambda}{v_{eq}} + (\rho_0 - \frac{\lambda}{v_{eq}})e^{-\frac{v_{eq}}{D}x} \tag{9}$$

Notice:

with $\rho_{ss} = \frac{\lambda}{v_{eq}}$, e.g.

$$\rho_{ss} = c_e^2 \frac{u^2}{1-u} + u$$

we get that

- if $c_a^2 > c_e^2$ then $\rho''(x) > 0$,
- if $c_a^2 < c_e^2$ then $\rho''(x) < 0$,
- utilization dependence:
  Scaling argument: $D \approx \sigma^2,\ \sigma^2 = cv_{eq}^2$
  $\lim_{u \to 1} v_{eq} \to 0$
  Hence from $e^{-\frac{v_{eq}}{D}x}$ the boundary layer ξ becomes $\xi \approx (1-u)$.

4) Factory Master Equation for Linear Factory from First Principles

Goal:
Derive Factory Master Equation from 'First Principles' with methods from gas-dynamics.
General idea:
Boltzmann equation for the density $f(x, y, t)$ of a particle at position x with attribute y at time t:

$$\partial_t f + \partial_x[u(x, y, t)f] + \partial_y(E(x, y, t)f) = Q[f] \; , \tag{10}$$

$\partial_y(Ef)$ models a continuous change in attribute,
$Q[f]$ models a random and discontinuous change in the attribute
Note that

$$\int Q[f](x, y, t) \; dy = 0, \; \forall f \; . \tag{11}$$

Define part density $\rho(x,t)$ and flux density $F(x,t)$ by

$$\begin{aligned}\rho(x,t) &= \int f(x,y,t)\ dy,\\ F(x,t) &= \int u(x,y,t) f(x,y,t)\ dy\end{aligned}$$

integrating we get the conservation law

$$\partial_t \rho + \partial_x F = 0\ .$$

as the zero order moment equation.
Goal: Determine E and Q from detailed underlying kinetic behavior and extend to higher moments.

Kinetic density for N particles:

$$f(x,y,t) = \sum_{n=1}^{N} H(t-a_n)\delta(x-\xi_n(t))\delta(y-\eta_n(t))$$

$f(x,y,t)$ satisfies the Liouville equation for a single particle density weakly in x:

$$\partial_t f + \partial_x[uf] + \partial_y[Ef] = 0,$$

Boundary condition:

$$f(x=0,y,t) = \sum_{n=1}^{N} \delta(y-r_n)\delta(t-a_j),$$

Moment expansion gives the heuristic equation: e.g: assume

$$f(x,y,t) = \rho(x,t)\delta(y-Y(x,t))$$

gives the conservation law with $v(x,t) = u(x, Y(x,t), t)$.

Deriving the state equation

Assume: M stages,
identify the attributes with velocity i.e.

$$u(x, y, t) = y$$

update the velocity every time a particle enters a new stage out of a probability distribution $P(x, z, t)$ where

$$f(x, y, t)\ dxdy = d\mathcal{P}\{\xi(t) = x, \eta(t) = y\}\ .$$

defines a probability density to find a particle at position x with velocity y.
Boltzmann equation

$$\partial_t f + y\partial_x f = M[P(x, y, t)\int zf(x, z, t)\ dz - yf(x, y, t)] \quad (12)$$

Asymptotic expansion for many stages $M \to \infty$: Chapman-Enskog expansion

$$f(x, y, t) = \phi(\rho(x, t), y, t) \ .$$

i.e. the kinetic density f is a 'shape function' dependent on space only through the macroscopic density ρ.

Zero order expansion: continuity equation with a state equation

$$v(x, t) = \frac{1}{ME[\tau(x, t)]}$$

$E[\tau]$ denotes the expectation of the cycle time τ under the probability distribution P.

First order expansion - flux becomes:

$$F = \rho v(1 + \frac{R}{M}) - \frac{1}{M} v V[\frac{1}{y}] \partial_x \rho \ . \tag{13}$$

with $V[\frac{1}{y}]$ the variance of the cycle time.

Equation free model - Kevrekidis et al

Microscopic model: *Monte Carlo simulation*

$$
\begin{aligned}
(a) \qquad & x(t+\Delta t) = x(t) + \frac{\Delta t}{\tau(t)}, \\
& \tau(t+\Delta t) = \kappa(t)\tau(t) + (1-\kappa(t))\tau(t), \quad t \geq a_n, \\
(b) \qquad & \mathcal{P}\{\kappa(t)=1\} = \omega\Delta t, \quad \mathcal{P}\{\kappa(t)=0\} = 1-\omega\Delta t, \\
& d\mathcal{P}\{\tau(t) \leq r\} = \mathcal{T}(r,t)dr, \\
(c) \qquad & x(a_n) = 0, \quad d\mathcal{P}\{\tau(a_n) \leq r\} = \mathcal{T}(r,a_n)dr,
\end{aligned}
\tag{14}
$$

microscopic model simulated for 20 short timesteps $\delta t_s = 10^{-3}$
Extract Euler time step for the macroscopic time evolution of the density $\rho(x,t)$.
Coarse time step $\delta t_c = 0.2$

Experiment

Influx:

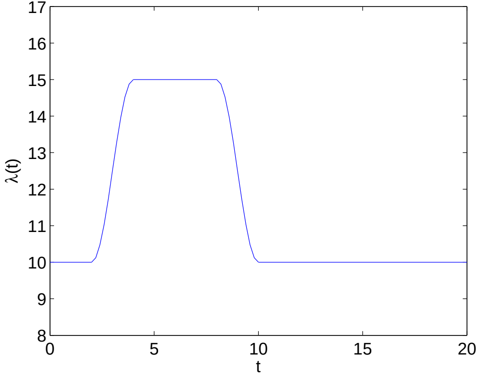


Figure: 13 Influx

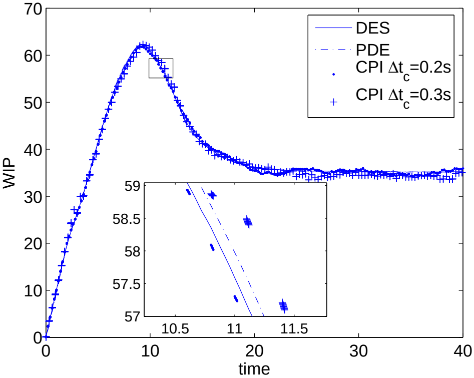


Figure: 15 WIP profile for equation free and PDE simulation

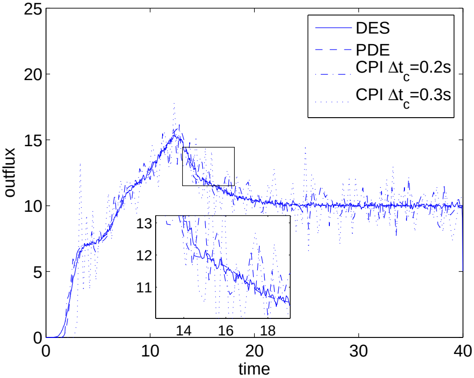


Figure: 15 Outflux for equation free and PDE simulation

5) Open problem: Boundary control of hyperbolic nonlinear wave equations

Consider adiabatic model:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial F}{\partial x} &= 0 \\ F &= \rho v_{eq} \end{aligned}$$

Optimal control problem: get from one steady state to another in fastest time.
Control parameter: influx at one boundary $\lambda(t)$.
Simplest problem: $v_{eq} = 1$ - linear wave equation.

Solution: Choose $\lambda = \lambda_{new}$.
After $t = 1$ the transition wave will have moved through the interval $[0, 1]$ and we have reached the new steady state

How can this be proven in a way that generalizes to nonlinear and non-local velocities?

Problem: No Pontryagin Maximum principle exists for PDEs.
Attempt: Convert PDE into network of ODEs via semi-discrete methods
Choose: $\Delta x = \frac{1}{n}$

Then

$$\begin{aligned} \dot{x_1} &= n(\lambda - x_1) \\ \dot{x_j} &= n(x_{j-1} - x_j) \end{aligned}$$

with an initial condition $x_1 = x_j = \lambda_1$
and a final condition $x_1 = x_j = \lambda_{new}$.
and a cost function $C = \int_0^t ds$

Solution due to Pontryagin's maximal princple: find minimum of Hamiltonian

$$H = 1 * \lambda(t) + \Sigma_j \xi_j(t) x_j(t)$$

with respect to the control parameter. Since H is linear in λ we get *bang-bang* -control:
λ switches between a minimal and maximal value.

Two fundamental problems with this approach:

- The intuitive result for the PDE suggests that the optimal control parameter is $\lambda = \lambda_{new}$.
  How does the ODE bang -bang control approach the PDE?
  Weak limit?
  There should be theory for that but I can't find any.
- **Numerical issues:**
  Once bang-bang control is established the open problem is to determine the swtiching manifolds. For n ODEs there will be $n - 1$ switches. How to find those?

Current approach:

Linear optimization problem with nonlinear constraints: Define τ_i to be the time between two switches.
Find

$$\min \Sigma_j \tau_j$$

subject to the flows $\phi_j(t, x_j)$ which are either explictly given (linear PDE) or numerically calculated.
Works for up to $n = 9$
not a very good way to numerical check a convergence to a weak limit.

Conclusions

PDE models of production flows are highly effective simulation tools. They can be adjusted to the desired level of accuracy and modeling sophistication. In addition

- they have execution times in seconds
- they can be adjusted to include policies
- they can be adjusted to cover inhomogeneous production lines.
- they allow to simulate transient situations
- they can be linked to generate simulations for the whole supply chain
- they can be justified (in parts) from first principles

Open Problems

- Find Boltzman equations for queueing networks. Issues:
  - Gasdynamics: ergodicity assumption in time and space , i.e. strong interactions
  - Queueing: ergodicity assumption in time and samples, i.e. weak interactions
  - Queueing networks not well understood - only M/M/n queues lead to good analytical solutions.
- Use the PDE models to solve business problems, i.e. optimization problems.

References:

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- D. Armbruster, C. Ringhofer, Thermalized kinetic and fluid models for reentrant supply chains, SIAM J. on Multiscale modeling and Simulation, **3**(4), pp 782 - 800, (2005)
- D. Armbruster, P. Degond, C. Ringhofer: A Model for the Dynamics of large Queuing Networks and Supply Chains, SIAM J. Applied Mathematics **66**(3) pp. 896-920. (2006)